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MITx MicroMasters Program in SCM Key Concepts

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Part I

SC0x - Supply Chain Analytics

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Supply Chain Analytics

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1.1 Supply Chain Management Overview

1.1.1 Summary

Supply Chain Basics is an overview of the concepts of Supply Chain Management and logistics. It demonstrates that, product supply chains as varied as bananas, to women's shoes, to cement, have common supply chain elements. There are many definitions of supply chain management but, ultimately, supply chains are the physical, financial, and information flow between trading partners that ultimately fulfill a customer request. The primary purpose of any supply chain is to satisfy a customer's need at the end of the supply chain. Essentially supply chains seek to maximize the total value generated which is defined as, the amount the customer pays minus the cost of fulfilling the need along the entire supply chain. All supply chains include multiple firms.

1.1.2 Key Concepts

While Supply Chain Management is a new term (first coined in 1982 by Keith Oliver from Booz Allen Hamilton in an interview with the Financial Times), the concepts are ancient and date back to ancient Rome. The term "logistics" has its roots in the Roman military. Additional definitions:

- Logistics involves... "managing the flow of information, cash and ideas through the coordination of supply chain processes and through the strategic addition of place, period and pattern values" – MIT Center for Transportation and Logistics
- "Supply Chain Management deals with the management of materials, information and financial flows in a network consisting of suppliers, manufacturers, distributors, and customers" - Stanford Supply Chain Forum
- "Call it distribution or logistics or supply chain management. By whatever name, it is the sinuous, gritty, and cumbersome process by which companies move materials, parts and products to customers" – Fortune 1994

Logistics vs. Supply Chain Management

According to the Council of Supply Chain Management Professionals...

- Logistics management is that part of supply chain management that plans, implements, and controls the efficient, effective, forward and reverse flow and storage of goods, services and related information between the point of origin and the point of consumption in order to meet customers' requirements.

Supply chain management encompasses the planning and management of all activities involved in sourcing and procurement, conversion, and all logistics management activities. Importantly, it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third party service providers, and customers. In essence, supply chain management integrates supply and demand management within and across companies.

Supply Chain Perspectives

Supply chains can be viewed in many different perspectives including process cycles (Chopra & Meindl 2013) and the SCOR model (Supply Chain Council).

The Supply Chain Process has four Primary Cycles: Customer Order Cycle, Replenishment Cycle, Manufacturing Cycle, and Procurement Cycle. Not every supply chain contains all four cycles.

The Supply Chain Operations Reference (SCOR) Model is another useful perspective. It shows the four major operations in a supply chain: source, make, deliver, plan, and return. (See Figure below)

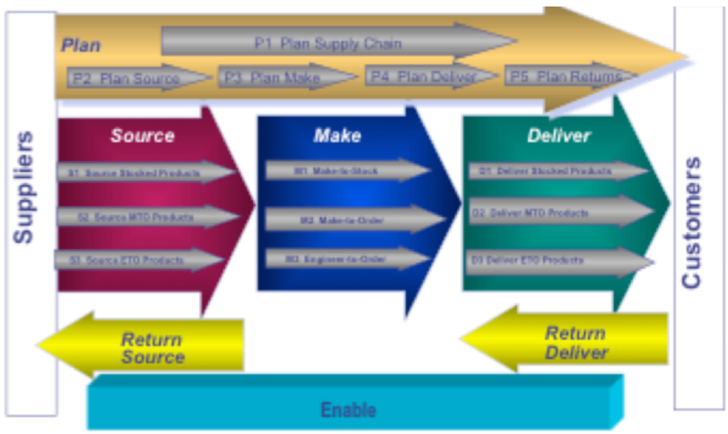


FIGURE 1.1: SCOR Model. Source: Supply Chain Council

Additional perspectives include:

- Geographic Maps - showing origins, destinations, and the physical routes.
- Diagrams – showing the flow of materials, information, and finance between echelons.
- Macro-Process or Software – dividing the supply chains into three key areas of management: Supplier Relationship, Internal, and Customer Relationship.
- Traditional Functional Roles – where supply chains are divided into separate functional roles (Procurement, Inventory Control, Warehousing, Materials Handling, Order Processing, Transportation, Customer Service, Planning, etc.). This is how most companies are organized.
- Systems Perspective – where the actions from one function are shown to impact (and be impacted by) other functions. The idea is that you need to manage the entire system rather than the individual siloed functions. As one expands the scope of management, there are more opportunities for improvement, but the complexity increases dramatically.

Supply Chain as a System

It is useful to think of the supply chain as a complete system. This means one should:

- Look to maximize value across the supply chain rather than a specific function such as transportation
- Note that while this increases the potential for improvement, complexity and coordination requirements increase as well
- Recognize new challenges such as:
  - Metrics — how will this new system be measured?
  - Politics and power — who gains and loses influence, and what are the effects?
  - Visibility — where data is stored and who has access?
  - Uncertainty — compounds unknowns such as lead times, customer demand, and manufacturing yield

  – Global Operations — most firms source and sell across the globe

Supply chains must adapt by acting as both a bridge and a shock absorber to connect functions as well as neutralize disruptions.

Learning Objectives

- Gain multiple perspectives of supply chains to include process and system views.
- Identify physical, financial, and information flows inherent to supply chains.
- Recognize that all supply chains are different but have common features.
- Understand importance of analytical models to support supply chain decision-making.

1.2 Mathematical Functions

1.2.1 Summary

This section provides an overview of the building blocks of the analytical models used frequently in supply chain management for decision-making. Each model serves a role; it all depends on how the techniques match with need. First, a classification of the types of models offers perspectives on when to use a model and what type of output they generate. Second, a review of the main components of models, beginning with an overview of types of functions, the quadratic and how to find its root(s), logarithms, multivariate functions, and the properties of functions. These "basics" will be used continuously throughout the remainder of the course.

1.2.2 Key Concepts

Models

Decision-making is at the core of supply chain management. Analytical models can aid in decision-making to questions such as "which transportation option should I use?" or "how much inventory should I have?" They can be classified into several categories based on degree of abstraction, speed, and cost.

Models can be further categorized into three based on their approach:

- Descriptive – What has happened?
- Predictive – What could happen?
- Prescriptive – What should we do?

Function

A Function is one of the main parts of a model. It is defined as "a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output."(Wikipedia)
$y=f(x)$

Linear Function

With a Linear function, "y changes linearly with x." A graph of a linear function is a straight line and has one independent variable and one dependent variable. (See figure below)

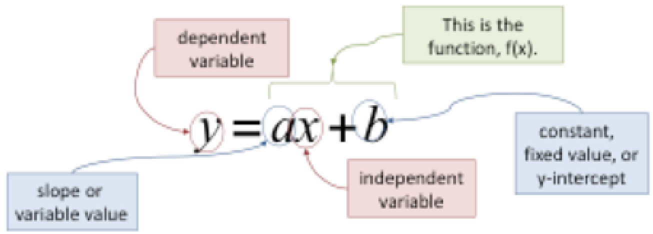


FIGURE 1.2: Components of a linear function

Quadratic Functions

A quadratic function takes the form of $y = ax^2 + bx + c$, where a, b, and c are numbers and are not equal to zero. The graph of a quadratic function is a curve that is called a parabola, forming the shape of a "U". (See figures below)

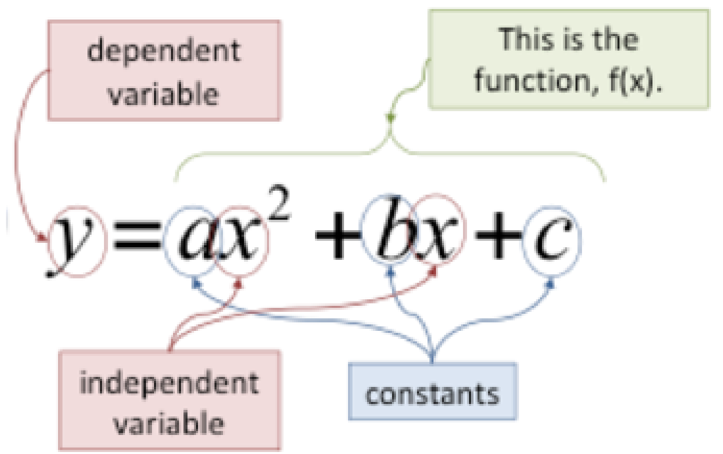


FIGURE 1.3: Components of a quadratic function

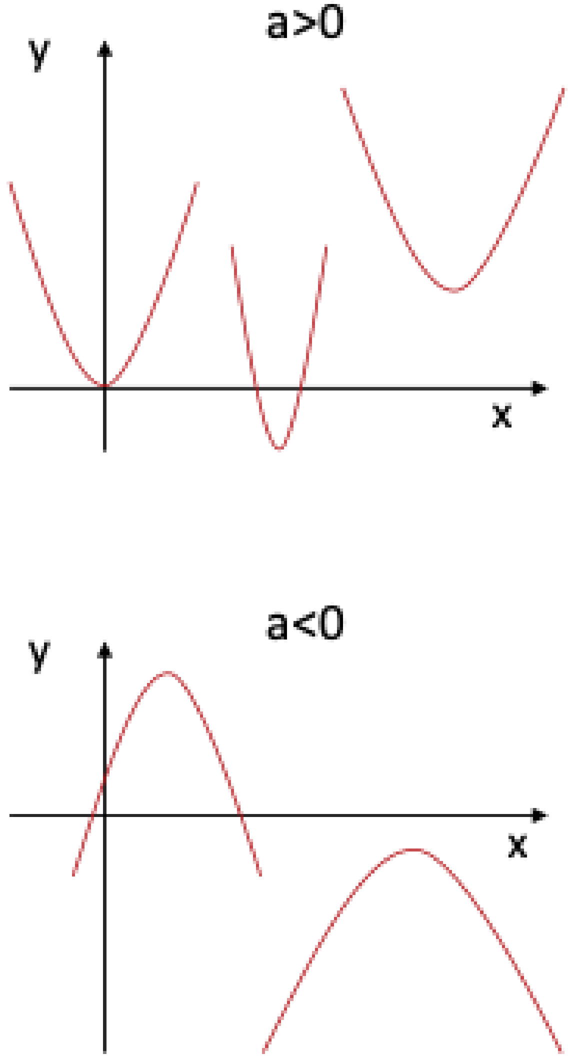


FIGURE 1.4: Graph of quadratic functions

Roots of a Quadratic function

A root, or solution, is that which satisfies a quadratic function. The equation can have 2, 1, or 0 roots. The roots must be a real number. There are two methods for finding roots:

Factoring: Find r_1 and r_2 such that $ax^2 + bx + c = a(x - r_1)(x - r_2)$

Quadratic equation $r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Other Common Functional Forms

- **Power Function** A power function is a function where $a \neq zero$, is a constant, and b is a real number. The shape of the curve is dictated by the value of b. $y = f(x) = ax^b$

- **Exponential Functions** Exponential functions have fast growth. In exponential functions, the variable is the power. $y = ab^x$
- **Multivariate Functions** Function with more than one independent x variable (x1, x2, x3).

> Euler's number, or e, is a constant number that is the base of the natural logarithm. e = 2.7182818... $y = e^x$

> Logarithm: A logarithm is a quantity representing the power to which a fixed number must be raised to produce a given number. It is the inverse function of an exponential.
>
> $y = b^x$
>
> $\log_b(y) = x$

Convexity and Continuity

Properties of functions:

- Convexity: Does the function "hold water"?
- Continuity: Function is continuous if you can draw it without lifting pen from paper!

Learning Objectives

- Recognize that decision-making is core to supply chain management.
- Gain perspectives on when to use analytical models.
- Understand building blocks that serve as foundation to analytical models.

1.3 Data Management

In data management, supply chain managers face immense complexity. This complexity is influenced by volume (how much), velocity (pace), variety (spread), and veracity (accuracy). Each of these components will influence how data is treated and used in the supply chain.

There are several recurring issues that supply chain managers must be aware of as they are working with data:

- Is the data clean?
- Is the data complete?
- What assumptions are being made about the data?
- Are the results making sense? How can they be checked?

Cleaning data is one of the most important, yet time consuming processes in data analysis. It can greatly influence the outcome of an analysis if not conducted properly. Therefore, Supply Chain professionals should always plan enough time for basic data checks (meaning if you get garbage in, you will get garbage out).

There are several typical checks that should always be looked at:

- Invalid values - negative, text, too small, too big, missing
- Mismatches between related data sets - # of rows, # of columns
- Duplication – non-unique identifiers
- Human error – wrong dates, invalid assumptions
- Always explore the outliers – they are the most interesting ones!

When cleaning data, one should be organized. This means to version the documents and keep track of data changes.

1.4 Probability

1.4.1 Summary

We review two very important topics in supply chain management: probability and distributions. Probability is an often-recurring theme in supply chain management due to the common conditions of uncertainty. On a given day, a store might sell 2 units of a product, on another, 50. To explore this, the probability review section includes an introduction of probability theory, probability laws, and proper notation. Summary or descriptive statistics are shown for capturing central tendency and the dispersion of a distribution. We also introduce two theoretical discrete distributions: Uniform and Poisson.

We then introduce three common continuous distributions: Uniform, Normal, and Triangle. The review then goes through the difference between discrete vs. continuous distributions and how to recognize these differences. The remainder of the review is an exploration into each type of distribution, what they look like graphically and what are the probability density functions and cumulative density functions of each.

1.4.2 Key Concepts

Probability basics
Probability defines the extent to which something is probable, or the likelihood of an event happening. It is measured by the ratio of the case to the total number of cases possible.

Probability Theory

- Mathematical framework for analyzing random events or experiments.
- Experiments are events we cannot predict with certainty (e.g., weekly sales at a store, flipping a coin, drawing a card from a deck, etc.).
- Events are a specific outcome from an experiment (e.g., selling less than 10 items in a week, getting 3 heads in a row, drawing a red card, etc.)

Notation

- P (A) – the probability that event A occurs
- P ($A^{'}$) = complement of P (A) – probability some other event that is not A occurs. This is also the probability that something other than A happens.
  $\cup$ = Union of sets (OR)
  $\cap$ = Intersection of sets (AND)
  $\varnothing$ = Null or Empty Set

Probability Laws

1. The probability of any event is between 0 and 1, that is 0 $\leq$ P(A)$\leq$ 1
2. If A and B are mutually exclusive events, then P (A or B) = P (A U B) = P (A) + P (B).
3. If A and B are any two events, then

$$P(A|B) = \frac{P(AandB)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$

   Where P(A | B) is the conditional probability of A occurring given B has already occurred.
4. If A and B are independent events, then

$$P(A|B) = P(A)$$

$$P(AandB) = P(A \cap B) = P(A|B) * P(B) = P(A) * P(B)$$

   Where events A and B are independent if knowing that B occurred does not influence the probability of A occurring.

Summary Statistics

Descriptive or summary statistics play a significant role in the interpretation, presentation, and organization of data. It characterizes a set of data. There are many ways that we can characterize a data set, we focus on two: Central Tendency and Dispersion or Spread.

Central Tendency

This is, in rough terms, the "most likely" value of the distribution. It can be formally measured in a number of different ways:

- Mode – the specific value that appears most frequently.
- Median – the value in the "middle" of a distribution that separates the lower from the higher half. This is also called the 50^{th} percentile value.
- Mean (μ) – the sum of values multiplied by their probability (called the expected value). Also the sum of values divided by the total number of observations (called the average).

  Expected value of X, E[X] is equal to:

$$E[X] = \bar{x} = \mu = \sum_{i=1}^{n} p_i x_i$$

Dispersion or Spread

This captures the degree to which the observations "differ" from each other. The most common dispersion metrics are:

- Range – the maximum value minus the minimum value.
- Inner Quartiles – 75^{th} percentile value minus the 25^{th} percentile value - captures the "central half" of the entire distribution.
- Variance (σ^2) – the expected value of the squared deviation around the mean; also called the Second Moment around the mean.

$Var[X] = \sigma^2 = \sum_{i=1}^{n} p_i(x_i - \bar{X})^2 = \sum_{i=1}^{n} p_i(x_i - \mu)^2$

- Standard Deviation (σ) – the square root of the variance. This has the same units as the expected value or mean.
- Coefficient of Variation (CV) – the ratio of the standard deviation over the mean = σ/μ. This is a common comparable metric of dispersion across different distributions. As a general rule:
  - 0 ≤ CV ≤ 0.75, low variability
  - 0.75 ≤ CV ≤ 1.33, moderate variability
  - CV > 1.33, high variability

Population versus Sample Variance

In practice, we usually do not know the true mean of a population. Instead, we need to estimate the mean from a sample of data pulled from the population. When calculating the variance, it is important to know whether we are using all of the data from the entire population or using just a sample of the population's data. In the first case we want to find the population variance while in the second case we want to find the sample variance.

The only difference between calculating the population variance and the sample variance (and thus their corresponding standard deviations) is that for the population variance, σ^2, we divide the sum of the observations by n (the number of observations) while for the sample variance, s^2, we divide by n-1.

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}$$

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} sl$$

Note that the sample variance will be slightly larger than the population variance for small values of n. As n gets larger, this difference essentially disappears. The reason for the use of n-1 is to include a degree of freedom in calculating the average ($\bar{x}$) from the same sample that we are estimating the variance from. It leads to an unbiased estimate of the population variance. In practice, you should just use the sample variance and standard deviation unless you are dealing with specific probabilities, like flipping a coin.

Spreadsheet Functions for Summary Statistics

All of these summary statistics can be calculated quite easily in any spreadsheet tool. The table below summarizes the functions for three widely used packages.

Function	Microsoft Excel	Google Sheets	Libre Office –> Calc
Minimum	=MIN(array)	=MIN(array)	=MIN(array)
Median	=MEDIAN(array)	=MEDIAN(array)	=MEDIAN(array)
Mode	=MODE(array)	=MODE(array)	=MODE(array)
Mean(μ)	=AVERAGE(array)	=AVERAGE(array)	=AVERAGE(array)
Maximum	=MAX(array)	=MAX(array)	=MAX(array)
Percentile	=PERCENTILE.INC(array,k)	=PERCENTILE.INC(array,percentile)	=PERCENTILE.INC(array,alpha)
Population Variance (σ^2)	=VAR.P(array)	=VARP(array)	=VAR.P(array)
Sample Variance (σ^2)	=VAR.S(array)	=VARA(array)	=VAR.S(array)
Pop.Std Deviation(σ)	=STDEV.P(array)	=STDEVP(array)	=STDEV.P(array)
Sample Std Deviation(σ)	=STDEV.S(array)	=STDEVA(array)	=STDEV.S(array)

TABLE 1.1: Spreadsheet functions for descriptive statistics

1.4.3 Discrete Distributions

Probability distributions can either be empirical (based on actual data) or theoretical (based on a mathematical form). Determining which is best depends on the objective of the analysis. Empirical distributions follow past history while theoretical distributions follow an underlying mathematical function. Theoretical distributions do tend to allow for more robust modeling since the empirical distributions can be thought of as a sampling of the population data. The theoretical distribution can be seen as better describing the assumed population distribution. Typically, we look for the theoretical distribution that best fits the data.

We presented five distributions. Two are discrete (Uniform and Poisson) and three are continuous (Uniform, Normal, and Triangle). Each is summarized in turn below.

Discrete Uniform Distribution ∼ U(a,b)

Finite number (N) of values observed with a minimum value of a and a maximum value of b. The probability of each possible value is 1/N where N= b – a +1

Probability Mass Function (pmf)

P[X=x]= f(x $|a,b) = \begin{cases} \frac{1}{n} \; for \; a \leq x \leq b \\ 0 \;\; otherwise \end{cases}$

Summary Metrics

- Mean =$(a + b)/2$
- Median = $(a + b)/2$
- Mode N/A (all values are equally likely)
- Variance = $((b - a + 1)^2 - 1)/12$

Poisson Distribution ∼ P(λ)

Discrete frequency distribution that gives the probability of a number of independent events occurring in a fixed time where the parameter λ = mean = variance. Widely used to model arrivals, slow moving inventory, etc. Note that the distribution only contains non-negative integers and can capture non-symmetric distributions. As the number of observation increases, the distribution becomes "bell like" and approximates the Normal Distribution.

Probability Mass Function (pmf)

$$P[X=x]= f(x \mid \lambda) = \begin{cases} \frac{e^{-\lambda}\lambda^{x}}{x!} for\ x = 0, 1, 2, ... \\ 0\ \ otherwise \end{cases}$$

where

- e= Euler's number ≈ 2.71828....
- λ = mean value(parameter)
- $x!$ = factorial of x, e.g., 3! = 3*2*1 = 6 and 0! = 1

Spreadsheet	Function	Prob 1	Prob 2
Microsoft Excel	=POISSON.DIST(x, mean, cumulative)	=POISSON.DIST(0, 2.2, 0)	=POISSON.DIST(2, 2.2, 1)
Google Sheets	=POISSON(x, mean, cumulative)	=POISSON(0, 2.2, 0)	=POISSON(2, 2.2, 1)
LibreOffice–Calc	=POISSON(Number; Mean; C)	=POISSON(0; 2.2; 0)	=POISSON(2; 2.2; 1)

TABLE 1.2: Spreadsheet functions for Poisson distribution

1.4.4 Continuous Distributions

Probability Density Function (pdf)

The pdf is the function of a continuous variable. The probability that X lies between values a and b is equal to the area under the curve between a and b. Total area under the curve equals 1, but for P(X=t) = 0 for any specific value of t.

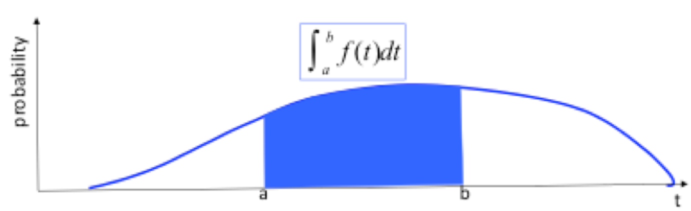


FIGURE 1.5: Probability density function

Cumulative Density Function (cdf)

- F(t) = P(X ≤ t) or the probability that X does not exceed t
- 0 ≤ F(t) ≤ 1
- F(b) >F(a)

Simple Rules

- P(X ≤ t)=F(t)
- P(X > t)=1–F(t)

- P(c $\leq$ X $\leq$ d)=F(d)–F(c)
- P(X=t)=0

Continuous Uniform Distribution $\sim$ U(a,b)
Sometimes also called a rectangular distribution

- "X if uniformly distributed over the range a to b, or X $\sim$ U (a,b)"

pdf: $f(t\ |a,b) = \begin{cases} \dfrac{1}{b-a} & if \ \ a \leq t \leq b \\ 0 & otherwise \end{cases}$

cdf: $F(t\ |a,b) = \begin{cases} 0 \ if \ t < a \\ \dfrac{t-a}{b-a} \ if \ a \leq t \leq b \\ 1 \ if \ t > b \end{cases}$

Summary Metrics

- Mean $=(a+b)/2$
- Median $= (a+b)/2$
- Mode N/A (all values are equally likely)
- Variance $= (b-a)^2/12$

Normal Distribution $\sim$ N(μ,σ) Widely used bell-shaped, symmetric continuous distribution with mean μ and standard deviation σ. Most commonly used distribution in practice.

Summary Metrics

- Mean $= \mu$
- Median $= \mu$
- Mode $= \mu$
- Variance $= \sigma^2$

pdf: $f(x|\mu,\sigma) = \dfrac{1}{\sigma(2\pi)^{1/2}} \ e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$

Common dispersion values $\sim$ N (μ, σ)

- P (X w/in 1σ around μ) = 0.6826
- P (X w/in 2σ around μ) = 0.9544
- P (X w/in 3σ around μ) = 0.9974

- +/- 1.65 σ around μ = 0.900
- +/- 1.96 σ around μ = 0.950
- +/- 2.81 σ around μ = 0.995

Unit or Standard Normal Distribution Z ~ N(0,1)

- The transformation from any ~ N(μ,σ) to the unit normal distribution = Z=(x- μ)/σ
- Z score (standard score) gives the number of standard deviations away from the mean
- Allows for use of standard tables and is used extensively in inventory theory for setting level

Function	Microsoft Excel	Google Sheets	Libre Office –> Calc
CDF of Normal Distribution	=NORM.DIST(X,μ,σ,1)	=NORMDIST(X,μ,σ,1)	=NORM.DIST(X,μ,σ,1)
PDF of Normal Distribution	=NORM.DIST(X,μ,σ,0)	=NORMDIST(X,μ,σ,0)	=NORM.DIST(X,μ,σ,0)
Inverse of Normal Distribution	=NORM.INV(Probability,μ,σ)	=NORMINV(Probability,μ,σ)	=NORM.INV(Probability,μ,σ)
Standard Normal cdf	=NORM.S.DIST(z,1)	=NORMSDIST(z)	=NORM.S.DIST(z,1)
Inverse Standard Normal cdf	=NORM.S.INV(Probability)	=NORMSINV(Probability)	=NORM.S.INV(Probability)

TABLE 1.3: Spreadsheet functions for Normal distribution

Triangle Distribution ~ T(a,b,c)

This is a continuous distribution with a minimum value of a, maximum value of b, and a mode of c. It is a good distribution to use when dealing with an anecdotal or unknown distribution. It can also handle non-symmetric distributions with long tails.

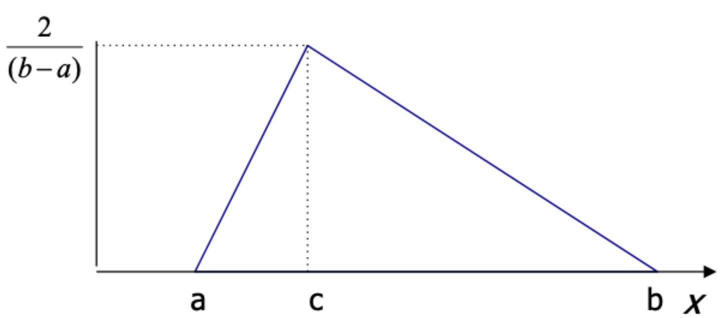


FIGURE 1.6: Triangle distribution

$$\text{pdf} = \text{f(x)} = \begin{cases} 0 \;\; if \;\; x < a \\ \dfrac{2(x-a)}{(b-a)(c-a)} \;\; if \;\; a \le x \le c \\ \dfrac{2(b-x)}{(b-a)(b-c)} \;\; if \;\; c \le x \le b \\ 0 \;\; if \;\; x > b \end{cases}$$

$$\text{cdf} = \text{f(x)} = \begin{cases} 0 \;\; if \;\; x \leq a \\ \dfrac{(x-a)^2}{(b-a)(c-a)} \;\; if \;\; a < x \leq c \\ 1 - \dfrac{(b-x)^2}{(b-a)(b-c)} \;\; if \;\; c \leq x \leq b \\ 1 \;\; if \;\; b \leq x \end{cases}$$

Summary Metrics

- E[x] = $\frac{a+b+c}{3}$
- Var[x] = $(\frac{1}{18})(a^2+b^2+c^2-ab-ac-bc)$
- P[x≤d] = $\frac{(d-a)^2}{(b-a)(c-a)}$ for a < d ≤ c
- P[x≤d] = 1 - $\frac{(b-d)^2}{(b-a)(b-c)}$ for c ≤ d < b
- P[x>d] = 1 - P[x≤d]
- d = a + $\sqrt{P[x \leq d](b-a)(c-a)}$ for a < d ≤ c
- d = b - $\sqrt{[1-P(x \leq d)](b-a)(b-c)}$ for c ≤ d < b

Differences between Continuous and Discrete Distributions

Just like variables, distributions can be classified into continuous (pdf) and discrete (pmf) probability distributions. While discrete distributions have a probability for each outcome, the probability for a specific point in a continuous distribution makes no sense and is zero. Instead for continuous distributions we look for the probability of a random variable falling within a specific interval. Continuous distributions use a function or formula to describe the data and thus instead of summing (as we did for discrete distributions) to find the probability, we integrate over the region.

Discrete Distributions

$\mu = E(X) = \sum_{i=1}^{n} p_i x_i$

$\sigma^2 = \sum_{i=1}^{n} p_i (x_i - \mu)^2$

Continuous Distributions

$\mu = \int_a^b t f(t) dt$

$\sigma^2 = \int_a^b (t-\mu)^2 f(t) dt$

Learning Objectives

- Understand probabilities, importance and application in daily operations and extreme circumstances.

- Understand and apply descriptive statistics.
- Understand difference between continuous vs. discrete random variable distributions.
- Review major distributions: Uniform (discrete and continuous), Poisson, Normal and Triangle.
- Understand the difference between discrete vs. continuous distributions.
- Recognize and apply probability mass functions (pmf), probability density functions (pdf), and cumulative density functions (cdf).

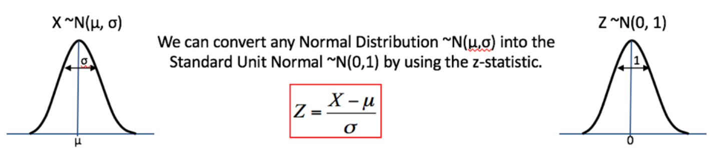
1.5 Statistics

1.5.1 Statistical testing and the central limit theorem

Central Limit Theorem
The central limit theorem states that the sample distribution of the mean of any independent random variable will be (nearly) normally distributed, if the sample size is large enough. Large enough is based on a few factors – one is accuracy (more sample points will be required) as well as the shape of the underlying population. Many statisticians suggest that 30, sometimes 40, is considered sufficiently large. The central limit theorem is important because it suggests that it does not matter what distributions the random variable follows. If we have a sufficient sample size we can assume a normal distribution.

The central limit theorem can be interpreted as follows:

- $X_i, ..X_n$ are identical and independently (iid) distributed random variables with mean=μ and standard deviation = σ:
  - The sum of the n random variables is S_n=Σ X_i.
  - The mean of the n random variables is $\bar{X} = S_n$/n.
- Then, if n is "large"(say > 30):
  - S_n is Normally distributed with mean = nμ and standard deviation $\sigma\sqrt{n}$
  - $\bar{X}$ is Normally distributed with mean = μ and standard deviation $\sigma/\sqrt{n}$



1.5.2 Sampling and confidence intervals

Sampling
An inference is a conclusion reached on the basis of evidence and reasoning. To make inferences about the population we need to collect a sample and ask testable questions such as, for example, if data fits a specific distribution? or if two variables are correlated? If sampling is done correctly, the sample mean should be an estimator of the population mean as well as corresponding parameters.

- Population: is the entire set of units of observation.

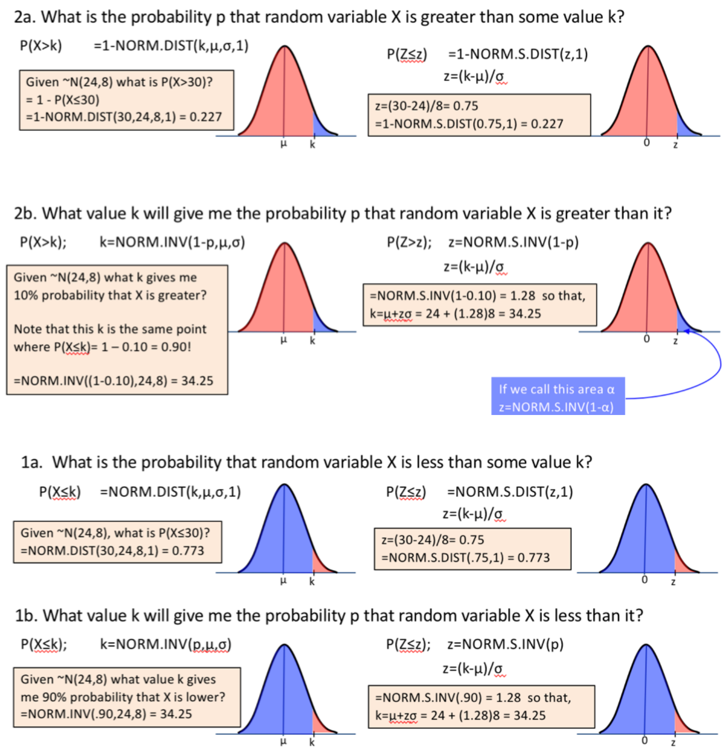


FIGURE 1.7: Convert a Normal Distribution to standard unit normal

Category	To find?	Normal Distribution N(μ, σ) If n $\geq$ 30	To find?	Standard unit normal Z $\sim N(0,1)$ If n $<$ 30
Find probability that random variable X is less than a given value k	P(X$\leq k$)	=NORM.DIST(k,μ, σ, 1)	P(Z$\leq z$)	=NORM.S.DIST(z,1)where z=(k-μ)/σ
Find probability that random variable X is more than a given value k	P(X $>$ k)	=1-NORM.DIST(k,μ, σ, 1)	P(Z $>$ z)	=1-NORM.S.DIST(z,1)
Find value k that will give the probability p that random variable X is less than it	k	=NORM.INV(p,μ, σ)	z	=NORM.S.INV(p)
Find value k that will give the probability p that random variable X is greater than it	k	=NORM.INV(1-p,μ, σ)	z	=NORM.S.INV(1-p)
Probability that random variable X lies between two points µ+b and µ-b	P($\mu - b \leq X \leq \mu + b$)	=NORM.DIST($\mu + b, \mu, \sigma, 1) - NORM.DIST(\mu - b, \mu, \sigma, 1)$	P(-z$\leq Z \leq z$)	=NORM.S.DIST(z,1)-NORM.S.DIST(-z,1)

TABLE 1.4: Summary of questions to be answered with probability distributions

- Sample: subset of the population.
- Parameters: describes the distribution of random variable.
- Random Sample: is a sample selected from the population that ensures that results are representative and not skewed from selection of the sample.

Things to keep in mind

- X is a random variable (μ, σ, . . .) for the entire population.
- X1, X2, . . . Xn are independent and identically distributed.
- $\bar{X}$ is an estimate of the population parameter (the mean or μ).

- Remember that $\bar{X}$ is also a random variable by itself!
- x1, x2, . . . xn, are the realizations or observations of random variable X.
- $\bar{x}$ is the sample statistic – the mean.
- We want to find how $\bar{x}$ relates to $\bar{X}$ relates to μ.

Why do we care?

- We can show that E[$\bar{X}$] = μ and that S = $\sigma/\sqrt{n}$.
- Note: standard deviation S decreases as sample size gets bigger!
- Also, the Central Limit Theorem says that sample mean $\bar{X}$ is $\sim$ N(μ, σ / $\sqrt{n}$).

Confidence Intervals

Confidence intervals are used to describe the uncertainty associated with a sample estimate of a population parameter.

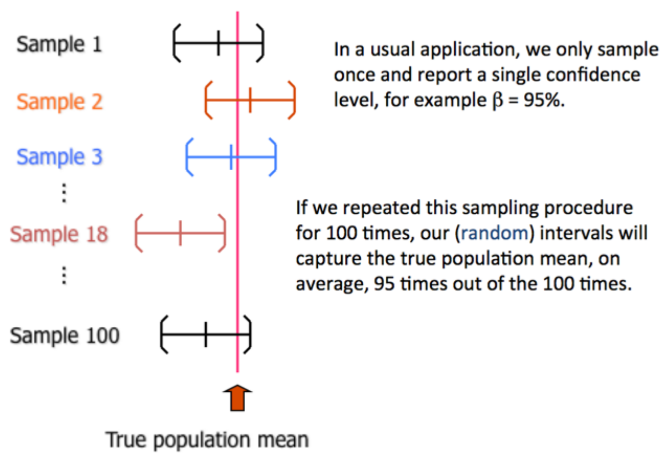


FIGURE 1.8: Confidence Interval

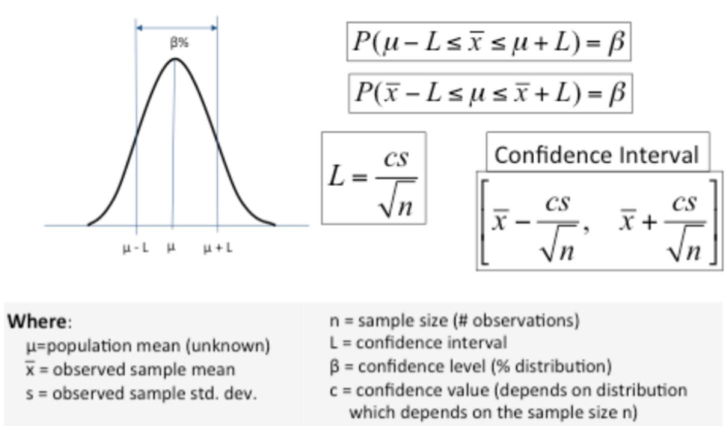


FIGURE 1.9: Confidence Interval

There are some important insights for confidence intervals around the mean. Recall that: There are trade-offs between interval (L), sample size (n) and confidence (c):

- When n is fixed, using a higher confidence level c leads to a wider interval, L.
- When confidence level c is fixed, increasing sample size n, leads to smaller interval, L.
- When both n and confidence level c are fixed, we can obtain a tighter interval, L, by reducing the variability (i.e. s).

When interpreting confidence intervals, a few things to keep in mind:

- Repeatedly taking samples and finding confidence intervals leads to different intervals each time, but c% of the resulting intervals would contain the true mean.
- To construct a c% confidence interval that is within (+/-) L of μ , the required sample size is: $n = z^2 * s^2/L^2$

After finding the z-value c using the table 1.4, use the formula below to derive the confidence intervals:

$$\left[\bar{x} - \frac{cs}{\sqrt{n}}, \bar{x} + \frac{cs}{\sqrt{n}}\right]$$

where s is the standard deviation and n is the number of samples.

Confidence vs Prediction Intervals

Prediction Interval

$$[\bar{x} - cs, \bar{x} + cs]$$

If we randomly choose other data points from this population, we would expect 90% of them to fall within this interval.

Confidence Interval

$$\left[\bar{x} - \frac{cs}{\sqrt{n}}, \bar{x} + \frac{cs}{\sqrt{n}}\right]$$

If we repeat this sampling infinite number of times, the true mean of this population should appear in 90% of those confidence intervals.

Student t- Distribution

If our sample is small (generally for samples n $\leq$ 30) we use the student t-distribution instead of the Normal distribution.

The t-distribution:

- is bell-shaped and symmetric around 0.
- is flatter and has fatter tails than the Normal distribution.
- its shape is a function of k, the degrees of freedom, where k=n-1, and
- its mean=0 and its standard deviation = $\sqrt{k/(k-2)}$.

Find what ?	Microsof Excel	Google Sheets	Libre Office – > Calc
CDF of t-Distribution (returns α probability)	=T.DIST(X, k, cumulative) =T.DIST.2T(X,k)	=TDIST(X,k,cumulative)	=TDIST(X,k,cumulative) =T.DIST.2T(X,k) =T.DIST(X,k)
Inverse of t Distribution (returns the t-value for left tail, use 1-α for right tail)	=T.INV(α,k) =T.INV.2T(α,k)	=T.INV.2T(α,k)	=T.INV.2T(α,k)

TABLE 1.5: Summary of questions to be answered with probability distributions

1.5.3 Hypothesis Testing

Hypothesis testing is a method for making a choice between two mutually exclusive and collectively exhaustive alternatives. In this method, we make two hypotheses and only one can be true. Null Hypothesis (H0) and the Alternative Hypothesis (H1). We test, at a specified significance level, to see if we can reject the Null hypothesis, or we cannot reject the Null Hypothesis.

Two types of errors can occur in hypothesis testing:

- Type I: Reject the Null Hypothesis when in fact it is true (Alpha).
- Type II: Fail to reject the Null Hypothesis when in fact it is false (Beta).
- We focus on Type I errors when setting significance level (.05, .01).

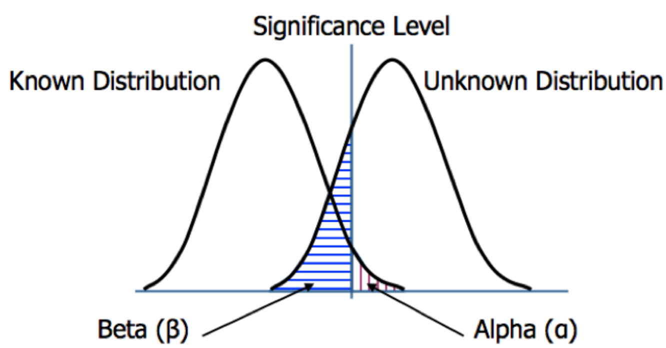


FIGURE 1.10: Significance Level

Three possible hypotheses or outcomes to a test:

- Unknown distribution is the same as the known distribution (Always H_0)
- Unknown distribution is ”higher” than the known distribution
- Unknown distribution is ”lower” than the known distribution

Example of Hypothesis Testing

Suppose we want to test whether a new information system has decreased my order cycle time. We know that historically, the average cycle time is 72.5 hours +/- 4.2 hours. We sampled 60 orders after the implementation and found the average to be 71.4 hours +/- 4.2 hours. We select a level of significance to be α =5%

1. Select the test statistic of interest:
   mean cycle time in hours – use Normal distribution (z-statistic).
2. Determine whether this is a one or two tailed test:
   One tailed test (test for direction).

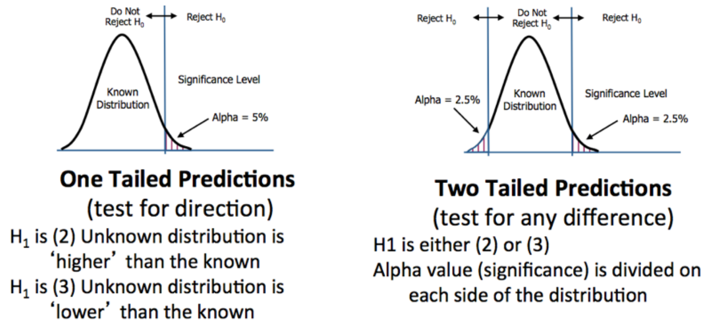


FIGURE 1.11: One-tailed and two-tailed predictions

3. Pick your significance level and critical value:
   α = 5%, therefore z= NORM.S.INV(.05) = -1.6448.
4. Formulate your Null and Alternative Hypotheses:
   H_0: New cycle time is not shorter than the old cycle time,
   H_1: New cycle time is shorter than the old cycle time.
5. Calculate the test statistic: z = $(\bar{X}$ - $\mu)/(\sigma/\sqrt{n})$ = (71.4 – 72.5)/(4.2/$\sqrt{60}$) = -2.0287.
6. Compare the absolute value of test statistic to the absolute value of critical value :
   z = 2.0287 > 1.6448 the test statistic > critical value, therefore, we can reject the null hypothesis.

Rather than just reporting that H0 was rejected at a 5% significance level, we might want to let people know how strongly we rejected it. If the p-value is the smaller than the level of level of significance α then we would reject the Null Hypothesis with our current set of data. Always report the p-value: p-value = NORM.S.DIST(-2.0287,1) = .0212.

Chi-Square test

The Chi-Square test is used to measure the goodness of fit and determine whether the data is, for example, normally distributed. To use a chi-square test, you typically will create a bucket of categories, c, count the expected and observed (actual) values in each category, and calculate the chi-square statistics to find the p-value. If the p-value is less than the level of significance, you will then reject the null hypothesis.

$$x^2 = \sum(\frac{(Observed - Expected)^2}{Expected})$$

where degrees of freedom df = c-1

Spreadsheet Functions:

Function	Returns p-value for Chi-Square Test
Microsoft Excel	=CHISQ.TEST(observed_values,expected_values)
Google Sheets	=CHITEST(observed_values,expected_values)
LibreOffice − > Calc	=CHISQ.TEST(observed_values;expected_values)

TABLE 1.6: Chi-Square Excel functions

1.5.4 Multiple Random Variables

Most situations in practice involve the use and interaction of multiple random variables (RVs) or some combination of random variables. We need to be able to measure the relationship between these RVs as well as understand how they interact.

Covariance and Correlation

Covariance and Correlation measure a certain kind of dependence between variables. If random variables are positively correlated, higher than average values of X are likely to occur with higher than average values of Y. For negatively correlated random variables, higher than average values of X are likely to occur with lower than average values of Y. It is important to remember as the old, but necessary saying that goes, "correlation does not equal causality." This means that you are finding a mathematical relationship – not a causal one.

Spreadsheet Functions:

Function	**Microsoft Excel**	**Google Sheets**	**Libre Office –> Calc**
Covariance	=COVAR(array,array)	=COVAR(array,array)	=COVAR(array;array)
Correlation	=CORREL(array,array)	=CORREL(array,array)	=CORREL(array;array)

TABLE 1.7: Correlation and Covariance spreadsheet functions

Correlation Coefficient: is used to standardize the covariance in order to better interpret it. It is a measure between -1 and +1 that indicates the degree and direction of the relationship between two random variables or sets of data.

$$CORR(X,Y) = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

Covariance:

$$Cov(X,Y) = \sum_{i=1}^{n} P(X = x_i, Y = y_i)[(x_i - \mu_X)(y_i - \mu_Y)] = \frac{\sum_{i=1}^{n}((x_i - \mu_X)(y_i - \mu_Y))}{n}$$

Linear Function of Random Variables

A linear relationship exists between X and Y when a one-unit change in X causes Y to change by a fixed amount, regardless of how large or small X is. Formally, this is: Y = aX + b.

The summary statistics of a linear function of a Random Variable are:

Expected value: E[Y] $= \mu_Y = a\mu_X + b$

Variance: VAR[Y] $=\sigma_Y^2 = a^2\sigma_X^2$

Standard Deviation $=\sigma_Y = |a|\sigma_X$

Sums of Random Variables If X and Y are random variables where W = aX + bY, then the summary statistics are:

Expected value: E[W] $= a\mu_x + b\mu_y$

Variance: VAR[W] $= a^2\sigma_x^2 + b^2\sigma_y^2 + 2abCOV(X,Y)$

$= a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\sigma_X\sigma_Y CORR(X,Y)$

Standard Deviation $=\sigma_W = \sqrt{\text{VAR[W]}}$

These relations hold for any X and Y. However, if X and Y are ∼N, then W is ∼N as well! If X and Y are independent variables, then the correlation and covariance terms will not exist.

1.6 Regression

1.6.1 Summary

In this section, we expand our tool set of predictive models to include ordinary least squares regression. This equips us with the tools to build, run and interpret a regression model. We first introduce how to work with multiple variables and their interaction. This includes correlation and covariance, which measures how two variables change together. As we review how to work with multiple variables, it is important to keep in mind that the data sets supply chain managers will deal with are largely samples, not a population. This means that the subset of data must be representative of the population. The later part of the lesson introduces hypothesis testing, which allows us to draw inferences from the data.

We then tackle linear regression. Regression is an important practice for supply chain professionals because it allows us to take multiple random variables and find relationships between them. In some ways, regression becomes more of an art than a science. There are four main steps to conducting regression analysis: choosing which independent variables to include, collecting data, running the regression, and analyzing the output (the most important step).

1.6.2 Ordinary Least Squares Linear Regression

Regression is a statistical method that allows users to summarize and study relationships between a dependent (Y) variable and one or more independent (X) variables. The dependent variable Y is a function of the independent variables X. It is important to keep in mind that variables have different scales (nominal/ordinal/ratio). For linear regression, the dependent variable is always a ratio. The independent variables can be combinations of different types.

Linear Regression Model

The data (x_i, y_i) are the observed pairs from which we try to estimate the β coefficients to find the "best fit." The error term, ε, is the "unaccounted" or "unexplained" portion.

Linear Model:

$$y_i = \beta_0 + \beta_1 x_i$$

$$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \text{ for } i = 1, 2, ...n$$

Residuals A linear regression model is not always appropriate for the data, one should assess the appropriateness of the model by defining residuals. The difference between the observed value of the dependent variable and predicted value is called the residual.

$$\hat{y_i} = b_0 + b_1 x_i \text{ for } i = 1, 2, ...n$$

$$e_i = y_i - \hat{y_i} = y_i - b_0 - b_1 x_i \text{ for } i = 1, 2, ...n$$

Ordinary Least Squares (OLS) Regression Ordinary least squares is a method for estimating the unknown parameters in a linear regression model. It finds the optimal value of the coefficients (b0 and b1) that minimize the sum of the squares of the errors:

$$\sum_{i=1}^{n}(e_i^2) = \sum_{i=1}^{n}(y_i - \hat{y_i})^2 = \sum_{i=1}^{n}(y_i - b_0 - b_1 x_i)^2$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$b_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}(x_i - \bar{x})^2}$$

Multiple Variables

These relationships translate also to multiple variables.

$$Y_i = \beta_0 + \beta_0 x_{1i} + ... + \beta_k x_{ki} + \varepsilon_i fori = 1, 2, ...n$$

$$E(Y|x_1, x_2, ..., x_k) = \beta_0 + \beta_0 x_1 + ... + \beta_k x_k$$

$$StdDev(Y|x_1, x_2, ..., x_k) = \sigma$$

$$\sum_{i=1}^{n}(e_i^2) = \sum_{i=1}^{n}(y_i - \hat{y_i})^2 = \sum_{i=1}^{n}(y_i - b_0 - b_1 x_{1i} - .... - b_k x_{ki})^2$$

Validating Model

All statistical software packages will provide statistics for evaluation (names and format will vary by package) but the model output typically includes: model statistics (regression statistics or summary of fit), analysis of variance (ANOVA), and parameter statistics (coefficient statistics).

Overall Fit
Overall fit - how much variation in the dependent variable (y), can we explain?
Total variation of CPL – find the dispersion around the mean.

Total Sum of Squares

TSS $= \sum_{i=1}^{n}(y_i - \bar{y})^2$

Make estimate for y for each x

Error or Residual Sum of Squares
$e_i = y_i - \hat{y}$

$RSS = \sum e_i^2 = \sum(y_i - \hat{y_i})^2$

Model explains % of total variation of the dependent variables.

Coefficient of Determination or Goodness of Fit ($\mathbf{R}^2$)

$$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y}_i)^2}$$

Adjusted R^2 corrects for additional variables

$$\text{adj R}^2 = 1 - \frac{RSS}{TSS}\left(\frac{n-1}{n-k-1}\right)$$

Individual Coefficients

Each Independent variable (and b_0) will have:

- An estimate of coefficient (b_1)
- A standard error (s_{bi})
  - s_e = Standard error of the model
  - s_x = Standard deviation of the independent variables
  - n = Number of observations

$$\text{b}_1 \pm t_{\alpha/2} s_{b_1}$$
$$v = n - 2$$

$$\text{s}_e = \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{N-2}}$$

$$\text{s}_{b_1} = \frac{s_e}{\sqrt{(n-1)s_x^2}}$$

The t-statistic

- k = number of independent variables
- b_i = estimate or coefficient of independent variable

$$\text{t} = \frac{b_1 - \beta_1}{s_{b_1}}$$

Corresponding p-value – Testing the Slope

- We want to see if there is a linear relationship i.e. we want to see if the slope (b_1) is something other than zero. So: H_0: $b_1 = 0$ and H_1: $b_1 \neq 0$.
- Confidence intervals – estimate an interval for the slope parameter.

$$\mathrm{b}_1 \pm t_{\alpha/2} s_{b_1}$$

$$v = n - 2$$

Multi-Collinearity, Autocorrelation and Heteroscedasticity

Multi-Collinearity is when two or more variables in a multiple regression model are highly correlated. The model might have a high R^2 but the explanatory variables might fail the t-test. It can also result in strange results for correlated variables.

Autocorrelation is a characteristic of data in which the correlation between the values of the same variables is based on related objects. It is typically a time series issue.

Heteroscedasticity is when the variability of a variable is unequal across the range of values of a second variable that predicts it. Some tell-tale signs include: observations are supposed to have the same variance. Examine scatter plots and look for "fan-shaped" distributions.

Learning Objectives

- Understand how to work with multiple variables.
- Be aware of data limitations with size and representation of population.
- Identify how to test a hypothesis.
- Review and apply steps in the practice of regression.
- Be able to perform regression analysis and recognize issues.

1.7 Optimization

1.7.1 Summary

This is an introduction and overview of optimization. It starts with an overview of unconstrained optimization and how to find extreme point solutions, keeping in mind first order and second order conditions. It also reviews the rules in functions such as the power rule. Next, this section reviews constrained optimization that shares similar objectives of unconstrained optimization but adds additional decision variables and constraints on resources. To solve constrained optimization problems the lesson introduces mathematical programs that are widely used in supply chain for many practices such as designing networks, planning production, selecting transportation providers, allocating inventory, scheduling port and terminal operations, fulfilling orders, etc. The overview of linear programming includes how to formulate the problem, how to graphically represent them, how to analyze the solution and conduct a sensitivity analysis.

In real supply chains, you cannot have .5 bananas in an order or in a shipment. This means that we must add additional constraints for integer programming where either all of the decision variables must either be integers, or be in a mixed integer programming where some, but not all variables are restricted to be an integer. We review the types of numbers you will encounter. Then we introduce integer programs and how they are different. We then review the steps to formulating an integer program and conclude with conditions for working with binary variables.

1.7.2 Key Concepts

Unconstrained Optimization

Unconstrained optimization considers the problem of minimizing or maximizing an objective function that depends on real variables with no restrictions on their values.

Extreme points:

- Extreme points are when a function takes on an extreme value - a value that is much smaller or larger in comparison to nearby values of the function.
- They are typically a min or a max (either global or local), or inflection points.
- Extreme points occur where slope (or rate of change) of function = 0.
- Test for Global vs. Local
  - Global min/max – for whole range
  - Local min/max – only in certain area

Finding Extreme Point Solutions

Use differential calculus to find extreme point solutions, look for where slope is equal to zero.

To find the extreme point, there is a three-step process:

- Take the first derivative of your function
- Set it equal to zero, and
- Solve for x*, the value of x at extreme point.

This is called the First Order Condition.

Instantaneous slope (or first derivative) occurs when

- dy/dx is the common form, where d means the rate of change.

δ (delta) = rate of change.

$$y' = f'(x) = dy/dx$$

The Product Rule: If function is constant, it doesn't have any effect.

$$y = f(x) = a- > y' = f'(x) = 0$$

Power Rule is commonly used for finding derivatives of complex functions.

$$y = f(x) = ax^n- > y' = f'(x) = anx^{n-1}$$

First and Second Order Conditions

In order to determine x* at the max/min of an unconstrained function

- First Order (necessary) condition – the slope must be 0 i.e. $f'(x*) = 0$
- Second order (sufficiency) condition - determines where extreme point is min or max by taking the second derivative, $f"(x)$.
  - If $f"(x) > 0$ extreme point is a local min
  - If $f"(x) < 0$ extreme point is a local max
  - If $f"(x) = 0$ it is inconclusive
- Special cases
  - If $f(x)$ is convex $->$ global min
  - If $f(x)$ is concave $->$ global max

1.7.3 Constrained Optimization

Similarities with unconstrained optimization

- Requires a prescriptive model
- Uses an objective function
- Solution is an extreme value

Differences

- Multiple decision variables
- Constraints on resources

Math Programming: Math programming is a powerful family of optimization methods that is widely used in supply chain analytics. It is readily available in software tools, but is only as good as the data input. It is the best way to identify the "best" solution under limited resources.

Some types of math programming in SCM:

- Linear Programming (LPs)
- Integer Programming (IPs)
- Mixed Integer and Linear Programming (MILPs)
- Non-linear Programming (NLPs)

1.7.4 Linear Programs

Decision Variables:

- What are you trying to decide?
- What are their upper or lower bounds?

Formulate objective function:

- What are we trying to minimize or maximize?

- Must include the decision variables and the form of the function determines the approach (linear for LP)

Formulate each constraint:

- What is my feasible region? What are my limits?
- Must include the decision variable and will almost always be a linear function

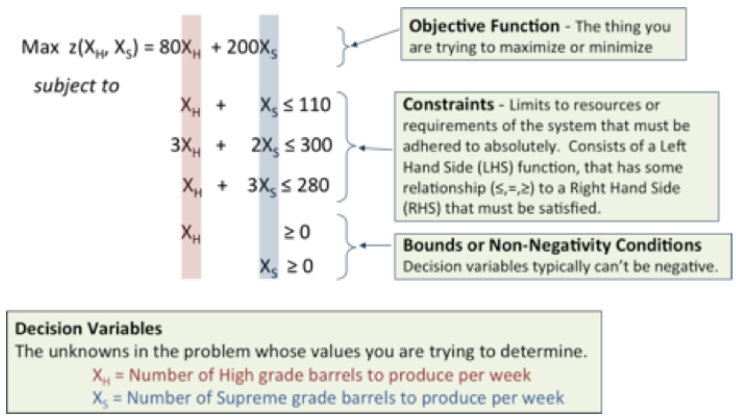


FIGURE 1.12: Linear program example

Solution

The solution of a linear program will always be in a "corner" of the Feasible Region:

- Linear constraints form a convex feasible region.
- The objective function determines which corner has the solution.

The Feasible Region is defined by the constraints and the bounds on the decision variables (See Figure).

Analysis of the Results

Sometimes the original question is the least interesting one, it is often more interesting to dive a little deeper into the structure of the problem.

Additional Questions:

- Am I using all of my resources?
- Where do I have a slack?
- Where I am constrained?
- How robust is my solution?

Sensitivity Analysis:

What happens when data values are changed.

- Shadow Price or Dual Value of a Constraint: What is the marginal gain in the profitability for an increase of one on the right-hand side of the constraint?

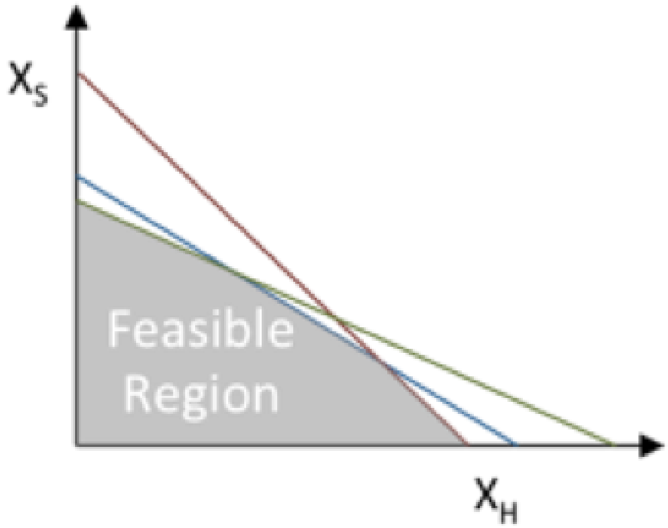


FIGURE 1.13: Graphical representation of the feasible region

- Slack Constraint – For a given solution, a constraint is not binding if total value of the left-hand side is not equal to the right-hand side value. Otherwise it is a binding constraint.
- Binding Constraint – A constraint is binding if changing it also changes the optimal solution.

Anomalies in Linear Programming:

- Alternative or Multiple Optimal Solutions (see figure below)
- Redundant Constraints - Does not affect the Feasible Region; it is redundant
- Infeasibility - There are no points in the Feasible Region; constraints make the problem infeasible

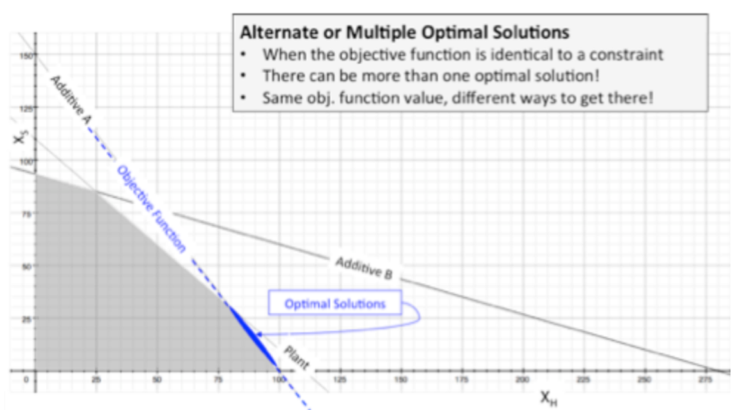


FIGURE 1.14: Alternative of multiple optimal solutions

1.7.5 Integer and Mixed Integer Programs

Although in some cases a linear program can provide an optimal solution, in many it cannot. For example, in warehouse location selection, batch orders, or scheduling, fractional answers are not acceptable. In addition, the optimal solution cannot always be found by rounding the linear program solution. This is where integer programs are important. However, integer program solutions are never better than a linear program solution, they lower the objective function. In general, formulating integer programs is much harder than formulating linear programs.

Numbers

- N = Natural, Whole or Counting numbers 1, 2, 3, 4
- Z - Integers = -3, -2, -1, 0, 1, 2, 3
- Q = Rational Number, continuous numbers = Any faction of integers 1/2 , -5/9
- R = Real Numbers = all Rational and Irrational Numbers, ex: e, π
- Binary Integers = 0, 1

- To identify the solution in integer programs – the Feasible Region becomes a collection of points, it is no longer a convex hull.
- In addition, we cannot rely on "corner" solutions anymore – the solution space is much bigger.

Mass enumeration - Unlike linear programs, integer programs can only take a finite number of integer values. This means that one approach is to enumerate all of these possibilities – calculating the objective function at each value and choosing the one with the optimal value. As the problem size increases, this approach is not feasible.

Formulating Integer Programs

To formulate an integer program, we follow the same approach for formulating linear programs – variables, constraints and an objective function. The only significant change to formulating integer programs is in the definition of the variables. See example formulation below with integer specification.

Max $z(X_{HL}, X_{SL}) = 8X_{HL} + 20X_{SL}$

$s.t.$

$Plant\ X_{HL} + X_{SL} \leq 11$

$Add.\ A\ 3X_{HL} + 2X_{SL} \leq 30$

$Add.\ B\ X_{HL} + 3X_{SL} \leq 28$

$X_{HL}, X_{SL} \geq 0\ \ Integers$

Binary Variables

Suppose you had the following formulation of a minimization problem subject to capacity at plants and meeting demand for individual products.

$Minz = \sum_i \sum_j c_{ij} x_{ij}$

s.t.

$\sum_i x_{ij} \leq C_j \;\; \forall_j$

$\sum_j x_{ij} \geq D_i \;\; \forall_i$

$x_{ij} \geq 0 \;\; \forall_{ij}$

where: x_{ij} = Number of units of product i made in plant j

c_{ij} = Cost per unit of product i made at plant j

C_j = Capacity in units at plant j

D_i = Demand for product i in units

We could add binary variables to this formulation to be able to model several different logical conditions. Binary variables are integer variables that can only take the values of 0 or 1. Generally, a positive decision (do something) is represented by 1 and the negative decision (do nothing) is represented by the value of 0.

Introducing a binary variable to this formulation, we would have:

$Minz = \sum_i \sum_j c_{ij} x_{ij} + \sum_j f_j y_j$

s.t.

$\sum_i x_{ij} \leq C_j \;\; \forall_j$

$\sum_j x_{ij} \geq D_i \;\; \forall_i$

$\sum_j x_{ij} - M y_j \leq 0 \;\; \forall_j$

$x_{ij} \geq 0 \;\; \forall_{ij}$

$\mathrm{y}_j = \{0, 1\}$

$where$:x_{ij} = Number of units of product i made in plant j

y_j = 1 if plant j is opened; = 0 otherwise

f_j = Fixed cost for producing at plant j

c_{ij} = Cost per unit of product i made at plant j

C_j = Capacity in units at plant j

D_i = Demand for product i in units

Note that not only have we added the binary variable in the objective function, we also have added a new constraint (the third one). This is known as a linking constraint or a logical constraint. It is required to enforce an "if-then" condition in the model. Any positive value of x_{ij} will force the y_j variable to be equal to one. The "M" value is a big number – it should be as big as the values of the sum of the $x_{ij}s$ can be. There are also other technical tricks that can be used to tighten this formulation.

We can also introduce Either/Or Conditions- where there is a choice between two constraints, only one of which has to hold; it ensures a minimum level, L_j, if $y_j = 1$.

$$\sum_i x_{ij} - My_j \leq 0 \quad \forall_j$$

$$\sum_i x_{ij} - L_j Y_j \geq 0 \quad \forall_j$$

For example:

$\sum_i x_{ij} \leq C_j \ \ \forall_j$

$\sum_j x_{ij} \geq D_j \ \ \forall_i$

$\sum_i x_{ij} - My_j \leq 0$

where: x_{ij} = Number of units of product i made in plant j

$y_j = 1$ if plant j is opened; $= 0$ o.w.

M = a big number (such as C_j in this case)

C_j = Maximum capacity in units at plant j

L_j = Minimum level of production at plant j

D_i = Demand for product i in units

We need to add a constraint that ensures that if we DO use plant j, that the volume is between the minimum allowable level, L_j, and the maximum capacity, C_j. This is sometimes called an Either-Or condition.

$\sum_i x_{ij} \leq My_j \ \ \forall_j$

$\sum_i x_{ij} \geq L_j Y_j \ \ \forall_j$

where: x_{ij} = Number of units of product i made in plant j

$y_j = 1$ if plant j is opened; $= 0$ o.w.

M = a big number (such as C_j in this case)

L_j = Minimum level of production at plant j

Finally, we can create a "Select From" Condition that allows us to select the best N choices. Note that this can be formulated as "choose at least N" or "choose no more than N" by changing the inequality sign on the second constraint.

$$\sum_i x_{ij} - My_j \leq 0 \quad \forall_j$$
$$\sum_j y_j \leq N \quad \forall_j$$

Difference between Linear Programs and Integer Programs/Mixed Integer Programs

- Integer programs are much harder to solve since the solution space expands.
  - For linear programs, a correct formulation is generally a good formulation.
  - For integer programs a correct formulation is necessary but not sufficient to guarantee solvability.
- Integer programs require solving multiple linear programs to establish bounds – relaxing the Integer constraints.
- While it seems the most straightforward approach, you often can't just "round" the linear program solution – it might not be feasible.
- When using integer (not binary) variables, solve the linear program first to see if it is sufficient.

Learning Objectives

- Learn the role of optimization.
- Understand how to optimize with unconstrained conditions.
- Identify how to find Extreme Point Solutions.
- Understand how to formulate problems with decision variables and resource constraints and graphically present them.
- Review how to interpret results and conduct sensitivity analysis.
- Understand the different "types" of numbers and how they change the approach to problems.
- Review the approach of formulating integer and mixed integer program problems and solving them.

1.8 Networks and Non-Linear Programming

Summary

This review concludes the learning portion on optimization with an overview of some frequently used advanced optimization models. Network models are key for supply chain professionals. The review begins by first defining the terminology used frequently in these networks. It then introduces common network problems including the Shortest Path, Traveling Salesman Problem (TSP), and Flow problems. These are used frequently in supply chain management and understanding when to apply and how to solve them is essential. We then introduce non-linear optimization, highlighting its differences with linear programming, and an overview on how to solve non-linear problems. The review concludes with practical recommendations of for conducting optimization, emphasizing that supply chain professionals should: know their problem, their team and their tool.

1.8.1 Network Models

Network Terminology

- Node or vertices – a point (facility, DC, plant, region)
- Arc or edge – link between two nodes (roads, flows, etc.) may be directional
- Network or graph – a collection of nodes and arcs

Common Network Problems

Shortest Path – Easy and fast to solve (LP or special algorithms) - Result of shortest part problem is used as the base of other analyses. It connects physical to operational network.

- Given: One origin, one destination.
- Find: Shortest path from single origin to single destination.
- Challenges: Time or distance? Impact of congestion or weather? How frequently should we update the network?
- Integrality is guaranteed.

- Caveat: Other specialized algorithms leverage the network structure to solve much faster.

Traveling Salesman Problem (TSP) – Hard to solve (heuristics)

- Given: One origin, many destinations, sequential stops, one vehicle.
- Objective: Starting from an origin node, find the minimum distance required to visit each node once and only one and return to the origin.
- Importance: TSP is at the core of all vehicle routing problems; local routing and last mile deliveries are both common and important.
- Challenges: It is exceptionally hard to solve due to its size; possible solutions increase exponentially with number of nodes.
- Primary approach: special algorithms for exact solutions (smaller problems) – Heuristics (many available).
  - Two examples: Nearest Neighbor, Cheapest Insertion

Nearest Neighbor Heuristic: This algorithm starts with the salesman at a random city who visits the city which is nearest and continues this process until all have been visited. It yields a short tour, but typically not the optimal one.

- Select any node to be the active node.
- Connect the active node to the closest unconnected node; make that the new active node.
- If there are more unconnected nodes go to step 2, otherwise connect to the origin and end.

Cheapest Insertion Heuristic: One approach to the TSP is to start with a subtour – tour of small subsets of nodes, and extend this tour by inserting the remaining nodes one after the other until all nodes have been inserted. There are several decisions to be made in how to construct the initial tour, how to choose the next node to be inserted, where to insert the chosen node.

- Form a sub tour from the convex hull.
- Add to the tour an unconnected node that increases the cost the least; continue until all nodes are connected.

Flow Problems (Transportation Transshipment) – Widely used (MILPs)

- Given: Multiple supply and demand nodes with fixed costs and capacities on nodes and/or arcs.
- Objective: Find the minimum cost flow of product from supply nodes that satisfy demand at destination nodes.
- Importance: Transportation problems are everywhere; transshipment problems are at the heart of larger supply chain network design models. In transportation problems, shipments are between two nodes. For transshipment problems, shipments may go through intermediary nodes, possibly changing mode of transport. Transshipment problems can be converted into transportation problems.
- Challenges: data requirements can be extensive; difficult to draw the line on “realism” vs. “practicality”.
- Primary approaches: mixed integer linear programs; some simulation – usually after optimization.

1.8.2 Non-Linear Optimization

A nonlinear program is similar to a linear program in that it is composed of an objective function, general constraints, and variable bounds. The difference is that a nonlinear program includes at least one nonlinear function, which could be the objective function, or some or all of the constraints.

- Many systems are nonlinear – important to know how to handle them.
- Harder to solve than linear programs – lose 'corner' solutions (see figure below).
- Shape of objective function and constraints dictate approach and difficulty.

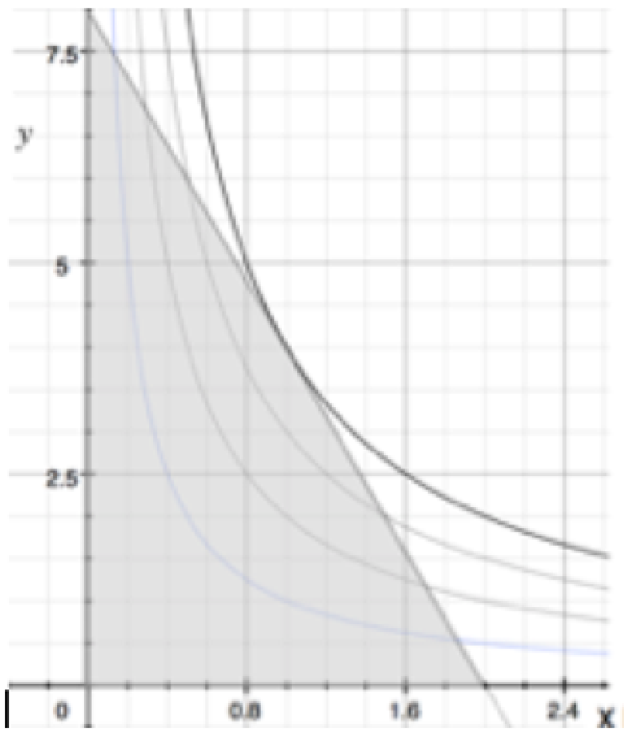


FIGURE 1.15: Example of NLP with linear constraint and non-linear objective function (z=xy).

Practical Tips for Optimization in Practice

- Know your problem:
  - Determining what to solve is rarely readily apparent or agreed upon by all stakeholders.
  - Establish and document the over-riding objective of a project early on.
- Level of detail and scope of model:
  - Models cannot fully represent reality, models will never represent all factors, determine problem boundaries and data aggregation levels.
- Input data:
  - Collecting data is hardest, least appreciated, and most time-consuming task in an optimization project.
  - Data collected is neither completely clean nor totally correct.

  - Ever hour spent on data collection, cleaning and verification saves days later on in the project.

- Sensitivity and Robustness Analysis
  - These are all deterministic models – data assumed perfect and unchanging.
  - Optimization models will do anything to save a dollar, yuan, peso, euro, etc.
  - Run multiple "what-if" scenarios changing uncertain input values and testing different conditions.

- Models vs. People (models don't make decisions, people do!)
  - Optimization models are good at making trade-offs between complicated options and uncovering unexpected insights and solutions.
  - People are good at:
    * Considering intangible and non-quantifiable factors.
    * Identifying underlying patterns.
    * Mining previous experiences and insights.
    * Models should be used for decision support not for decision making.

Learning Objectives

- Introduction to advanced optimization methods.
- Understand the conditions and when to apply network models.
- Differentiate nonlinear optimization and when it should be used.
- Review recommendations for running optimization in practice – emphasizing importance of knowing the problem, team and tool.

1.9 Algorithms and Approximations

1.9.1 Summary

In this lesson, we will be reviewing Algorithms and approximations. The first half of the lesson will be a review of algorithms – which you technically have already been introduced to, but perhaps not in these terms. We will be reviewing the basics of algorithms, their components, and how they are used in our everyday problem solving. To demonstrate these, we will be looking at a few common supply chain problems such as the Shortest Path problem, Traveling Salesman Problem, and Vehicle Routing Problem while applying the appropriate algorithm to solve them.

In the next part of the lesson, we will be reviewing approximations. Approximations are good first steps in solving a problem because they require minimal data, allow for fast sensitivity analysis, and enable quick scoping of the solution space. Recognizing how to use approximation methods are important in supply chain management because commonly optimal solutions require large amounts of data and are time consuming to solve. So, if that level of granularity is not needed, approximation methods can provide a basis to work on and to see whether further analysis is needed.

1.9.2 Algorithms

Algorithm - a process or set of rules to be followed in calculations or other problem-solving operations, especially by a computer.

Desired properties of an Algorithm:

- should be unambiguous.
- require a defined set of inputs.
- produce a defined set of outputs.
- should terminate and produce a result, always stopping after a finite time.

Algorithm Example: find max

Inputs:

- L = array of N integer variables
- v(i) = value of the i^{th} variable in the list

Algorithm:

- set max = 0 and i = 1
- select item i in the list
- if v(i) > max, then set max = v(i)
- if i < N, then set i = i+1 and go to step 2, otherwise go to step 5
- end

Output: maximum value in array L (max)

1.9.3 Shortest Path Problem

Objective: Find the shortest path in a network between two nodes.

Importance: Its result is used as base for other analysis, and connects physical to operational network.

Primary approaches:

- Standard Linear Programming (LP)
- Specialized Algorithms (Dijkstra's Algorithm)

Minimize: $\sum_i \sum_j c_{ij} x_{ij}$

Subject to:

$\sum_i x_{ji} = 1 \quad \forall_j = S$

$\sum_i x_{ji} - \sum_i x_{ij} = 0 \quad \forall_j \neq s, j \neq t$

$\sum_i x_{ij} = 1 \quad \forall_j = t$

$x_i j \geq 0$

where:

x_{ij} = Number of units flowing from node i to node j

c_{ij} = Cost per unit for flow from node i to node j

s = Source node – where flow starts

t = Terminal node – where flow ends

1.9.4 Dijkstra's Algorithm

Djikstra's algorithm (named after its discoverer, E.W. Dijkstra) solves the problem of finding the shortest path from a point in a graph (the source) to a destination.

L_j = length of path from source node s to node j
P_j = preceding node for j in the shortest path
S_j = 1 if node j has been visited, = 0 otherwise
d_{ij} = distance or cost from node i to node j

Inputs:

- Connected graph with nodes and arcs with positive costs, d(ij)
- Source (s) and Terminal (t) nodes

Algorithm:

1. For all nodes in graph, set L()=∞, P()=Null, S()=0.
2. Set s to i, S(i)=1, and L(i)=0.
3. For all nodes, j, directly connected (adjacent) to node i, if L(j) > L(i) + $d(ij)$, then set L(j) = L(i) + $d(ij)$ and P(j)=i.
4. For all nodes where S()=0, select the node with lowest L() and set it to i, set S(i)=1.
5. Is this node t, the terminal node? If so, go to end. If not, go to step 3.
6. end – return L(t).

Output: L(t) and P array

To find path from s to t, start at the end.

- Find P(t) – say it is j
- If j=source node, stop, otherwise, find P(j)
- keep tracing preceding nodes until you reach source node

1.9.5 Traveling Salesman Problem (TSP)

Starting from an origin node, find the minimum distance required to visit each node once and only once and return to the origin.

Nearest Neighbor Heuristic

1. Select any node to be the active node.
2. Connect the active node to the closest unconnected node, make that the new active node.
3. If there are more unconnected nodes go to step 2, otherwise connect to the starting node and end.

2-Opt Heuristic

1. Identify pairs of arcs (i-j and k-l), where d(ij) + d(kl) > d(ik) + d(jl) – usually where they cross.
2. Select the pair with the largest difference, and re-connect the arcs (i-k and j-l).
3. Continue until there are no more crossed arcs.

1.9.6 Vehicle Routing Problem

Find minimum cost tours from single origin to multiple destinations with varying demand using multiple capacitated vehicles.

Heuristics

- Route first Cluster second
  - Any earlier TSP heuristic can be used.

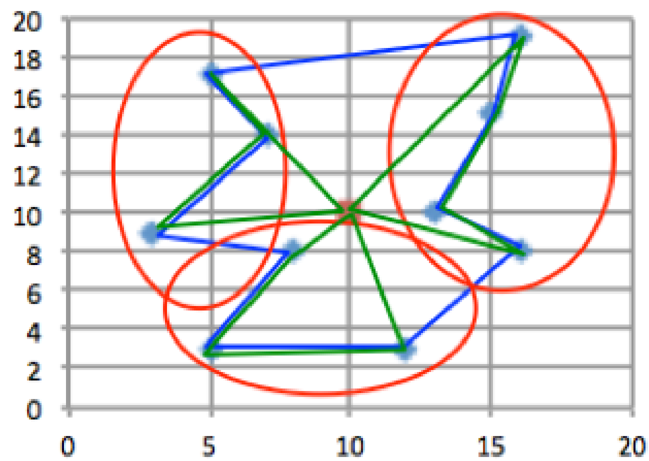


FIGURE 1.16: Route first cluster second.

Optimal

- Mixed Integer Linear Program (MILP)
- Select optimal routes from potential set

Cluster first Route second

- Sweep Algorithm
- Savings (Clarke-Wright)

VRP Sweep Heuristic

Find minimum cost tours from DC to 10 destinations with demand as shown using up to 4 vehicles of capacity of 200 units.

Sweep Heuristic

1. Form a ray from the DC, select an angle and direction (CW vs CCW) to start
2. Select a new vehicle, j, that is empty, w_j=0, and has capacity, c_j.
3. Rotate the ray in selected direction until it hits a customer node, i, or reaches the starting point (go to step 5).
4. If the demand at i (D_i) plus current load already in the vehicle (w_j) is less than the vehicle capacity, add it to the vehicle, w_j=D_i + w_j and go to step 3. Otherwise, close this vehicle, and go to step 2 to start a new tour.
5. Solve the TSP for each independent vehicle tour.

Different starting points and directions can yield different solutions! Best to use a variety or a stack of heuristics.

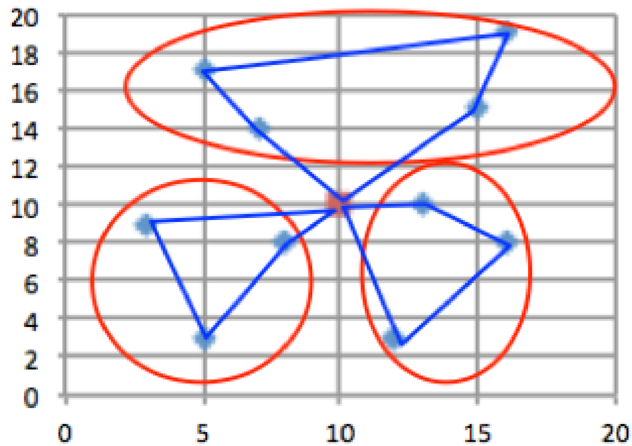


FIGURE 1.17: Cluster first route second.

1.9.7 Clark-Wright Savings Algorithm

The Clarke and Wright savings algorithm is one of the most known heuristic for VRP. It applies to problems for which the number of vehicles is not fixed (it is a decision variable).

- Start with a complete solution (out and back).
- Identify nodes to link to form a common tour by calculating the savings.

> **Example:**
> Joining node 1 and 2 into a single tour
> Current tours cost = $2c_{O1} + 2c_{O2}$
> Joined tour costs = $c_{O1} + c_{12} + c_{2O}$
> So,if $2c_{O1} + 2c_{O2} > c_{O1} + c_{12} + c_{2O}$ then join
> them That is: $c_{O1} + c_{20} - c_{12} > 0$

- This savings value can be calculated for every pair of nodes.
- Run through the nodes pairing the ones with the highest savings first.
- Need to make sure vehicle capacity is not violated.
- Also, “interior tour” nodes cannot be added – must be on end.

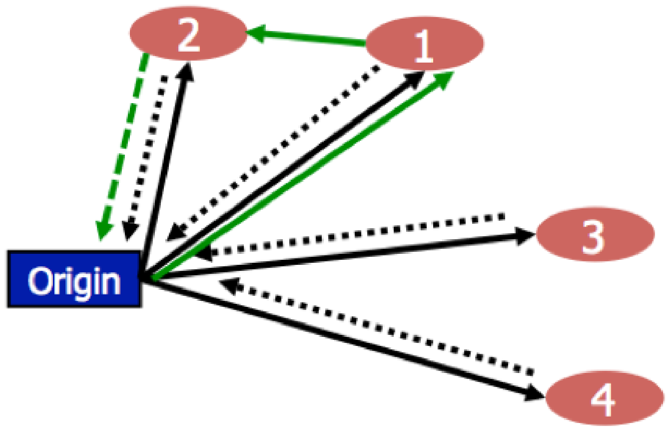


FIGURE 1.18: Savings algorithm.

1.9.8 Savings Heuristic

1. Calculate savings $s_{i,j} = c_{O,i} + c_{O,j}$ - $c_{i,j}$ for every pair (i, j) of demand nodes.
2. Rank and process the savings s_i, j in descending order of magnitude.
3. For the savings $s_{i,j}$ under consideration, include arc (i, j) in a route only if:

   •No route or vehicle constraints will be violated by adding it in a route and

   •Nodes i and j are first or last nodes to/from the origin in their current route.

4. If the savings list has not been exhausted, return to Step 3, processing the next entry in the list; otherwise, stop.

1.9.9 Solving VRP with MILP

Potential routes are an input and can consider different costs, not just distance.

Mixed integer linear programs are used to select routes where:

- Each column is a route
- Each row is a node/stop
- Total cost of each route is included

Min $\sum_j C_j Y_j$

Such that:

$\sum_{j=1}^{J} a_{ji} Y_j \geq D_i \forall_i$

$\sum_{j=1}^{J} Y_j \leq V$

$Y_j = 0, 1 \quad \forall_j$

Indices

- Demand nodes i
- Vehicle routes j

Input Data

- C_j = Total cost of route j ($)
- D_i = Demand at node i (number of visits)
- V = Maximum vehicles
- a_{ij} =1 if node i is in route j;= 0 otherwise

Decision Variables

- Y_j =1 if route j is used; =0 otherwise

1.9.10 Approximation Methods

In this second half, we will discuss examples of approximation and estimation. In particular we will review estimation of One-to-Many Distribution through line haul distance, traveling salesman and vehicle routing problems.

Approximation : a value or quantity that is nearly but not exactly correct.

Estimation: a rough calculation of the value, number, quantity, or extent of something. synonyms: estimate, approximation, rough calculation, rough guess, evaluation, back-of-the envelope.

Why use approximation methods?

- Faster than more exact or precise methods.
- Uses minimal amounts of data.
- Can determine if more analysis is needed: Goldilocks Principle: Too big, Too little, Just right.

Always try to estimate a solution prior to analysis!

Quick Estimation Simple Estimation Rules:

1. Break the problem into pieces that you can estimate or determine directly.
2. Estimate or calculate each piece independently within an order of magnitude.
3. Combine the pieces back together paying attention to units.

Example: How many piano tuners are there in Chicago?

- There are approximately 9,000,000 people living in Chicago.
- On average, there are two persons in each household in Chicago.
- Roughly one household in twenty has a piano that is tuned regularly.
- Pianos that are tuned regularly are tuned on average about once per year.
- It takes a piano tuner about two hours to tune a piano, including travel time.
- Each piano tuner works eight hours in a day, five days in a week, and 50 weeks in a year.

Tunings per Year = (9,000,000 ppl) ÷ (2 ppl/hh) (1 piano/20 hh) (1 tuning/piano/year)
= 225,000
Tunings per Tuner per Year = (50 wks/yr) ÷ (5 day/wk) (8 hrs/day) ÷ (2 hrs to tune)
= 1000
Number of Piano Tuners = (225,000 tunings / year) ÷ (1000 tunings / year / tuner)
= 225

Actual Number = 290

Estimation of One to Many Distribution

Single Distribution Center:

- Products originate from one origin.
- Products are demanded at many destinations.
- All destinations are within a specified Service Region.
- Ignore inventory (same day delivery).

Assumptions:

- Vehicles are homogeneous.
- Same capacity, Q_{MAX}.
- Fleet size is constant.

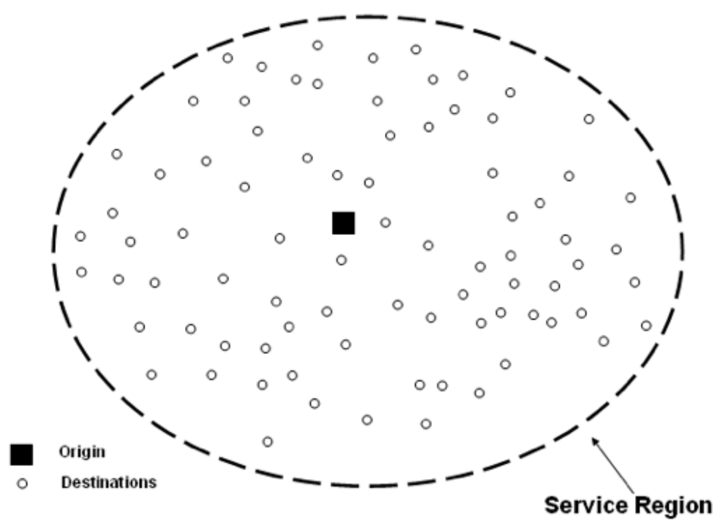


FIGURE 1.19: Estimation of one-to-many distribution

Finding the estimated total distance:

- Divide the Service Region into Delivery Districts
- Estimate the distance required to serve each district

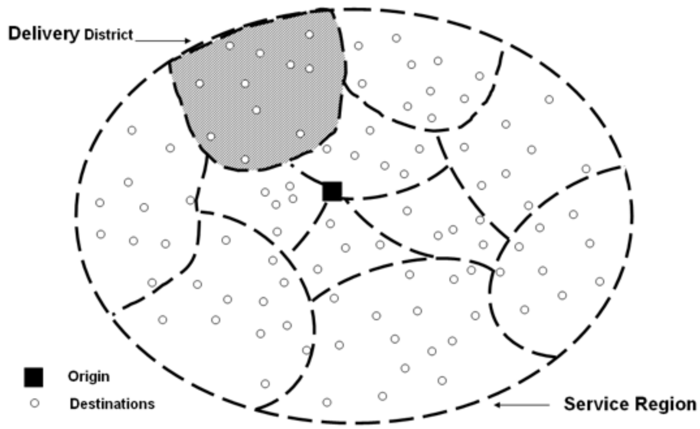


FIGURE 1.20: Finding the estimated total distance

Route to serve a specific district:

- Line haul from origin to the 1^{st} customer in the district.
- Local delivery from 1^{st} to last customer in the district.
- Back haul (empty) from the last customer to the origin.

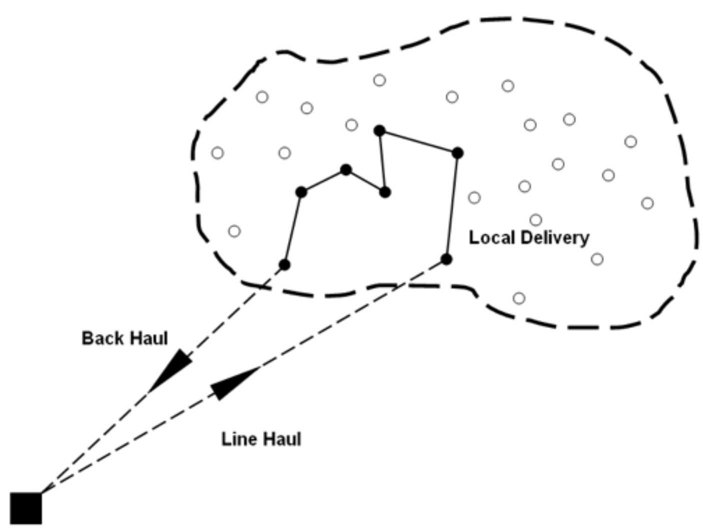


FIGURE 1.21: Route to serve a specific district

$d_{TOUR} \approx 2d_{LineHaul} + d_{Local}$

$d_{LineHaul}$ = Distance from origin to center of gravity (centroid) of delivery district

$d_{L}ocal$ = Local delivery between customers in one district

How do we estimate distances?

- Point to Point
- Routing or within a Tour

Estimating Point to Point Distances

Depends on the topography of the underlying region

Euclidean Space: $d_{A-B} = \sqrt{[(x_A - x_B)^2 + (y_A - y_B)^2]}$

Grid: $d_{A-B} = |x_A - x_B| + |y_A - y_B|$

Random Network: different approach

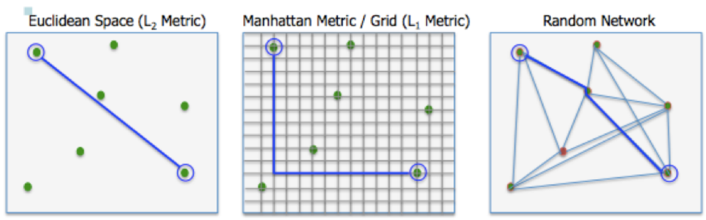


FIGURE 1.22: Estimating point to point distances

For Random (real) Networks use:

$D_{A-B} = k_{CF} d_{A-B}$

Find d_{A-B} - the "as crow flies" distance.

- Euclidean: for really short distances $d_{A-B} = \sqrt{((x_A - x_B)^2 + (y_A - y_B)^2)}$
- Great Circle: for locations within the same hemisphere

  $d_{A-B} = 3959(arccos[sin[LAT_A]sin[LAT_B] + cos[LAT_A]cos[LAT_B]cos[LONG_A - LONG_B]])$
- Where:
  - LAT_i = Latitude of point i in radians
  - $LONG_i$ = Longitude of point i in radians
  - Radians = (Angle in Degrees) * ($\pi/180$)

  Apply an appropriate circuity factor (k_{CF})
- How do you get this value?
- What do you think the ranges are?
- What are some cautions for this approach?

1.9.11 Estimating Local Route Distances

Traveling Salesman Problem

- Starting from an origin, find the minimum distance required to visit each destination once and only once and return to origin.
- The expected TSP distance, d_{TSP}, is proportional to $\sqrt{(nA)}$ where n= number of stops and A=area of district.
- The estimation factor (k_{TSP}) is a function of the topology.

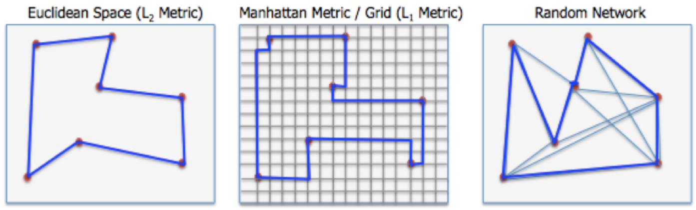


FIGURE 1.23: Estimating local route distances

One to Many System

What can we say about the expected TSP distance to cover n stops in district with an area of A? A good approximation, assuming a "fairly compact and fairly convex" area, is:

A = Area of district

n = Number of stops in district

δ = Density (stops/Area)

k_{TSP} = TSP network factor $(unit - less)$

d_{TSP}= Traveling Salesman Distance

d_{stop}= Average distance per stop

$$d_{TSP} \approx k_{TSP}\sqrt{nA} = k_{TSP}\sqrt{n(\frac{n}{\delta})} = k_{TSP}\frac{n}{\sqrt{\delta}}$$

$$d_{stop} \approx \frac{k_{TSP}\sqrt{nA}}{n} = k_{TSP}\sqrt{\frac{A}{n}}$$

What values of k_{TSP} should we use?

- Lot of research has been done on this. The value depends on district shape, approach to routing, etc.
- Euclidean Networks
  - k_{TSP} = 0.57 to 0.99 depending on clustering and size of N (MAPE ≈ 4%, MPE ≈ -1 %)
  - k_{TSP} = 0.765 commonly used and is a good approximation!
- Grid Networks
  - k_{TSP} = 0.97 to 1.15 depending on clustering and partitioning of district

1.9.12 Estimating Vehicle Tour Distances

Finding the total distance traveled on all tours, where:

- l = number of tours
- c = number of customer stops per tour and
- n=total number of stops = c*l

$$d_{TOUR} = 2d_{LineHaul} + \frac{ck_{TSP}}{\sqrt{\delta}} d_{AllTours} = ld_{TOUR} = 2ld_{LineHaul} + \frac{nk_{TSP}}{\sqrt{\delta}}$$

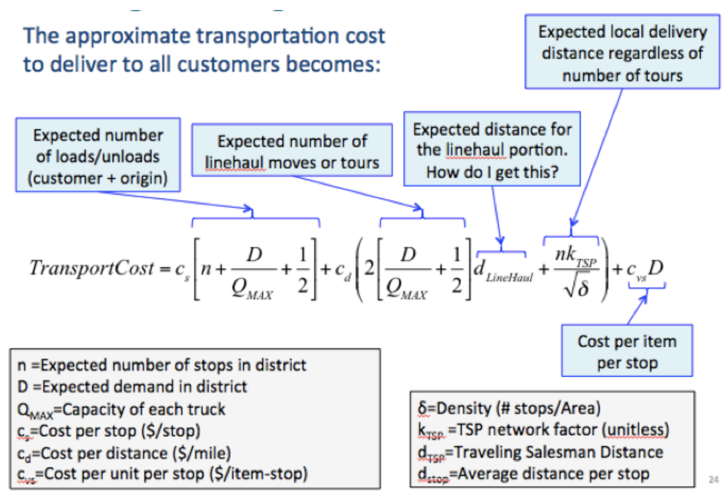
Minimize number of tours by maximizing vehicle capacity

$$l = [\frac{D}{Q_{MAX}}]^{+} d_{AllTours} = 2[\frac{D}{Q_{MAX}}]^{+} d_{LineHaul} + \frac{nk_{TSP}}{\sqrt{\delta}}$$

$[x]^{+}$ = lowest integer value > x. This is a step function

Estimate this with continuous function:

$[x]^{+} \approx$x + (1/2)



$$TransportCost = c_s\left[n + \frac{D}{Q_{MAX}} + \frac{1}{2}\right] + c_d\left(2\left[\frac{D}{Q_{MAX}} + \frac{1}{2}\right]d_{LineHaul} + \frac{nk_{TSP}}{\sqrt{\delta}}\right) + c_{vs}D$$

Key Points

- Review the basic components of algorithms.
- Recognize desired properties of an algorithm.
- Review different network algorithms.
- Recognize how to solve the Shortest Path Problem.
- Recognize which algorithms to use for the Traveling Salesman Problem.
- Recognize how to solve a Vehicle Routing Problem (Cluster First – Route Second).
- Review how to use approximations.
- Recognize steps to quick estimation.

1.10 Simulation

1.10.1 Summary

This section provides an overview of simulation. After a review of deterministic, prescriptive modeling (optimization, math programming) and predictive models (regression), simulation offers an approach for descriptive modeling. Simulations let you experiment with different decisions and see their outcomes. Supply chain managers typically use simulations to assess the likelihood of outcomes that may follow from different actions. This section outlines the steps in simulation from defining the variables, creating and running the model to refining the model. It offers insights into the benefits of using simulation for open ended questions but warns of its expensive and time-consuming nature.

Over the duration of the course we have reviewed several types of models including optimization, regression, and simulation. Optimization (LP, IP, MILP, NLP) is a prescriptive form of model that finds the "best solution" and/or provides a recommendation. Regression is a predictive form of model that measures the impact of independent variables on dependent variables. We now cover simulation, which captures the outcomes of different policies with an uncertain or stochastic environment.

Simulation

Simulation can be used in a variety of contexts; it is most useful in capturing complex system interactions, modeling system uncertainty, or generating data to analyze, describe and visualize interactions, outcomes, and sensitivities. There are several classes of simulation models including: system dynamics, Monte Carlo Simulation, discrete time simulation, and agent based simulations.

There are five main steps in developing a simulation study. Formulate and plan the study, collect data and define a model, construct the model and validate, make experimental runs, and analyze the output. The following will review the steps individually.

1.10.2 Steps in a Simulation Study

Formulate and plan the study

Once it has been determined that simulation is the appropriate tool for the problem under investigation, the next step is to formulate the plan and study. This involves a few main steps:

- Define the goals of the study and determine what needs to be solved.
- Develop a model where: for example, daily demand varies and a "production policy" will be applied.
- Based on the inputs, calculate the outputs: for example, based on demand and policy, calculate the profitability.
- Assess profitability and performance metrics of different policies.

Collect data and define a model

Once the plan has been formulated, the data needs to be collected and a model should be defined. If you face a situation where you have lack of sample data, you will need to determine the "range" of the variable(s) by talking to the stakeholders or experts to identify possible values. Some data can be derived from known distributions such as Poisson or Uniform/Triangular Distributions when little to no information is available.

If sample data is available, conduct steps as previously reviewed such as, creating histograms and calculating summary statistics. Then conduct a Chi-Square test to fit the sample to "traditional" distributions or use a "custom" empirical distribution such as discrete empirical (use % of observation as probabilities), or continuous – use histogram to compute probabilities of each range and then "uniform" within the range.

In Excel:

= CHISQ.TEST (ObservedRange,ActualRange) – returns p-value

= 1-CHISQ.DIST (Chi-square, Degrees of freedom, Cumulative) – returns p-value

Next steps are to:

- Determine relationship between various variables.
- Determine performance metrics.
- Collect data and estimate probability.

Construct model and validate
Make necessary inputs random, add a data table to automate runs of model, add summary statistics based on results from data table.

Generating random variables with the underlying principle of generating a random (or pseudo-random) number and transforming it to fit the desired distribution:

- Manual techniques: rolling die, turning a roulette wheel, random number tables.
- Excel
  - RAND () = continuous variable between 0 and 1
    - Generate random number u
    - For each random u, calculate a value y whose cumulative distribution function is equal to u; assign value y as the generated number:
      F(y) = P(y)=u
- Uniform Distribution $\sim U(a,b)$
  $\sim U(a,b) = a + (b–a)*RAND()$
  $NormalDistribution \sim N(\mu, \sigma)$
  $\sim N(\mu, \sigma)$ = NORM.INV(RAND(), μ, σ)

Validation of Model

Validation of Model – is the process of determining the degree to which a model and its associated data are an accurate representation of the real world for the intended use of the model. Different ways of validating model include comparing to historical data or getting an expert input. One primary method is parameter variability and sensitivity analysis:

- Generate statistical parameters with confidence intervals.
- Hypothesis Testing.

Make experimental runs

You will need to make multiple runs for each policy. Use hypothesis testing to evaluate the results. If spreadsheets contain a random input, we can use our data table to repeatedly analyze the model. An additional column for runs can be made.

Analyze output

Analyzing output deals with drawing inferences about the system model based on the simulation output. Need to ensure that the model output has maintained a proper balance between establishing model validation and statistical significance. Depending on the system examined, simulation will potentially be able to provide valuable insights with the output. The ability to draw inferences from results to make system improvements will be discussed further in future courses.

Learning Objectives

- Review the steps in developing a simulation model.
- To understand when to use a simulation method, and when not to.
- Recognize different kinds of simulation based approaches and when to apply them.

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Part II

SC1x - Supply Chain Fundamentals

2

Supply Chain Fundamentals

CONTENTS

2.1 Core Supply Chain Concepts

2.1.1 Summary

Virtually all supply chains are a combination of push and pull systems. A push system is where execution is performed ahead of an actual order so that the forecasted demand, rather than actual demand, has to be used in planning. A pull system is where execution is performed in response to an order so that the actual demand is known with certainty. The point in the process where a supply chain shifts from being push to pull is sometimes called the push/pull boundary or push/pull point. In manufacturing, the push/pull point is also known as the decoupling point (DP) or customer order decoupling point (CODP). The CODP coincides with an important stock point, where the customer order arrives (switching inventory based on a forecast to actual demand), and also allows to differentiate basic production systems: make-to-stock, assemble-to-order, make-to-order, or engineer-to-order.

Postponement is a common strategy to combine the benefits of push (product ready for demand) and pull (fast customized service) systems. Postponement is where the undifferentiated raw or components are "pushed" through a forecast, and the final finished and customized products are then "pulled".

Segmentation is a method of dividing a supply chain into two or more groupings where the supply chains operate differently and more efficiently and/or effectively. While there are no absolute rules for segmentation, there are some rules of thumb, such as: items should be homogeneous within the segment and heterogeneous across segments; there should be critical mass within each segment; and the segments need to be useful and communicable. A segment only makes sense if it does something different (planning, inventory, transportation etc.) from the other segments. The most common segmentation is for products using an ABC classification.

In an ABC segmentation, the products driving the most revenue (or profit or sales) are Class A items (the important few). Products driving very little revenue (or profit or sales) are Class C items (the trivial many), and the products in the middle are Class B. A common breakdown is the top 20% of items (Class A) generate 80% of the revenue, Class B is 30% of the products generating 15% of the revenue, and the Class C items generate less than 5% of the revenue while constituting 50% of the items.

Supply chains operate in uncertainty. Demand is never known exactly, for example. In order to handle and be able to analyze systems with uncertainty, we need to capture the distribution of the variable in question. When we are describing a random situation, say, the expected demand for pizzas on a Thursday night, it is helpful to describe the potential outcomes in terms of the central tendency (mean or median) as well as the dispersion (standard deviation, range). We will often characterize the distribution of potential outcomes as following a well-known function such as Normal and Poisson.

2.1.2 Key Concepts

Pull vs. Push Process

- Push — work performed in anticipation of an order (forecasted demand)
- Pull — execution performed in response to an order (demand known with certainty)
- Hybrid or Mixed — push raw products, pull finished product (postponement or delayed differentiation)
- Push/pull boundary point — point in the process where a supply chain shifts from being push to pull
- In manufacturing, also known as "decoupling point" (DP) or "customer order decoupling point" (CODP) — the point in the material flow where the product is linked to a specific customer
- Mass customization / Postponement — to delay the final assembly, customization, or differentiation of a product until as late as possible

Segmentation

- Differentiate products in order to match the right supply chain to the right product

- Products typically segmented on
  - Physical characteristics (value, size, density, etc.)
  - Demand characteristics (sales volume, volatility, sales duration, etc.)
  - Supply characteristics (availability, location, reliability, etc.)
- Rules of thumb for number of segments
  - Homogeneous—products within a segment should be similar
  - Heterogeneous—products across segments should be very different
  - Critical Mass—segment should be big enough to be worthwhile
  - Pragmatic—segmentation should be useful and communicable
- Demand follows a power law distribution, meaning a large volume of sales is concentrated in few products

Power Law

The distribution of percent sales volume to percent of SKUs (Stock Keeping Units) tends to follow a Power Law distribution ($y = ax^k$) where y is percent of demand (units or sales or profit), x is percent of SKUs, and a and k are parameters. The value for k should obviously be less than 1 since if k=1 the relationship is linear. In addition to segmenting according to products, many firms segment by customer, geographic region, or supplier. Segmentation is typically done using revenue as the key driver, but many firms also include variability of demand, profitability, and other factors, to include:

- Revenue = Average sales * Unit sales price
- Profit = Average sales * Margin
- Margin = Unit sales price – Unit cost

Handling Uncertainty

Uncertainty of an outcome (demand, transit time, manufacturing yield, etc.) is modeled through a probability distribution. We discussed two in the lesson: Poisson and Normal.

Normal Distribution ∼N(μ, σ)

This is the Bell Shaped distribution that is widely used by both practitioners and academics. While not perfect, it is a good place to start for most random variables that you will encounter in practice such as transit time and demand. The distribution is both continuous (it can take any number, not just integers or positive numbers) and is symmetric around its mean or average. Being symmetric additionally means the mean is also the median and the mode. The common notation that we will use to indicate that some value follows a Normal Distribution is ∼N(μ, σ) where mu, μ , is the mean and sigma, σ, is the standard deviation. Some books use the notation ∼N(μ, σ) showing the variance, σ^2, instead of the standard deviation. Just be sure which notation is being followed when you consult other texts.

The normal distribution is formally defined as:

$$f_x(x_0) = \frac{e^{\frac{-(x_0-\mu)^2}{2\sigma_x^2}}}{\sigma_x\sqrt{2\pi}}$$

We will also make use of the Unit Normal or Standard Normal Distribution. This is ∼N(0,1) where the mean is zero and the standard deviation is 1 (as is the variance, obviously). The chart below shows

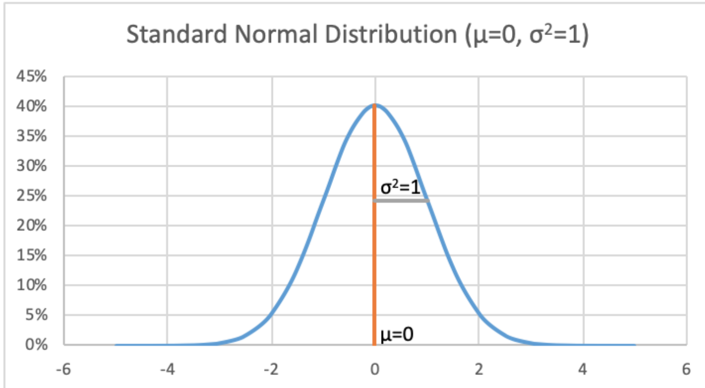

FIGURE 2.1: Standard Normal Distribution

the standard or unit normal distribution. We will be making use of the transformation from any Normal Distribution to the Unit Normal (See Figure 1).

We will make extensive use of spreadsheets (whether Excel or LibreOffice) to calculate probabilities under the Normal Distribution. The following functions are helpful:

- In Excel 2007 and above, NORM.DIST(x, μ, σ, true) = the probability that a random variable is less than or equal to x under the Normal Distribution ~N(μ, σ). So, that NORM.DIST(25, 20, 3, 1) = 0.952 which means that there is a 95.2% probability that a number from this distribution will be less than 25. For Excel 2003, use the above function without the period, i.e, NORMDIST(25, 20, 3, 1).
- In Excel 2007 and above, NORM.INV(probability, μ, σ) = the value of x where the probability that a random variable is less than or equal to it is the specified probability. So, NORM.INV(0.952, 20, 3) = 25. For Excel 2003, use the above function without the period, i.e NORMINV(0.952, 20, 3).

To use the Unit Normal Distribution ~N(0,1) we need to transform the given distribution by calculating *a* k value where k=(x - μ)/σ. This is sometimes called *a* z value in statistics courses, but in almost all supply chain and inventory contexts it is referred to as *a* k value. So, in our example, k = (25 – 20)/3 = 1.67. Why do we use the Unit Normal? Well, the k value is a helpful and convenient piece of information. The k is the number of standard deviations the value x is above (or below if it is negative) the mean. We will be looking at *a* number of specific values for k that are widely used as thresholds in practice, specifically:

- Probability ($x \leq 0.90$) where k = 1.28
- Probability ($x \leq 0.95$) where k = 1.645
- Probability ($x \leq 0.99$) where k = 2.33

Because the Normal Distribution is symmetric, there are also some common confidence intervals:

- $\mu \pm \sigma$ 68.3% — meaning that 68.3% of the values fall within 1 standard deviation of the mean,
- $\mu \pm 2\sigma$ 95.5% — 95.5% of the values fall within 2 standard deviations of the mean, and
- $\mu \pm 3\sigma$ 99.7% — 99.7% of the values fall within 3 standard deviations of the mean.

In a spreadsheets you can use the functions:

- **NORM.S.DIST(k,1)=** the probability that a random variable is less than k units above (or below) mean. For example, NORM.S.DIST(2.0, 1) = 0.977 meaning the 97.7% of the distribution is less than 2 standard deviations above the mean. For excel 2003, use the function without period, i.e, NORMDIST(k).

- **NORM.S.INV(probability)=** the value corresponding to the given probability. So that NORM.S.INV(0.977) = 2.0. If I then wanted to find the value that would cover 97.7% of a specific distribution, say where N(279, 46) I would just transform it. Since k=(x - μ)/σ for the transformation, I can simply solve for x and get: x = μ + kσ = 279 + (2.0)(46) = 371. This means that the random variable ~N(279, 46) will be equal or less than 371 for 97.7% of the time. For older versions of Excel, use NORMSINV(probability).

Poisson distribution ~Poisson (λ)

We will also use the Poisson (pronounced pwa-SOHN) distribution for modeling things like demand, stock outs, and other less frequent events. The Poisson, unlike the Normal, is discrete (it can only be integers ≥ 0), always positive, and non-symmetric. It is skewed right – that is, it has a long right tail. It is very commonly used for low value distributions or slow moving items. While the Normal Distribution has two parameters (mu and sigma), the Poisson only has one, lambda, λ.

Formally, the Poisson Distribution is defined as shown below:

$$p[x_0] = \text{Prob}[x = x_0] = \frac{e^{-\lambda}\lambda^{x_0}}{x_0!} \quad \text{for} \quad x_0 = 0, 1, 2, ...$$

$$F[x_0] = \text{Prob}[x \leq x_0] = \sum_{x=0}^{x_0} \frac{e^{-\lambda}\lambda^{x}}{x!}$$

The chart below (Figure 2) shows the Poisson Distribution for λ =3. The Poisson parameter λ is both the mean and the variance for the distribution! Note that λ does not have to be an integer.

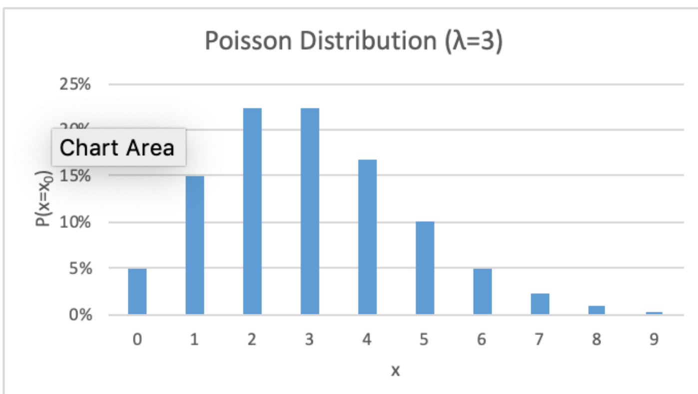

FIGURE 2.2: Poisson Distribution

In spreadsheets, the following functions are helpful:

- **POISSON(x_0, λ, false)** $\Longrightarrow P(\text{x} = \text{x}_0)$ = the probability that a random variable is equal to x_0 under the Poisson Distribution ~P(λ). So, that POISSON(2, 1.56, 0) = 0.256 which means that there is a 25.6% probability that a number from this distribution will be equal to 2.

- **POISSON(x_0, λ, true)** $\Longrightarrow P(\text{x} \leq x_0)$ = the probability that a random variable is less than or equal to x_0 under the Poisson Distribution ~P(λ). So, that POISSON(2, 1.56, 1) = 0.793 which means that there is a 79.3% probability that a number from this distribution will be less than or equal to 2. This is simply just the cumulative distribution function.

Uniform distribution ~U (a,b)

We will sometimes use the Uniform distribution, which has two parameters: a minimum value a and a maximum value b. Each point within this range is equally likely to occur. To find the cumulative probability for some value C, the probability that $x \leq c = (c-a)/(b-a)$, that is, the area from a to c minus the total area from a to b. The expected value or the mean is simply $(a+b)/2$ while the standard deviation is $= (b-a)/\sqrt{12}$.

Learning Objectives

- Identify and understand differences between push and pull systems.
- Understand why and how to segment supply chains by products, customers, etc.
- Ability to model uncertainty in supply chains, primarily, but not exclusively, in demand uncertainty.

References

Push/Pull Processes: Chopra & Meindl Chpt 1; Nahmias Chpt 7; Segmentation: Nahmias Chpt 5; Silver, Pyke, & Peterson Chpt 3; Ballou Chpt 3 Probability Distributions: Chopra & Meindl Chpt 12; Nahmias Chpt 5; Silver, Pyke, & Peterson App B

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2.2 Demand Forecasting

2.2.1 Summary

Forecasting is one of three components of an organization's Demand Planning, Forecasting, and Management process. Demand Planning answers the question "What should we do to shape and create demand for our product?" and concerns things like promotions, pricing, packaging, etc. Demand Forecasting then answers "What should we expect demand for our product to be given the demand plan in place?" The final component, Demand Management, answers the question, "How do we prepare for and act on demand when it materializes?" This concerns things like Sales& Operations Planning (S&OP) and balancing supply and demand.

Within the Demand Forecasting component, you can think of three levels, each with its own time horizon and purpose. Strategic forecasts (years) are used for capacity planning, investment strategies, etc. Tactical forecasts (weeks to months to quarters) are used for sales plans, short-term budgets, inventory planning, labor planning, etc. Finally, operations forecasts (hours to days) are used for production, transportation, and inventory replenishment decisions. The time frame of the action dictates the time horizon of the forecast.

Forecasting methods can be divided into being subjective (most often used by marketing and sales) or objective (most often used by production and inventory planners). Subjective methods can be further divided into being either Judgemental (someone somewhere knows the truth), such as sales force surveys, Delphi sessions, or expert opinions, or Experimental (sampling local and then extrapolating), such as customer surveys, focus groups, or test marketing. Objective methods are either Causal (there is an underlying relationship or reason) such as leading indicators, etc. or Time Series (there are patterns in the demand) such as exponential smoothing, moving average, etc. All methods have their place and their role. We will spend a lot of time on the objective methods but will also discuss the subjective ones as well.

Regardless of the forecasting method used, you will want to measure the quality of the forecast. The two major dimensions of quality are bias (a persistent tendency to over- or under-predict) and accuracy (closeness to the actual observations). No single metric does a good job capturing both dimensions, so it is worth having multiple.

2.2.2 Key Concepts

Forecasting Truisms

1. *Forecasts are always wrong* – Yes, point forecasts will never be completely perfect. The solution is to not rely totally on point forecasts. Incorporate ranges into your forecasts. Also you should try to capture and track the forecast errors so that you can sense and measure any drift or changes.
2. *Aggregated forecasts are more accurate than dis-aggregated forecasts* – The idea is that combining different items leads to a pooling effect that will in turn lessen the variability. The peaks balance out the valleys. The coefficient of variation (CV) is commonly used to measure variability and is defined as the standard deviation over the mean (CV=σ/μ). Forecasts are generally aggregated by SKU (a family of products versus an individual one), time (demand over a month versus over a single day), or location (demand for a region versus a single store).
3. *Shorter horizon forecasts are more accurate than longer horizon forecasts* – Essentially this means that forecasting tomorrow's temperature (or demand) is easier and probably more accurate than forecasting for a year from tomorrow. This is not the same as aggregating. It is all about the time between making the forecast and the event happening. Shorter is always better. This is where postponement and modularization helps. If we can somehow shorten the forecasting time for an end item, we will generally be more accurate.

Forecasting Metrics

There is a cost trade-off between cost of errors in forecasting and cost of quality forecasts that must be balanced. Forecast metric systems should capture bias and accuracy.

Notation:

A_t: Actual value for observation t
F_t: Forecasted value for observation t
e_t: Error for observation t, $e_t = A_t - F_t$
n: number of observations
μ: mean
σ: standard deviation
CV: Coefficient of Variation – a measure of volatility – CV=$\frac{\sigma}{\mu}$

Formulas:

Mean Deviation: MD= $\frac{\sum_{t=1}^{n} e_t}{n}$

Mean Absolute Deviation: MAD= $\frac{\sum_{t=1}^{n} |e_t|}{n}$

Mean Squared Error: MSE=$\frac{\sum_{t=1}^{n} {e_t}^2}{n}$

Root Mean Squared Error: RMSE=$\sqrt{\frac{\sum_{t=1}^{n} {e_t}^2}{n}}$

Mean Percent Error: MPE=$\frac{\sum_{t=1}^{n}\frac{e_t}{A_t}}{n}$

Mean Absolute Percent Error: MAPE=$\frac{\sum_{t=1}^{n}\frac{|e_t|}{A_t}}{n}$

Statistical Aggregation: $\sigma_{agg}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + ... + \sigma_n^2$

$\sigma_{agg} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + ... + \sigma_n^2}$

$\mu_{agg} = \mu_1 + \mu_2 + \mu_3 + ... + \mu_n$

Statistical Aggregation of n Distributions of Equal Mean and Variance:
$\sigma_{agg} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + ... + \sigma_n^2} = \sigma_{ind}\sqrt{n}$

$\mu_{agg} = \mu_1 + \mu_2 + \mu_3 + ... + \mu_n = n\mu_{ind}$

$CV_{agg} = \frac{\sigma\sqrt{n}}{\mu n} = \frac{\sigma}{\mu\sqrt{n}} = \frac{CV_{ind}}{\sqrt{n}}$

2.2.3 Time Series Analysis

Time Series is an extremely widely used forecasting technique for mid-range forecasts for items that have a long history or record of demand. Time series is essentially pattern matching of data that are distributed over time. For this reason, you tend to need a lot of data to be able to capture the components or patterns. Business cycles are more suited to longer range, strategic forecasting time horizons. Three important time series models:

- Cumulative – where everything matters and all data are included. This results in a very calm forecast that changes very slowly over time – thus it is more stable than responsive.
- Naïve – where only the latest data point matters. This results in very nervous or volatile forecast that can change quickly and dramatically – thus it is more responsive than stable.
- Moving Average – where we can select how much data to use (the last M periods). This is essentially the generalized form for both the Cumulative ($M = \infty$) and Naïve (M=1) models.

All three of these models are similar in that they assume stationary demand. Any trend in the underlying data will lead to severe lagging. These models also apply equal weighting to each piece of information that is included. Interestingly, while the M-Period Moving Average model requires M data elements for each SKU being forecast, the Naïve and Cumulative models only require 1 data element each.

Components of time series

- Level (a)
  - Value where demand hovers (mean)
  - Captures scale of the time series
  - With no other pattern present, it is a constant value
- Trend (b)
  - Rate of growth or decline
  - Persistent movement in one direction

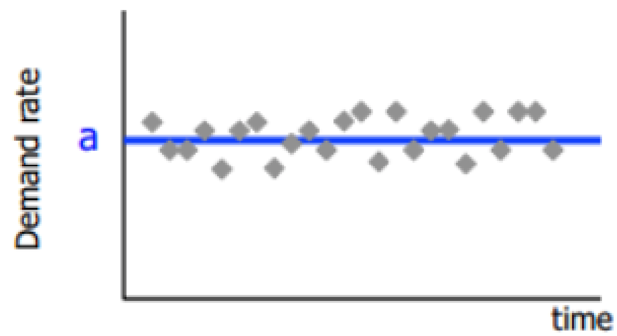

FIGURE 2.3: Level (a)

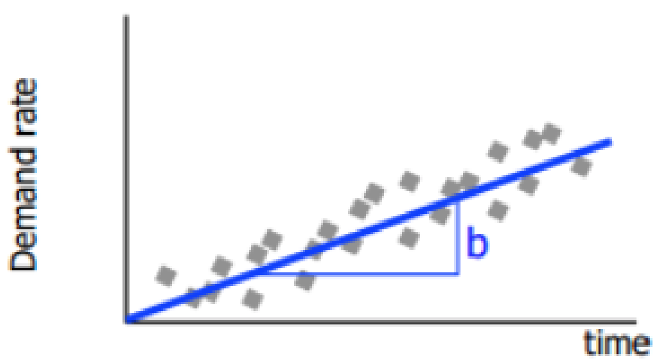

FIGURE 2.4: Trend (b)

  - Typically linear but can be exponential, quadratic, etc.

- Season Variations (F)
  - Repeated cycle around a known and fixed period
  - Hourly, daily, weekly, monthly, quarterly, etc. direction
  - Can be caused by natural or man-made forces

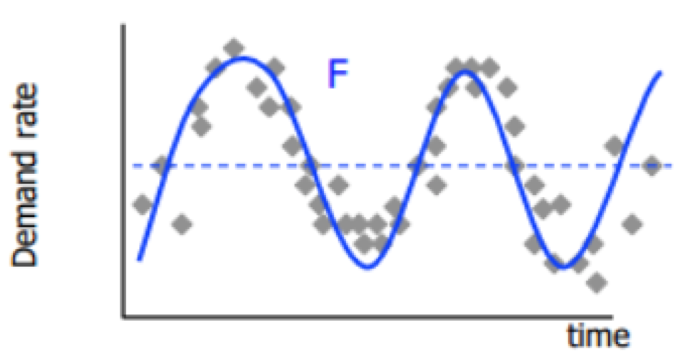

FIGURE 2.5: Season Variations (F)

- Random Fluctuation (e or ε)

- Remainder of variability after other components
- Irregular and unpredictable variations, noise

Notation:

x_t: Actual demand in period t
$\hat{x}_{t,t+1}$: Forecast for time t+1 made during time t
a: Level component
b: Linear trend component
Ft: Season index appropriate for period t
e_t: Error for observation t, $e_t = A_t - F_t$
t: Time period (0, 1, 2,. . . n)
Level Model: $x_t = a + e_t$
Trend Model: $x_t = a + bt + e_t$
Mix Level-Seasonality Model: $x_t = aF_t + e_t$
Mix Level-Trend-Seasonality Model: $x_t = (a + bt)F_t + e_t$

Formulas:

Time Series Models (Stationary Demand only):
Cumulative Model: $\hat{x}_{t,t+1} = \frac{\sum_i^t x_i}{t}$
Naïve Model: $\hat{x}_{t,t+1} = x_t$
M-Period Moving Average Forecast Model: $\hat{x}_{t,t+1} = \frac{\sum_{i=t+1-M}^t x_i}{M}$

- If M=t, we have the cumulative model where all data is included
- If M=1, we have the naïve model, where the last data point is used to predict the next data point

2.2.4 Exponential Smoothing

Exponential smoothing, as opposed to Cumulative, Naïve, and Moving Average, treats data differently depending on its age. The idea is that the value of data degrades over time so that newer observations of demand are weighted more heavily than older observations. The weights decrease exponentially as they age. Exponential models simply blend the value of new and old information.

The alpha factor (ranging between 0 and 1) determines the weighting for the newest information versus the older information. The "α" value indicates the value of "new" information versus "old" information:

- As $\alpha \to 1$, the forecast becomes more nervous, volatile, and naïve
- As $\alpha \to 0$, the forecast
- t becomes more calm, staid, and cumulative
- α can range from $0 \leq \alpha \leq 1$, but in practice, we typically see $0 \leq \alpha \leq 0.3$

The most basic exponential model, or Simple Exponential model, assumes stationary demand. Holt's Model is a modified version of exponential smoothing that also accounts for trend in addition to level. A new smoothing parameter, β, is introduced. It operates in the same way as the α.
We can also use exponential smoothing to dampen trend models to account for the fact that trends usually do not remain unchanged indefinitely as well as for creating a more stable estimate of the forecast errors.

Notation:

x_t : Actual demand in period t
$x_{t,t+1}$: Forecast for time t+1 made during time t
α : Exponential smoothing factor for level ($0 \leq \alpha \leq 1$)
β : Exponential smoothing factor for trend ($0 \leq \beta \leq 1$)
ϕ : Exponential smoothing factor for dampening ($0 \leq \phi \leq 1$)
ω : Mean Square Error trending factor ($0.01 \leq \omega \leq 0.1$)

Formulas:

Simple Exponential Smoothing Model (Level Only) – This model is used for stationary demand. The "new" information is simply the latest observation. The "old" information is the most recent forecast since it encapsulates the older information.

$$\hat{x}_{t,t+1} = \alpha x_t + (1 - \alpha)\hat{x}_{t-1,t}$$

Exponential Smoothing for Level & Trend – also known as Holt's Method, assumes a linear trend. The forecast for time t+τ made at time t is shown below. It is a combination of the latest estimates of the level and trend. For the level, the new information is the latest observation and the old information is the most recent forecast for that period – that is, the last period's estimate of level plus the last period's estimate of trend. For the trend, the new information is the difference between the most recent estimate of the level minus the second most recent estimate of the level. The old information is simply the last period's estimate of the trend.

$$\hat{x}_{t,t+\tau} = \hat{a}_t + \tau\hat{b}_t$$

$$\hat{a}_t = \alpha x_t + (1 - \alpha)(\hat{a}_{t-1} + \hat{b}_{t-1})$$

$$\hat{b}_t = \beta(\hat{a}_t - \hat{a}_{t-1}) + (1 - \beta)\hat{b}_{t-1}$$

Damped Trend Model with Level and Trend – We can use exponential smoothing to dampen a linear trend to better reflect the tapering effect of trends in practice.

$$\hat{x}_{t,t+\tau} = \hat{a}_t + \sum_{i}^{\tau} \varphi^i \hat{b}_t$$

$$\hat{a}_t = \alpha x_t + (1 - \alpha)(\hat{a}_{t-1} + \varphi\hat{b}_{t-1})$$

$$\hat{b}_t = \beta(\hat{a}_t - \hat{a}_{t-1}) + (1 - \beta)\varphi\hat{b}_{t-1}$$

Mean Square Error Estimate – We can also use exponential smoothing to provide a more robust or stable value for the mean square error of the forecast.

$$MSE_t = \omega(x_t - \hat{x}_{t-1,t})^2 + (1 - \omega)MSE_{t-1}$$

2.2.5 Exponential Smoothing with Holt-Winter

Seasonality

- For multiplicative seasonality, think of the F_i as "percent of average demand" for a period i
- The sum of the F_i for all periods within a season must equal P
- Seasonality factors must be kept current or they will drift dramatically. This requires a lot more book-keeping, which is tricky to maintain in a spreadsheet, but it is important to understand

Forecasting Model Parameter Initialization Methods

- While there is no single best method, there are many good ones
- Simple Exponential Smoothing
  - Estimate level parameter a_0 by averaging demand for first several periods
- Holt Model (trend and level)—must estimate both a_0 and b_0
  - Find a best fit linear equation from initial data
  - Use least squares regression of demand for several periods variations, noise
    * Dependent variable = demand in each time period = x_t
    * Independent variable = slope = β_1
    * Regression equation: $x_t = \beta_0 + \beta_1 t$
- Seasonality Models
  - Much more complicated, you need at least two season of data but preferably four or more
  - First determine the level for each common season period and then the demand for all periods
  - Set initial seasonality indices to ratio of each season to all periods

Notation:

x_t : Actual demand in period t
$x_{t,t+1}$: Forecast for time t+1 made during time t
α : Exponential smoothing factor $(0 \leq \alpha \leq 1)$
β : Exponential smoothing trend factor $(0 \leq \beta \leq 1)$
γ : Seasonality smoothing factor $(0 \leq \gamma \leq 1)$
F_t : Multiplicative seasonal index appropriate for period t
P : Number of time periods within the seasonality (note: $\sum_{i=1}^{P} \hat{F}_i = P$)

Forecasting Models

Holt-Winter Exponential Smoothing Model (Level, Trend, and Seasonality) – This model assumes a linear trend with a multiplicative seasonality effect over both level and trend. For the level estimate, the new information is again the "de-seasoned" value of the latest observation, while the old information is the old estimate of the level and trend. The estimate for the trend is the same as for the Holt model. The Seasonality estimate is the same as the Double Exponential smoothing model.

$$\hat{x}_{t,t+\tau} = (\hat{a}_t + \tau\hat{b}_t)\hat{F}_{t+\tau-P}$$
$$\hat{a}_t = \alpha(\frac{x_t}{\hat{F}_{t-P}}) + (1-\alpha)(\hat{a}_{t-1} + \hat{b}_{t-1})$$
$$\hat{b}_t = \beta(\hat{a}_t - \hat{a}_{t-1}) + (1-\beta)\hat{b}_{t-1}$$
$$\hat{F}_t = \gamma(\frac{x_t}{\hat{a}_t}) + (1-\gamma)\hat{F}_{t-P}$$

Double Exponential Smoothing (Seasonality and Level) – This is a multiplicative model in that the seasonality for each period is the product of the level and that period's seasonality factor. The new information for the estimate of the level is the "de-seasoned" value of the latest observation; that is, you are trying to remove the seasonality factor. The old information is simply the previous most recent estimate for level. For the seasonality estimate, the new information is the "de-leveled" value of the latest observation; that is, you try to remove the level factor to understand any new seasonality. The old information is simply the previous most recent estimate for that period's seasonality.

$$\hat{x}_{t,t+\tau} = \hat{a}_t \hat{F}_{t+\tau-P}$$
$$\hat{a}_t = \alpha(\frac{x_t}{\hat{F}_{t-P}}) + (1-\alpha)(\hat{a}_{t-1})$$
$$\hat{F}_t = \gamma(\frac{x_t}{\hat{a}_t}) + (1-\gamma)\hat{F}_{t-P}$$

Normalizing Seasonality Indices – This should be done after each forecast to ensure the seasonality does not get out of synch. If the indices are not updated, they will drift dramatically. Most software packages will take care of this – but it is worth checking.

$$\hat{F}_t^{NEW} = \hat{F}_t^{OLD}\left(\frac{P}{\sum_{i=t+1-P}^{t}\hat{F}_i^{OLD}}\right)$$

2.2.6 Special Cases

There are different types of new products and the forecasting techniques differ according to their type. The fundamental idea is that if you do not have any history to rely on, you can look for history of similar products and build one.

When the demand is very sparse, such as for spare parts, we cannot use traditional methods since the estimates tend to fluctuate dramatically. Croston's method can smooth out the estimate for the demand.

New Product Types

- Not all new products are the same. We can roughly classify them into the following six categories (listed from easiest to forecast to hardest):
  - Cost Reductions: Reduced price version of the product for the existing market
  - Product Repositioning: Taking existing products/services to new markets or applying them to a new purpose (aspirin from pain killer to reducing effects of a heart attack)
  - Line Extensions: Incremental innovations added to complement existing product lines (Vanilla Coke, Coke Zero) or Product Improvements: New, improved versions of existing offering targeted to the current market—replaces existing products (next generation of product)
  - New-to-Company: New market/category for the company but not to the market (Apple iPhone or iPod)
  - New-to-World: First of their kind, creates new market, radically different (Sony Walkman, Post-it notes, etc.)
- While they are a pain to forecast and to launch, firms introduce new products all the time – this is because they are the primary way to increase revenue and profits (See Figure 6)

New Product Development Process

All firms use some version of the process shown below to introduce new products. This is sometimes called the stage-gate or funnel process. The concept is that lots of ideas come in on the left and very few final products come out on the right. Each stage or hurdle in the process winnows out the winners from the losers and is used to focus attention on the right products. The scope and scale of forecasting changes along the process as noted in Figure 7.

Type of New Product	Percent of Introductions	Forecast Accuracy (1-MAPE)	Launch Cycle Length	Success Rate
New-to-World	8-10%	40%	104 weeks	38-65%
New-to-Company	17-20%	47%		
Line Extensions	21-26%	54-62%	62/29* weeks	55-77%
Product Improvements	26-36%	65%		
Product Re-Positioning	5-7%	54-65%	N/A	66-79%
Cost Reductions	10-11%	72%		

*Major revisions/incremental improvements about evenly split

FIGURE 2.6: New product introductions
Source: Adapted from Cooper, Robert (2001) Winning at New Products, Kahn, Kenneth (2006) New Product forecasting, and PDMA (2004) New Product Development Report.)

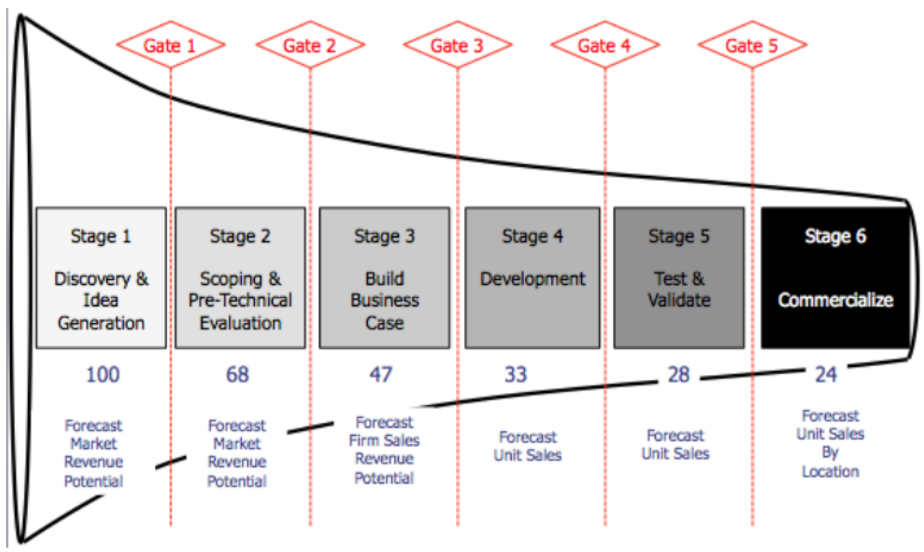

FIGURE 2.7: New product development process

Forecasting Models Discussed

New Product – "Looks-Like" or Analogous Forecasting

- Perform by looking at comparable product launches and create a week-by-week or month-by-month sales record.
- Then use the percent of total sales in each time increment as a trajectory guide.
- Each launch should be characterized by product type, season of introduction, price, target market demographics, and physical characteristics.

Intermittent or Sparse Demand – Croston's Method

- Used for products that are infrequently ordered in large quantities, irregularly ordered, or ordered in different sizes.
- Croston's Method separates out the demand and model—unbiased and has lower variance than simple smoothing.

- Cautions: infrequent ordering (and updating of model) induces a lag to responding to magnitude changes.

Notation:

x_t : Demand in period t
y_t : 1 if transaction occurs in period t, =0 otherwise
z_t : Size (magnitude) of transaction in time t
n_t : Number of periods since last transaction
α : Smoothing parameter for magnitude
β : Smoothing parameter for transaction frequency

Formulas

Croston's Method

We can use Croston's method when demand is intermittent. It allows us to use the traditional exponential smoothing methods. We assume the Demand Process is $x_t = y_t z_t$ and that demand is independent between time periods, so that the probability that a transaction occurs in the current time period is 1/n:

$$Prob(y_t = 1) = \frac{1}{n} \quad and \quad Prob(y_t = 0) = 1 - \frac{1}{n}$$

Updating Procedure for $\hat{z}_t$ and $\hat{n}_t$:
If $x_t = 0$ (no transaction occurs), then

$$\hat{z}_t = \hat{z}_{t-1} \quad and \quad \hat{n}_t = \hat{n}_{t-1}$$

If $x_t > 0$ (transaction occurs), then

$$\hat{z}_t = \alpha x_t + (1-\alpha)\hat{z}_{t-1}$$
$$\hat{n}_t = \beta n_t + (1-\beta)\hat{n}_{t-1}$$

Updating Procedure for n_t:

$$n_t = \begin{cases} 1 & \text{if} \quad x_{t-1} > 0 \\ n_{t-1} + 1 & if \quad x_{t-1} = 0 \end{cases}$$

Forecast:

$$\hat{x}_{t,t+1} = \frac{\hat{z}_t}{\hat{n}_t}$$

Learning Objectives

- Forecasting is part of the entire Demand Planning and Management process.
- Range forecasts are better than point forecasts, aggregated forecasts are better than dis-aggregated, and shorter time horizons are better than longer.
- Forecasting metrics need to capture bias and accuracy.
- Understand how to initialize a forecast.
- Understand that Time Series is a useful technique when we believe demand follows certain repeating patterns.
- Recognize that all time series models make a trade-off between being naïve (using only the last most recent data) or cumulative (using all of the available data).

- Understand how exponential smoothing treats old and new information differently.
- Understand how changing the alpha or beta smoothing factors influences the forecasts.
- Understand how seasonality can be handled within exponential smoothing.
- Understand why demand for new products need to be forecasted with different techniques.
- Learn how to use basic Diffusion Models for new product demand and how to forecast intermittent demand using Croston's Method.
- Understand how the typical new product pipeline process (stage-.-gate) works and how forecasting fits in.

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Also, I recommend checking out the Institute of Business Forecasting & Planning (https://ibf.org/) and their Journal of Business Forecasting.

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2.3 Inventory Management

2.3.1 Summary

Inventory management is at the core of all supply chain and logistics management. There are many reasons for holding inventory including minimizing the cost of controlling a system, buffering against uncertainties in demand, supply, delivery and manufacturing, as well as covering the time required for any process. Having inventory allows for a smoother operation in most cases since it alleviates the need to create product from scratch for each individual demand. Inventory is the result of a push system where the forecast determines how much inventory of each item is required.

There is, however, a problem with having too much inventory. Excess inventory can lead to spoilage, obsolescence, and damage. Also, spending too much on inventory limits the resources available for other activities and investments. Inventory analysis is essentially the determination of the right amount of inventory of the right product in the right location in the right form. Strategic decisions cover the inventory implications of product and network design. Tactical decisions cover deployment and determine what items to carry, in what form (raw materials, work-in-process, finished goods, etc.), and where. Finally, operational decisions determine the replenishment policies (when and how much) of these inventories.

We seek the Order Replenishment Policy that minimizes these total costs and specifically the Total Relevant Costs (TRC). A cost component is considered relevant if it impacts the decision at hand and we can control it by some action. A Replenishment Policy essentially states two things: the quantity to be ordered, and when it should be ordered. As we will see, the exact form of the Total Cost Equation used depends on the assumptions we make in terms of the situation. There are many different assumptions inherent in any of the models we will use, but the primary assumptions are made concerning the form of the demand for the product (whether it is constant or variable, random or deterministic, continuous or discrete, etc.).

2.3.2 Key Concepts

Reasons to Hold Inventory

- Cover process time
- Allow for uncoupling of processes
- Anticipation/Speculationelayed differentiation)
- Minimize control costs
- Buffer against uncertainties such as demand, supply, delivery, and manufacturing

Inventory Decisions

- Strategic supply chain decisions are long term and include decisions related to the supply chain such as potential alternatives to holding inventory and product design.
- Tactical are made within a month, a quarter or a year and are known as deployment decisions such as what items to carry as inventory, in what form to carry items and how much of each item to hold and where.
- Operational level decisions are made on daily, weekly or monthly basis and replenishment decisions such as how often to review inventory status, how often to make replenishment decisions and how large replenishment should be. The replenishment decisions are critical to determine how the supply chain is set up.

Inventory Classification

- Financial/Accounting Categories: Raw Materials, Work in Progress (WIP), Components/Semi-Finished Goods and Finished Goods. This category does not help in tracking opportunity costs and how one may wish to manage inventory.
- Functional (See Figure 8):
  - Cycle Stock – Amount of inventory between deliveries or replenishments
  - Safety Stock – Inventory to cover or buffer against uncertainties
  - Pipeline Inventory – Inventory when order is placed but has not yet arrived

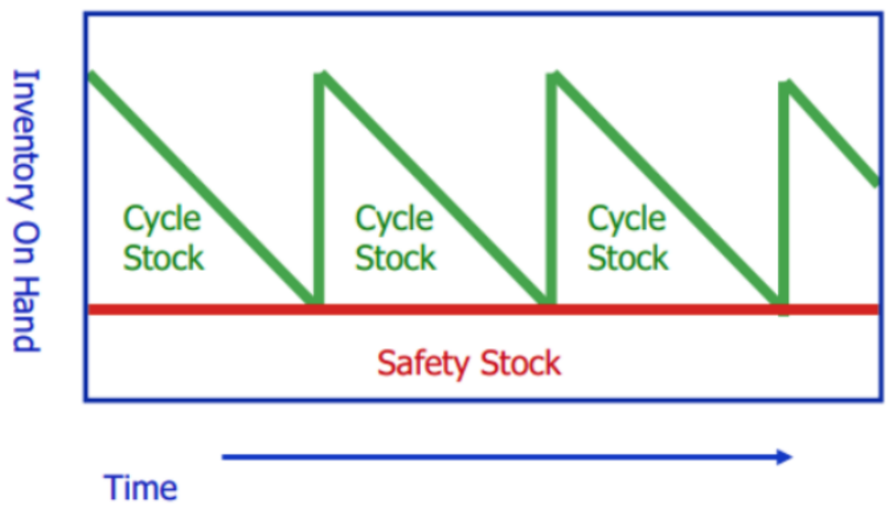

FIGURE 2.8: Inventory chart: Depiction of functional inventory classifications

Relevant Costs

The Total Cost (TC) equation is typically used to make the decisions of how much inventory to hold and how to replenish. It is the sum of the Purchasing, Ordering, Holding, and Shortage costs. The Purchasing costs are usually variable or per-item costs and cover the total landed cost for acquiring that product – whether from internal manufacturing or purchasing it from outside.

Total cost = Purchase (Unit Value) Cost + Order (Set Up) Cost + Holding (Carrying) Cost + Shortage (stock-out) Cost

- Purchase: Cost per item or total landed cost for acquiring product.
- Ordering: It is a fixed cost and contains cost to place, receive and process a batch of good including processing invoicing, auditing, labor, etc. In manufacturing this is the set up cost for a run.
- Holding: Costs required to hold inventory such as storage cost (warehouse space), service costs (insurance, taxes), risk costs (lost, stolen, damaged, obsolete), and capital costs (opportunity cost of alternative investment).
- Shortage: Costs of not having an item in stock (on-hand inventory) to satisfy a demand when it occurs, including backorder, lost sales, lost customers, and disruption costs. Also known as the penalty cost.

A cost is relevant if it is controllable and it applies to the specific decision being made.

Notation:

c : Purchase cost (\$/unit)
c_t : Ordering Costs (\$/order)
h : Holding rate – usually expressed as a percentage (\$/\$ value/time)
c_e : Excess holding Costs (\$/unit-time); also equal to ch
c_s : Shortage costs (\$/unit)
TRC : Total Relevant Costs – the sum of the relevant cost components
TC : Total Costs – the sum of all four cost elements

2.3.3 Economic Order Quantity (EOQ)

The Economic Order Quantity or EOQ is the most influential and widely used (and sometimes misused!) inventory model in existence. While very simple, it provides deep and useful insights. Essentially, the EOQ is a trade-off between fixed (ordering) and variable (holding) costs. It is often called Lot-Sizing as well. The minimum of the Total Cost equation (when assuming demand is uniform and deterministic) is the EOQ or Q^*. The Inventory Replenishment Policy becomes "Order Q^* every T^* time periods" which under our assumptions is the same as "Order Q^* when Inventory Position (IP)=0".

Like Wikipedia, the EOQ is a GREAT place to start, but not necessarily a great place to finish. It is a good first estimate because it is exceptionally robust. For example, a 50% increase in Q over the optimal quantity (Q^*) only increases the TRC by $\sim$ 8%!

While very insightful, the EOQ model should be used with caution as it has restrictive assumptions (uniform and deterministic demand). It can be safely used for items with relatively stable demand and is a good first-cut "back of the envelope" calculation in most situations. It is helpful to develop insights in understanding the trade-offs involved with taking certain managerial actions, such as lowering the ordering costs, lowering the purchase price, changing the holding costs, etc.

EOQ Model

- Assumptions
  - Demand is uniform and deterministic.
  - Lead time is instantaneous (0) – although this is not restrictive at all since the lead time, L, does not influence the Order Size, Q.
  - Total amount ordered is received.
- Inventory Replenishment Policy
  - Order Q^* units every T^* time periods.
  - Order Q^* units when inventory on hand (IOH) is zero.
- Essentially, the Q^* is the Cycle Stock for each replenishment cycle. It is the expected demand for that amount of time between order deliveries.

Notation:

c : Purchase cost (\$/unit)
c_t : Ordering Costs (\$/order)
c_e : Excess holding Costs (\$/unit/time); equal to ch
c_s : Shortage costs (\$/unit)
D : Demand (units/time)
DA : Actual Demand (units/time)
DF : Forecasted Demand (units/time)

h : Carrying or holding cost ($/inventory $/time)
Q : Replenishment Order Quantity (units/order)
Q^* : Optimal Order Quantity under EOQ (units/order)
Q_A^* : Optimal Order Quantity with Actual Demand (units/order)
Q_F^* : Optimal Order Quantity with Forecasted Demand (units/order)
T : Order Cycle Time (time/order)
T^* : Optimal Time between Replenishments (time/order)
N : Orders per Time or 1/T (order/time)
TRC(Q) : Total Relevant Cost ($/time)
TC(Q) : Total Cost ($/time)

Formulas

Total Costs: TC = Purchase + Order + Holding + Shortage

This is the generic total cost equation. The specific form of the different elements depends on the assumptions made concerning the demand, the shortage types, etc.

$$TC(Q) = cD + c_t\frac{D}{Q} + c_e\frac{Q}{2} + c_s E[UnitsShort]$$

Total Relevant Costs: TRC = Order + Holding

The purchasing cost and the shortage costs are not relevant for the EOQ because the purchase price does not change the optimal order quantity (Q*) and since we have deterministic demand, we will not stock out.

$$TRC(Q) = c_t\frac{D}{Q} + c_e\frac{Q}{2}$$

Optimal Order Quantity (Q^*)

Recall that this is the First Order condition of the TRC equation – where it is a global minimum.

$$Q^* = \sqrt{\frac{2c_tD}{c_e}}$$

Optimal Time between Replenishments

Recall that $T^* = Q^*/D$. That is, the time between orders is the optimal order size divided by the annual demand. Similarly, the number of replenishments per year is simply $N^* = 1/T^* = D/Q^*$. Plugging in the actual Q^* gives you the formula below.

$$T^* = \sqrt{\frac{2c_t}{Dc_e}}$$

Note: Be sure to put T^* into units that make sense (days, weeks, months, etc.). Don't leave it in years!

Optimal Total Costs

Adding the purchase cost to the TRC(Q^*) costs gives you the TC(Q^*). We still assume no stock out costs.

$$TC(Q^*) = cD + \sqrt{2c_tc_eD}$$

Optimal Total Relevant Costs

Plugging the Q^* back into the TRC equation and simplifying gives you the formula below.

$$TRC(Q^*) = \sqrt{2c_t c_e D}$$

Sensitivity Analysis

The EOQ is very robust. The following formulas provide simple ways of calculating the impact of using a non-optimal Q, an incorrect annual Demand D, or a non-optimal time interval, T.

EOQ Sensitivity with Respect to Order Quantity
The equation below calculates the percent difference in total relevant costs to optimal when using a non-optimal order quantity (Q):

$$\frac{TRC(Q)}{TRC(Q^*)} = \left(\frac{1}{2}\right)\left(\frac{Q^*}{Q} + \frac{Q}{Q^*}\right)$$

Note: If optimal quantity does not make sense, it is always better to order little more rather ordering little less.

EOQ Sensitivity with Respect to Demand
The equation below calculates the percent difference in total relevant costs to optimal when assuming an incorrect annual demand (D_F) when in fact the actual annual demand is D_A:

$$\frac{TRC(Q_F^*)}{TRC(Q_A^*)} = \left(\frac{1}{2}\right)\left(\sqrt{\frac{D_A}{D_F}} + \sqrt{\frac{D_F}{D_A}}\right)$$

EOQ Sensitivity with Respect to Time Interval between Orders
The equation below calculates the percent difference in total relevant costs to optimal when using a non-optimal replenishment time interval (T). This will become very important when finding realistic replenishment intervals. The Power of Two Policy shows that ordering in increments of 2^k time periods, we will stay within 6% of the optimal solution. For example, if the base time period is one week, then the Power of Two Policy would suggest ordering every week (2^0) or every two weeks (2^1) or every four weeks (2^2) or every eight weeks (2^3) etc. Select the interval closest to one of these increments.

$$\frac{TRC(T)}{TRC(T^*)} = \left(\frac{1}{2}\right)\left(\frac{T}{T^*} + \frac{T^*}{T}\right)$$

2.3.4 Economic Order Quantity (EOQ) Extensions

The Economic Order Quantity can be extended to cover many different situations, three extensions include: lead-time, volume discounts, and finite replenishment or EPQ.

We developed the EOQ previously assuming the rather restrictive (and ridiculous) assumption that lead-time was zero. That is, instantaneous replenishment like on Star Trek. However, including a non-zero lead time while increasing the total cost due to having pipeline inventory will NOT change the calculation of the optimal order quantity, Q^*. In other words, lead-time is not relevant to the determination of the needed cycle stock.

Volume discounts are more complicated. Including them makes the purchasing costs relevant since they now impact the order size. We discussed three types of discounts: All-Units (where the discount applies to all items purchased if the total amount exceeds the break point quantity), Incremental (where the discount only applies to the quantity purchased that exceeds the breakpoint quantity), and One-Time (where a one-time-only discount is offered and you need to determine the optimal quantity to procure as an advance buy). Discounts are exceptionally common in practice as they are used to incentivize buyers to purchase more or to order in convenient quantities (full pallet, full truckload, etc.).

A price **break** point is the minimum quantity required to get a price discount.

Finite Replenishment is very similar to the EOQ model, except that the product is available at a certain production rate rather than all at once. In the lesson we show that this tends to reduce the average inventory on hand (since some of each order is manufactured once the order is received) and therefore increases the optimal order quantity.

- Lead time is greater than 0 (order not received instantaneously)
  - Inventory Policy:
    - Order Q^* units when IP=DL
    - Order Q^* units every T^* time periods
- Discounts
  - All Units Discount—Discount applies to all units purchased if total amount exceeds the break point quantity
  - Incremental Discount—Discount applies only to the quantity purchased that exceeds the break point quantity
  - One-Time-Only Discount—A one-time-only discount applies to all units you order right now (no quantity minimum or limit)
- Finite Replenishment
  - Inventory becomes available at a rate of P units/time rather than all at one time
  - If Production rate approach infinity, model converges to EOQ

Notation:

c : Purchase cost ($/unit)
c_e : Excess holding Costs ($/unit/time); Equal to ch
c_s : Shortage costs ($/unit)
c_g : One Time Good Deal Purchase Price ($/unit)
F_i : Fixed Costs Associated with Units Ordered below Incremental Discount Breakpoint i
D : Demand (units/time)
D_A : Actual Demand (units/time)
D_F : Forecasted Demand (untis/time)
h : Carrying or holding cost ($/inventory $/time)
L : Order Lead time
Q : Replenishment Order Quantity (units/order)
Q^* : Optimal Order Quantity under EOQ (units/order)
Q_i : Breakpoint for quantity discount for discount i (units per order)
Q_g : One Time Good Deal Order Quantity
P : Production (units/time)
T : Order Cycle Time (time/order)
T^*: Optimal Time between Replenishments (time/order)
N : Orders per Time or 1/T (order/time)
TRC(Q) : Total Relevant Cost ($/time)
TC(Q) : Total Cost ($/time)

Formulas

Inventory Position
Inventory Position (IP) = Inventory on Hand (IOH) + Inventory on Order (IOO) – Back Orders (BO) – Committed Orders (CO)

Inventory on Order (IOO) is the inventory that has been ordered, but not yet received. This is inventory in transit and also knows as Pipeline Inventory (PI).

Average Pipeline Inventory

Average Pipeline Inventory (API), on average, is the annual demand times the lead time. Essentially, every item spends L time periods in transit. Both D and L should have consistent time units.

$$API = DL$$

Total Cost including Pipeline Inventory

The TC equation changes slightly if we assume a non-zero lead time and include the pipeline inventory.

$$TC(Q) = cD + c_t\left(\frac{D}{Q}\right) + c_e\left(\frac{Q}{2} + DL\right) + c_s E[UnitsShort]$$

Note that as before, though, the purchase cost, shortage costs, and now pipeline inventory is not relevant to determining the optimal order quantity, Q^*:

$$Q^* = \sqrt{\frac{2c_t D}{c_e}}$$

Discounts

If we include volume discounts, then the purchasing cost becomes relevant to our decision of order quantity.

All Units Discounts

Discount applies to all units purchased if total amount exceeds the break point quantity. The procedure for a single range All Units quantity discount (where new price is c_1 if ordering at least Q_1 units) is as follows:

1. Calculate Q^*_{C0}, the EOQ using the base (non-discounted) price, and Q^*_{C1}, the EOQ using the first discounted price
2. If $Q^*_{C1} \geq Q_1$, the breakpoint for the first all units discount, then order Q^*_{C1} since it satisfies the condition of the discount. Otherwise, go to step 3.
3. Compare the TRC(Q^*_{C0}), the total relevant cost with the base (non-discounted) price, with TRC(Q_1), the total relevant cost using the discounted price (c_1) at the breakpoint for the discount. If TRC(Q^*_{C0}) < TRC(Q_1), select Q^*_{C0}, otherwise order Q_1.

Note that if there are more discount levels, you need to check this for each one.

$$c = c_0 \quad for \quad 0 \leq Q \leq Q_1 \quad and \quad c = c_1 \quad for \quad Q_1 \leq Q$$

$$TRC = Dc_0 + c_t\left(\frac{D}{Q}\right) + c_0\left(\frac{hQ}{2}\right) \quad for \quad 0 \leq Q \leq Q_1$$

$$TRC = Dc_1 + c_t\left(\frac{D}{Q}\right) + c_1\left(\frac{hQ}{2}\right) \quad for \quad Q_1 \leq Q$$

Note: All units discount tend to raise cycle stock in the supply chain by encouraging retailers to increase the size of each order. This makes economic sense for the manufacturer, especially when he incurs a very high fixed cost per order.

Incremental Discounts

Discount applies only to the quantity purchased that exceeds the break point quantity. The procedure for a multi-range Incremental quantity discount (where if ordering at least Q_1 units, the new price for the $Q - Q_1$ units is c_1) is as follows:

1. Calculate the Fixed cost per breakpoint, F_i,
2. Calculate the Q_i^* for each discount range i (to include the F_i)
3. Calculate the TRC for all discount ranges where the $Q_{i-1} < Q_i^* < Q_{i+1}$, that is, if it is in range.
4. Select the discount that provides the lowest TRC

The effective cost, c_i^e, can be used for the TRC calculations.

$$F_0 = 0; F_i = F_{i-1} + (c_{i-1} - c_i)Q_i$$

$$Q^* = \sqrt{\frac{2D(c_t + F_i)}{hc_i}}$$

$$c_i^e = c_i + \frac{F_i}{Q_i^*}$$

x

One Time Discount

This is a less common discount – but it does happen. A one time only discount applies to all units you order right now (no minimum quantity or limit). Simply calculate the Q_g^* and that is your order quantity. If $Q_g^* = Q^*$ then the discount does not make sense. If you find that $Q_g^* < Q^*$, you made a mathematical mistake – check your work!

$$TC = (CycleTime)(TC^* + PurchaseCost) = \left(\frac{Q_g}{D}\right)\sqrt{2c_thcD} + \left(\frac{Q_g}{D}\right)cD$$

$$Savings = TC - TC_{SP}$$

$$Savings = \left(\left(\frac{Q_g}{D}\right)\sqrt{2c_thcD} + \left(\frac{Q_g}{D}\right)cD\right) - \left(c_gQ_g + hc_g\left(\frac{Q_g}{2}\right)\left(\frac{Q_g}{D}\right) + c_t\right)$$

$$Q_g^* = \frac{Q^*ch + D(c - c_g)}{hc_g}$$

Finite Replenishment or Economic Production Quantity

One can think of the EPQ equations as generalized forms where the EOQ is a special case where P=infinity. As the production rate decreases, the optimal quantity to be ordered increases. However, note that if $P < D$, this means the rate of production is slower than the rate of demand and that you will never have enough inventory to satisfy demand.

$$TRC[Q] = \frac{c_tD}{Q} + \frac{Q\left(1 - \frac{D}{P}\right)hc}{2}$$

$$EPQ = \sqrt{\frac{2c_tD}{hc(1 - \frac{D}{P})}} = \frac{EOQ}{\sqrt{(1 - \frac{D}{P})}}$$

EOQ with Planned Back Orders

A planned backorder is where we stock out on purpose knowing that customers will wait, although we do incur a penalty cost, c_s, for stocking out. Why do we accept to stock out? Because, sometimes it is very costly to have enough inventory on hand to always satisfy demand, and customers are willing to wait. From this, we develop the idea of the critical ratio (CR), which is the ratio of the c_s (the cost of shortage or having too little product) to the ratio of the sum of c_s and c_e (the cost of having too much or an excess of product). The critical ratio, by definition, ranges between 0 and 1 and is a good metric of level of service. A high CR indicates a desire to stock out less frequently. If c_s is infinity, it means we do not allow for shortage and the model becomes the conventional EOQ. As cs gets smaller, the Q^*_{PBO} gets larger and a larger percentage is allowed to be backordered – since the penalty for stocking out gets reduced.

Formulas

EOQ with Planned Backorders
This is an extension of the standard EOQ with the ability to allow for backorders at a penalty of c_s.

$$TRC(Q,b) = c_t\left(\frac{D}{Q}\right) + c_e\left(\frac{(Q-b)^2}{2Q}\right) + c_s\left(\frac{b^2}{2Q}\right)$$

$$Q^*_{PBO} = \sqrt{\frac{2c_tD}{c_e}}\sqrt{\frac{c_s+c_e}{c_s}} = Q^*\sqrt{\frac{(c_s+c_e)}{c_s}} = Q^*\sqrt{\frac{1}{CR}}$$

$$b^* = \frac{c_eQ^*_{PBO}}{(c_s+c_e)} = \left(1-\frac{c_s}{(c_s+c_e)}\right)Q^*_{PBO}$$

$$T^*_{PBO} = \frac{Q^*PBO}{D}$$

Order Q^*_{PBO} when IOH = -b^*; Order Q^*_{PBO} every T^*_{PBO} time periods.
Where:
Q^*_{PBO} is the Optimal Order Quantity with Planned backorders

2.3.5 Introduction to Stochastic Demand

Deterministic versus Probabilistic Demand: Differences and Implications

Empirical Distribution

$$\mu = \sum_i^n p_i x_i$$

$$\sigma = \sqrt{\sum_i^n p_i(x_i-\mu)^2} = \sqrt{E[X^2]-E[X]^2}$$

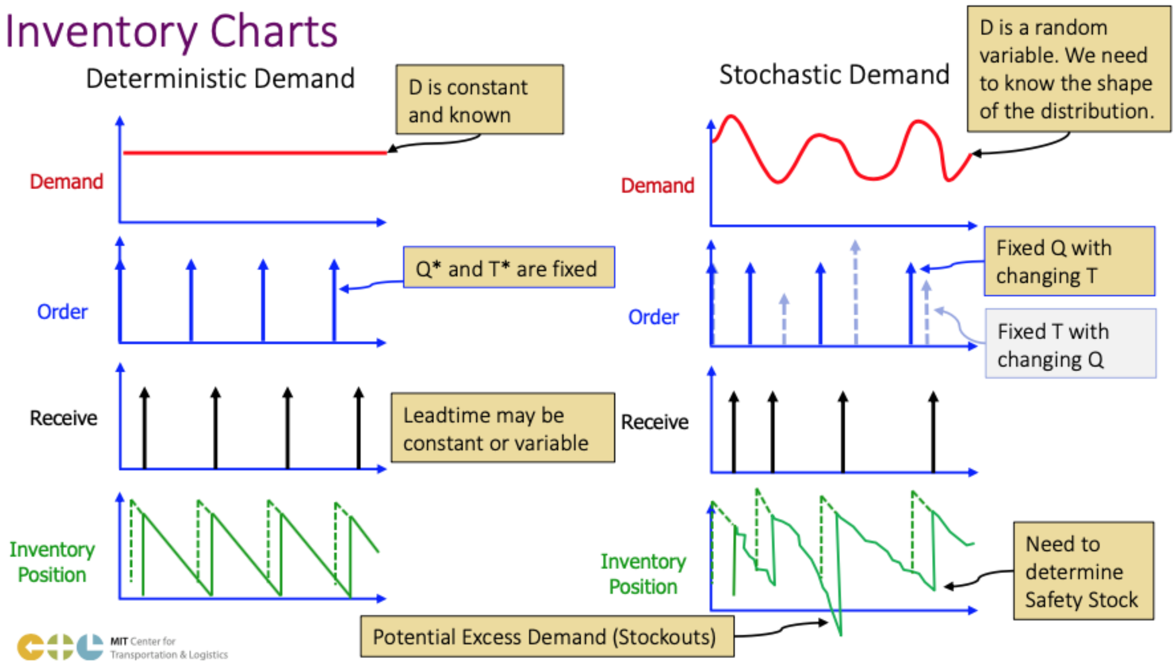

FIGURE 2.9: Inventory Under Deterministic vs Stochastic Demand

Theoretical Probability Distributions

Rules of Thumb for Selecting Demand Distributions

1. If Demand is all positive integers of low value (e.g., 1, 2, 3, 4 etc.) then use Poisson.
2. If Demand is larger numbers (says > 20), integrality is not critical and CV is small (∼<= 0.3) then use Normal.
3. Looks relatively even over a specified range then use Uniform.
4. Has no reliable data or there is only anecdotal memories, then use Triangle.

2.3.6 Single Period Inventory Models

The single period inventory model is second only to the economic order quantity in its widespread use and influence. Also referred to as the Newsvendor (Newsboy) model, the single period model differs from the EOQ in three main ways. First, while the EOQ assumes uniform and deterministic demand, the single-period model allows demand to be variable and stochastic (random). Second, while the EOQ assumes a steady state condition (stable demand with essentially an infinite time horizon), the single-period model assumes a single period of time. All inventories must be ordered prior to the start of the time period and they cannot be replenished during the time period. Any inventory left over at the end of the time period is scrapped and cannot be used at a later time. If there is extra demand that is not satisfied during the period, it too is lost. Third, for EOQ we are minimizing the expected costs, while for the single period model we are actually maximizing the expected profitability.

The critical ratio applies directly to the single period model as well. We show that the optimal order quantity, Q^*, occurs when the probability that the demand is less than Q^* = the Critical Ratio. In other words, the Critical Ratio tells me how much of the demand probability that should be covered in order to maximize the expected profits.

	Type of Distribution	Probability Density Function	Cumulative Density Function
1	Discrete Uniform Distribution Notation: U(a,b) a is minimum b is maximum n is number of values	$P[X=x]=f(x\mid a,b)=\begin{cases}\frac{1}{n} & \text{for } a\le x\le b\\ 0 & \text{otherwise}\end{cases}$	$P[X\le x]=F(x\mid a,b)$ $=\begin{cases}\frac{(x-a+1)}{n} & \text{for } a\le x\le b\\ 0 & \text{if } x<a\\ 1 & \text{if } x>b\end{cases}$
2	Poisson Distribution Notation: P (λ) λ is mean	$P[X=x]=f(x\mid\lambda)=\begin{cases}\frac{e^{-\lambda}\lambda^{x}}{x!} & \text{for } x=0,1,2,\ldots\\ 0 & \text{otherwise}\end{cases}$	$P[X\le x]=F(x\mid\lambda)=e^{-\lambda}\sum_{i=0}^{x}\frac{\lambda^{i}}{i!}$
3	Continuous Uniform Distribution Notation U(a,b) a is minimum b is maximum	$f(x\mid a,b)=\begin{cases}\frac{1}{b-a} & \text{if } a\le x\le b\\ 0 & \text{otherwise}\end{cases}$	$F(x\mid a,b)=P[X\le x]=\begin{cases}0 & \text{if } x<a\\ \frac{x-a}{b-a} & \text{if } a\le x\le b\\ 1 & \text{if } x>b\end{cases}$
4	Normal Distribution Notation: N(μ,σ) μ = Mean σ = Std Deviation	In Excel & Libre Office: PDF = NORM.DIST(X, μ, σ, 0) $f(x\mid\mu,\sigma)=\frac{1}{(2\pi)^{1/2}\sigma}e^{\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{2}\right]}$	In Excel & Libre Office: CDF = NORM.DIST(X, μ, σ, 1)
5	Triangle Distribution Notation: T(a, b, c) a is minimum b is maximum c is mode $E[x]=\frac{a+b+c}{3}$ $Var[x]=\left(\frac{a^2+b^2+c^2-ab-ac-bc}{18}\right)$	$f(x\mid a,b,c)=\begin{cases}\frac{2(x-a)}{(b-a)(c-a)} & \text{if } a\le x\le c\\ \frac{2(b-x)}{(b-a)(b-c)} & \text{if } c\le x\le b\\ 0 & \text{if } x<a \text{ or } x>b\end{cases}$	$F(x\mid a,b,c)=\begin{cases}0 & \text{if } x<a\\ \frac{(x-a)^2}{(b-a)(c-a)} & \text{if } a\le x\le c\\ 1-\frac{(b-x)^2}{(b-a)(b-c)} & \text{if } c\le x\le b\\ 1 & \text{if } x>b\end{cases}$

FIGURE 2.10: Density Functions for Discrete Uniform Distribution, Poisson Distribution, Continuous Uniform Distribution, Normal Distribution, and Triangle Distribution

Marginal Analysis: Single Period Model

Two costs are associated with single period problems

- Excess cost (ce) when $D < Q$ (\$/unit) i.e. too much product
- Shortage cost (cs) when $D > Q$ (\$/unit) i.e. too little product

If we assume continuous distribution of demand

	Type of Distribution	Units Sold: E[sold]	Units Short: E[short]
1	Continuous Uniform Distribution	$= \frac{Q^2 - a^2}{2(b-a)} + \frac{Q(b-Q)}{(b-a)}$	$= \frac{(b-Q)^2}{2(b-a)}$
2	Normal Distribution	=E[Demanded]-E[short]	= σ G(k)
3	Triangle Distribution	$If\ a \leq Q \leq c, then\ find\ E[sold]$ $= E[sold \mid x \leq Q] + Q * P[x \geq Q]$ $= \left(\frac{1}{3}\right)\left(\frac{(Q-a)^2(2Q+a)}{(b-a)(c-a)}\right) + Q\left(1 - \frac{(Q-a)^2}{(b-a)(c-a)}\right)$	$If\ c \leq Q \leq b, then\ find\ E[short]$ $= E[(x-Q) \mid x \geq Q] = \left(\frac{1}{3}\right)\frac{(b-Q)^3}{(b-a)(b-c)}$

FIGURE 2.11: Expected Units Short and Expected Units Sold for Continuous Uniform Distribution, Normal Distribution and Triangle Distribution

- $c_e P[X \leq Q]$ = expected excess cost of the Qth unit ordered
- $c_s(1 - P[X \leq Q])$ = expected shortage cost of the Qth unit ordered

This implies that if E[Excess Cost] < E[Shortage Cost] then increase Q and that we are at Q^* when E[Shortage Cost] = E[Excess Cost].
Solving this gives us: $P[x \leq Q] = \frac{c_s}{(c_e + c_s)}$
In words, this means that the percentage of the demand distribution covered by Q should be equal to the Critical Ratio in order to maximize expected profits.

Notation:

p : Selling price ($/unit)
c : Purchase cost ($/unit)
c_t : Ordering Costs ($/order)
c_e : Excess holding Costs ($/unit/time); Equal to ch
c_s : Shortage Costs ($/unit)
D : Average Demand (units/time)
B : Penalty for not satisfying demand beyond lost profit ($/unit)
b : Backorder Demand (units)
b^*: Optimal units on backorder when placing an order (unit)
g : Salvage value for excess inventory ($/unit)
h : Carrying or holding cost ($/inventory $/time)
L : Replenishment Lead Time (time)
Q : Replenishment Order Quantity (units/order)
Q^*_{PBO} : Optimal Order Quantity with Planned backorders
T : Order Cycle Time (time/order)
TRC(Q) : Total Relevant Cost ($/time)
TC(Q) : Total Cost ($/time)

Single Period (Newsvendor) Model

To maximize expected profitability, we need to order sufficient inventory, Q, such that the probability that the demand is less than or equal to this amount is equal to the Critical Ratio. Thus, the probability of stocking out is equal to 1 – CR.

$$P[x \leq Q] = \frac{c_s}{(c_e + c_s)}$$

For the simplest case where there is neither salvage value nor extra penalty of stocking out, these become:

$c_s = p–c$, that is the lost margin of missing a potential sale and,

$c_e = c$, that is, the cost of purchasing one unit. The Critical Ratio becomes:

$$CR = \frac{c_s}{(c_s + c_e)} = \frac{(p - c)}{(p - c + c)} = \frac{p - c}{p}$$

which is simply the margin divided by the price!

When we consider also salvage value (g) and shortage penalty (B), these become: c_s = p − c + B, that is the lost margin of missing a potential sale plus a penalty per item short and c_e = c − g, that is, the cost of purchasing one unit minus the salvage value I can gain back. Now the critical ratio becomes

$$CR = \frac{c_s}{(c_s + c_e)} = \frac{(p - c + B)}{(p - c + B + c - g)} = \frac{p - c + B}{p + B - g}$$

2.3.7 Single Period Inventory Models-Expected Profitability

We expand our analysis of the single period model to be able to calculate the expected profitability of a given solution. In the previous lesson, we learned how to determine the optimal order quantity, Q^*, such that the probability of the demand distribution covered by Q^* is equal to the Critical Ratio, which is the ratio of the shortage costs divided by the sum of the shortage and excess costs.

In order to determine the profitability for a solution, we need to calculate the expected units sold, the expected cost of buying Q units, and the expected units short, E[US]. Calculating the E[US] is tricky, but we show how to use the Normal Tables as well as spreadsheets to determine this value.

Notation:

p : Selling price ($/unit)
c : Purchase cost ($/unit)
c_t : Ordering Costs ($/order)
c_e : Excess holding Costs ($/unit); For single period problems this is not necessarily equal to ch, since that assumes that you can keep the inventory for later use.
c_s : Shortage Costs ($/unit)
D : Average Demand (units/time)
B : Penalty for not satisfying demand beyond lost profit ($/unit)
g : Salvage value for excess inventory ($/unit)
k : Safety Factor
x : Units Demanded
E[x] : Expected units demanded
E[US] : Expected Units Short (units)
Q : Replenishment Order Quantity (units/order)
TRC(Q) : Total Relevant Cost ($/period)
TC(Q) : Total Cost ($/period)

Formulas:

Profit Maximization

In words, the expected profit for ordering Q units is equal to the sales price, p, times the expected number of units demanded, E[x]), minus the cost of purchasing Q units, cQ, minus the expected number of units I would be short times the sales price. The difficult part of this equation is the expected units short, or the E[US].

$$E[Profit(Q)] = pE[x] - cQ - pE[UnitsShort]$$

Expected Profits with Salvage and Penalty

If we include a salvage value, g, and a shortage penalty, B, then this becomes:

$$\text{P(Q)=} \begin{cases} \text{-cQ+px+g(Q-x)} & \text{if} \quad \text{x} \leq Q \\ \text{-cQ+pQ-B(x-Q)} & \text{if} \quad \text{x} \geq Q) \end{cases}$$

$$E[P(Q)] = (p-g)E[x] - (c-g)Q - (p-g+B)E[US]$$

Rearranging this becomes:

$$E[P(Q)] = p(E[x] - E[US]) - cQ + g(Q - (E[x] - E[US])) - B(E[US])$$

In words, the expected profit for ordering Q units is equal to four terms. The first term is the sales price, p, times the expected number of units demanded, E[x]), minus the expected units short. The second term is simply the cost of purchasing Q units, cQ. The third term is the expected number of items that I would have left over for salvage, times the salvage value, g. The fourth and final term is the expected number of units short times the shortage penalty, B.

Expected Values

E[Units Demanded]

Continuous: $\int_{x=0}^{\infty} x f_x(x)dx = \hat{x}$

Discrete: $\sum_{x=0}^{\infty} xP[x] = \hat{x}$

E[Units Sold]

Continuous: $\int_{x=0}^{Q} x f_x(x)dx + Q\int_{x=Q}^{\infty} f_x(x)dx$

Discrete: $\sum_{x=0}^{Q} xP[x] + Q\sum_{x=Q+1}^{\infty} P[x]$

E[Units Short]

Continuous: $\int_{x=Q}^{\infty}(x-Q)f_x(x)dx$

Discrete: $\sum_{x=Q+1}^{\infty}(x-Q)P[x]$

E[Excess]

E[Excess] and E[US]/E[Units Short] are different things but they are related to each other.

$$E[UnitsShort] = \sigma * G(k)$$

$$E[Excess] = \sigma * (k + G(k))$$

E[Excess] is a distance from Q. So, on average, how much of Q we have left after we have sold all the units to be sold. E[Units Short] is a distance from the Demand. So, on average, how many units short of the Demand were we once we've sold all the units that were sold that period.
You don't have to use E[Excess]. Every problem can be solved using either one, and you can use either to calculate profits. If you want to see how we get from one to the other, here are some steps:

$$Q = E[UnitsSold] + E[Excess]$$

$$E[Excess] = Q - E[UnitsSold]$$
$$E[Excess] = Q - (E[x] - E[UnitsShort])$$
$$E[Excess] = Q - E[x] + E[UnitsShort]$$

$Q - E[x] = \sigma * k$ (you can see this if you plot this in the distribution)

Expected Units Short E[US]

This is a tricky concept to get your head around at first. Think of the E[US] as the average (mean or expected value) of the demand ABOVE some amount that we specify or have on hand. As my Q gets larger, then we expect the E[US] to get smaller, since I will probably not stock out as much.

Luckily for us, we have a nice way of calculating the E[US] for the Normal Distribution. The Expected Unit Normal Loss Function is noted as G(k). To find the actual units short, we simply multiply this G(k) times the standard deviation of the probability distribution.

$$E[US] = \int_{x=Q}^{\infty} (x - Q) f_x(x) dx = \sigma G(\frac{(Q - \mu)}{\sigma}) = \sigma G(k)$$

You can use the Normal tables to find the G(k) for a given k value or you can use spreadsheets with the equation below:
New Excel Version: $G(k) = NORM.S.DIST(k, 0) - k * (1 - NORM.S.DIST(k, 1))$
Old Excel Version: $G(k) = NORMDIST(k, 0, 1, 0) - k * (1 - NORMSDIST(k))$

2.3.8 Multi Period Inventory Models

We develop inventory replenishment models when we have uncertain or stochastic demand. We built off of both the EOQ and the single period models to introduce three general inventory policies: the Base Stock Policy, the (s,Q) continuous review policy and the (R,S) periodic review policy (the R,S model will be explained in the next lesson). These are the most commonly used inventory policies in practice. They are imbedded within a company's ERP and inventory management systems.

To put them in context, here is the summary of the five inventory models covered so far:

- **Economic Order Quantity** — Deterministic Demand with infinite horizon
  - Order Q^* every T^* periods
  - Order Q^* when IP = μ_{DL}
- **Single Period/ Newsvendor** — Probabilistic Demand with finite (single period) horizon
  - Order Q^* at start of period where P[$x \leq Q$]=CR
- **Base Stock Policy** — Probabilistic Demand with infinite horizon
  - Essentially a one-for-one replenishment
  - Order what was demanded when it was demanded in the quantity it was demanded

- **Continuous Review Policy (s,Q)** — Probabilistic Demand with infinite horizon
  - This is event-based – we order when, and if, inventory passes a certain threshold
  - Order Q^* when IP $\leq$ s
- **Periodic Review Policy (R,S)** — Probabilistic Demand with infinite horizon
  - This is a time-based policy in that we order on a set cycle
  - Order up to S units every R time periods

All of the models make trade-offs: EOQ between fixed and variable costs, Newsvendor between excess and shortage inventory, and the latter three between cost and level of service. The concept of level of service, LOS, is often murky and specific definitions and preferences vary between firms. However, for our purposes, we can break them into two categories: targets and costs. We can establish a target value for some performance metric and then design the minimum cost inventory policy to achieve the level of service. The two metrics covered are Cycle Service Level (CSL) and Item Fill Rate (IFR). The second approach is to place a dollar amount on a specific type of stock out occurring and then minimize the total cost function. The two cost metrics we covered were Cost of Stock Out Event (CSOE) and Cost of Item Short (CIS). They are related to each other.

Regardless of the metrics used, the end result is a safety factor, k, and a safety stock. The safety stock is simply $k\sigma_{DL}$. The term σ_{DL} is defined as the standard deviation of demand over lead time, but it is more technically the root mean square error (RMSE) of the forecast over the lead time. Most companies do not track their forecast error to the granular level that you require for setting inventory levels, so defaulting to the standard deviation of demand is not too bad of an estimate. It is essentially assuming that the forecast is the mean.

Notation:

$B1$: Cost associated with a stock out event ($/event)
c : Purchase cost ($/unit)
c_t : Ordering Costs ($/order)
c_e : Excess holding Costs ($/unit/time); Equal to ch
c_s : Shortage costs ($/unit)
D : Average Demand (units/time)
DS: Demand over short time period (e.g. week)
DL: Demand over long time period (e.g. month)
h: Carrying or holding cost ($/inventory $/time)
L : Replenishment Lead Time (time) Q : Replenishment Order Quantity (units/order)
T : Order Cycle Time (time/order) μ_{DL}: Expected Demand over Lead Time (units/time)
σ_{DL} : Standard Deviation of Demand over Lead Time (units/time)
k : Safety Factor
s : Reorder Point (units)
S: Order up to Point (units)
R: Review Period (time)
N: Orders per Time or 1/T (order/time)
IP: Inventory Position = Inventory on Hand + Inventory on Order – Backorders
IOH: Inventory on Hand (units)
IOO: Inventory on Order (units)
IFR: Item Fill Rate (%)
CSL: Cycle Service Level (%)
CSOE: Cost of Stock Out Event ($ / event)
CIS: Cost per Item Short; Equal to c_s
E[US]: Expected Units Short (units)
G(k): Unit Normal Loss Function

Base Stock Policy

The Base Stock policy is a one-for-one policy. If I sell four items, I order four items to replenish the inventory. The policy determines what the stocking level, or the base stock, is for each item. The base stock, S^*, is the sum of the expected demand over the lead time plus the RMSE of the forecast error over lead time multiplied by some safety factor k. The LOS for this policy is simply the Critical Ratio. Note that the excess inventory cost, c_e, in this case (and all models here) assumes you can use it later and is the product of the cost and the holding rate, ch.

- Optimal Base Stock, S^* : $S^* = \mu_{DL} + k_{LOS}\sigma_{DL}$
- Level of Service (LOS): $LOS = P[\mu_{DL} \leq S^*] = CR = \frac{c_s}{(c_s+c_e)}$

Continuous Review Policies (s,Q)

This is also known as the Order-Point, Order-Quantity policy and is essentially a two-bin system. The policy is "Order Q^* units when Inventory Position is less than the re-order point s". The re-order point is the sum of the expected demand over the lead-time plus the RMSE of the forecast error over lead-time multiplied by some safety factor k.

- Reorder Point: $s = \mu_{DL} + k\sigma_{DL}$
- Order Quantity (Q): Q is typically found through the EOQ formula

Formulas

Level of Service Metrics
Here are four methods for determining the appropriate safety factor, k, for use in any of the inventory models. They are Cycle Service Level, Cost per Stock Out Event, Item Fill Rate, and Cost per Item Short.

Cycle Service Level (CSL)

The CSL is the probability that there will not be a stock out within a replenishment cycle. This is frequently used as a performance metric where the inventory policy is designed to minimize cost to achieve an expected CSL of, say, 95%. Thus, it is one minus the probability of a stock out occurring. If I know the target CSL and the distribution (we will use Normal most of the time) then we can find the s that satisfies it using tables or a spreadsheet where s = NORM.INV (CSL, Mean, StandardDeviation) and k=NORM.S.INV(CSL).

$$CSL = 1 - P[Stockout] = 1 - P[X > s] = P[X \leq s]$$

Note that as k increases, it gets difficult to improve CSL and it will require enormous amount of inventory to cover the extreme limits.

Cost Per Stock out Event (CSOE) or B1 Cost

The CSOE is related to the CSL, but instead of designing to a target CSL value, a penalty is charged when a stock out occurs within a replenishment cycle. The inventory policy is designed to minimize the total costs – so this balances cost of holding inventory explicitly with the cost of stocking out. Minimizing the total costs for k, we find that as long as $\frac{B_1 D}{c_e \sigma_{DL} Q\sqrt{2\pi}} > 1$, then we should set:

$$k = \sqrt{2ln\frac{B_1 D}{c_e \sigma_{DL} Q\sqrt{2\pi}}}$$

If $\frac{B_1 D}{c_e \sigma_{DL} Q \sqrt{2\pi}} < 1$ we should set k as low as management allows.

Item Fill Rate (IFR)

The IFR is the fraction of demand that is met with the inventory on hand out of cycle stock. This is frequently used as a performance metric where the inventory policy is designed to minimize cost to achieve an expected IFR of, say, 90%. If I know the target IFR and the distribution (we will use Normal most of the time) then we can find the appropriate k value by using the Unit Normal Loss Function, G(k).

$$IFR = 1 - \frac{E[US]}{Q} = 1 - \frac{\sigma_{DL} G[k]}{Q}$$
$$G(k) = \frac{Q}{\sigma_{DL}}(1 - IFR)$$

G(k) is the Unit Normal Loss Function, which can be calculated in Spreadsheets as:

New Excel Version: $G(k) = NORM.S.DIST(k, 0) - k * (1 - NORM.S.DIST(k, 1))$ Old Excel Version: $G(k) = NORMDIST(k, 0, 1, 0) - k * (1 - NORMSDIST(k))$

Once we find the k using unit normal tables, we can plug the values in $s = \mu_{DL} + k\sigma_{DL}$ to frame the policy.

Cost per Item Short (CIS)

CIS represents the cost of shortage c_s, where there is a cost associated for each unit short within a replenishment cycle. The inventory policy related to CIS is designed to minimize the total costs – so it balances the cost of holding inventory with the cost of stocking out.

As long as $\frac{Qc_e}{Dc_s} \leq 1$, then the optimal k for the minimum Total Relevant Cost of this policy is obtained when:

$$P[StockOut] = P[x \geq k] = \frac{Qc_e}{Dc_s}$$

In a spreadsheet, this becomes k=NORM.S.INV(1 - $\frac{Qc_e}{Dc_s}$) or (k = NORMSINV(1 - $\frac{Qc_e}{Dc_s}$) for older versions).

If $\frac{Qc_e}{Dc_s} > 1$, we should set k as low as management allows.

Summary of the Metrics Presented:

	Metric	**How to find k**
% Service Based	Cycle Service Level (CSL)	K = NORM.S.INV(1 − P[X > s])
% Service Based	Item Fill Rate (IFL)	Find k from $G(k) = \frac{Q}{\sigma_{DL}}(1 - IFR)$
$ Cost Based	Cost per Stock Out Event (CSOE)	$k = \sqrt{2 \ln\left(\frac{B_1 D}{c_e \sigma_{DL} Q \sqrt{2\pi}}\right)}$
$ Cost Based	Cost per Item Short (CIS)	K = NORM.S.INV(1 - $\frac{Qc_e}{Dc_s}$)

FIGURE 2.12: Summary of metrics presented

A Tip on Converting Times

You will typically need to convert annual forecasts to weekly demand or vice versa or something in between. This is generally very easy – but some students get confused at times:

Converting long to short (n is number of short periods within long):

$$E[D_S] = E[D_L]/n$$
$$VAR[D_S] = VAR[D_L]/n$$
$$\sigma_s = \sigma_L/\sqrt{n}$$

Converting from short to long:

$$E[D_L] = nE[D_S]$$
$$VAR[D_L] = nVAR[D_S]$$
$$\sigma_L = \sqrt{n}\sigma_s$$

Periodic Review Policies

There are trade-offs between the different performance metrics (both cost- and service-based). We demonstrate that once one of the metrics is determined (or explicitly set) then the other three are implicitly set. Because they all lead to the establishment of a safety factor, k, they are dependent on each other. This means that once you have set the safety stock, regardless of the method, you can calculate the expected performance implied by the remaining three metrics.

Periodic Review policies are very popular because they fit the regular pattern of work where ordering might occur only once a week or once every two weeks. The lead-time and the review period are related and can be traded-off to achieve certain goals.

Notation:

B_1 : Cost associated with a stock out event
c : Purchase cost ($/unit)
c_t : Ordering Costs ($/order)
c_e : Excess holding Costs ($/unit/time); Equal to ch
c_s : Shortage costs ($/unit)
c_g : One Time Good Deal Purchase Price ($/unit)
D : Average Demand (units/time)
h : Carrying or holding cost ($/inventory $/time)
L : Replenishment Lead Time (time)
Q : Replenishment Order Quantity (units/order)
T : Order Cycle Time (time/order)
μ_{DL} : Expected Demand over Lead Time (units/time)
σ_{DL} : Standard Deviation of Demand over Lead Time (units/time)
$\mu_{DL} + R$: Expected Demand over Lead Time plus Review Period (units/time)
$\sigma_{DL} + R$: Standard Deviation of Demand over Lead Time plus Review Period (units/time)
k : Safety Factor
s : Reorder Point (units)
S : Order up to Point (units)
R : Review Period (time)
N : Orders per Time or 1/T (order/time)
IP : Inventory Position = Inventory on Hand + Inventory on Order − Backorders
IOH : Inventory on Hand (units)

IOO : Inventory on Order (units)
IFR : Item Fill Rate (%)
CSL : Cycle Service Level (%)
CSOE : Cost of Stock Out Event ($/ event)
CIS : Cost per Item Short; Equal to c_s
E[US] : Expected Units Short (units)
G(k) : Unit Normal Loss Function

Formulas

Inventory Performance Metrics Safety stock is determined by the safety factor, k. So that: $s = \mu_{DL} + k\sigma_{DL}$ and the expected cost of safety stock $= c_e k\sigma_{DL}$. Two ways to calculate k: Service based or Cost based metrics:

- Service Based Metrics—set k to meet expected level of service:
  - Cycle Service Level: $CSL = P[x \leq k]$
  - Item Fill Rate: $IFR = 1 - (\frac{\sigma_{DL} G[k]}{Q})$

Note: IFR is always higher than CSL for the same safety stock level.

- Cost Based Metrics—find k that minimizes total costs over a period of time:
  - Total Cost of Stock out Events (over a period of time): $E[CSOE] = (B_1)P[x \geq k](\frac{D}{Q})$
  - Total Cost of Items Short (over a period of time): $E[CIS] = c_s E[US](\frac{D}{Q}) = c_s \sigma_{DL} G(k)(\frac{D}{Q})$

Safety Stock Logic – relationship between performance metrics

The relationship between the four metrics (2 cost and 2 service based) is shown in the flowchart below (Figure 13). Once one metric (CSL, IFR, CSOE, or CIS) is explicitly set, then the other three metrics are implicitly determined.

Periodic Review Policy (R,S)

This is also known as the Order Up To policy and is essentially a two-bin system. The policy is "Order Up To S^* units every R time periods". The order up to point, S^*, is the sum of the expected demand over the lead-time and the replenishment time plus the RMSE of the forecast error over lead plus replenishment time multiplied by some safety factor k.

- Order Up To Point: $S = \mu_{DL+R} + k\sigma_{DL+R}$

Since the inventory position IP will not be the same every time the inventory is checked, this means the order quantity will not be constant in each replenishment cycle and will be equal to $S^* - IP$. However, on average, the order quantity for an (R,S) system equals the expected demand D over a replenishment cycle R, so $Q = D * R$.

Periodic (R,S) versus Continuous (s, Q) Review

- There is a convenient transformation of (s, Q) to (R, S)
  - (s,Q) = Continuous, order Q when IP $\leq$ s
  - (R, S) = Periodic, order up to S every R time periods
- Allows for the use of all previous (s, Q) decision rules

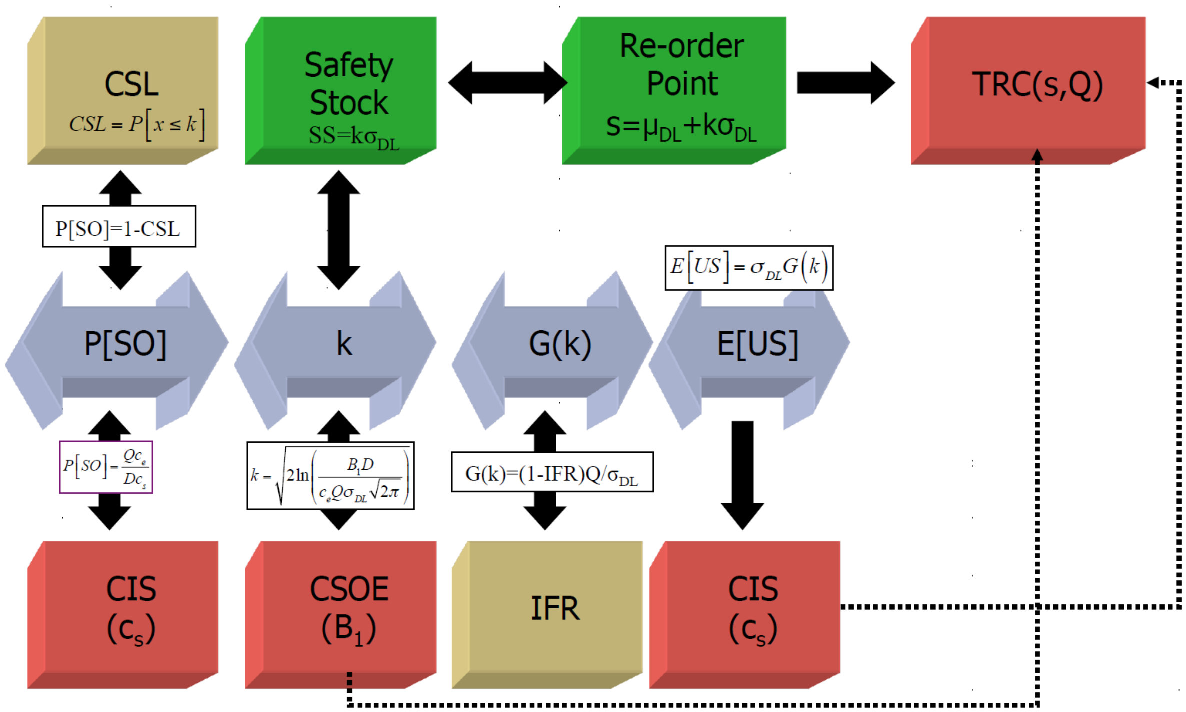

FIGURE 2.13: Summary of metrics presented

  - Reorder point, s, for continuous becomes Order Up To point, S, for periodic system
  - Q for continuous becomes D*R for periodic
  - L for a continuous becomes R+L for periodic

- Approach
  - Make transformations
  - Solve for (s, Q) using transformations
  - Determine final policy such that $S = X_{DL+R} + k\sigma_{DL+R}$

Relationship Between L & R
The lead time, L, and the review period, R, both influence the total costs. Note that the average inventory costs for a (R,S) system is $= c_e[\frac{DR}{2} + k\sigma_{DL+R} + LD]$. This implies that increasing Lead Time, L, will increase Safety Stock non-linearly and Pipeline Inventory linearly while increasing the Review Period, R will increase the Safety Stock non-linearly and the Cycle Stock linearly.

2.3.9 Inventory Models for Multiple Items & Locations

There are several problems with managing items independently, including:

- Lack of coordination—constantly ordering items
- Ignoring of common constraints such as financial budgets or space
- Missed opportunities for consolidation and synergies
- Waste of management time

Managing Multiple Items

There are two issues to solve in order to manage multiple items:

1. Can we aggregate SKUs to use similar operating policies?
   (a) Group using common cost characteristics or break points
   (b) Group using Power of Two Policies
2. How do we manage inventory under common constraints?
   (a) Exchange curves for cycle stock
   (b) Exchange curves for safety stock

Aggregation Methods

When we have multiple SKUs to manage, we want to aggregate those SKUs where we can use the same policies.

Grouping Like Items—Break Points

- Basic Idea: Replenish higher value items faster
- Used for situations with multiple items that have
  - Relatively stable demand
  - Common ordering costs, c_t, and holding charges, h
  - Different annual demands, D_i, and purchase cost c_i
- Approach
  - Pick a base time period, w_0, (typically a week)
  - Create a set of candidate ordering periods (w_1, w_2, etc.)
  - Find $D_i c_i$ values where TRC(w_j)=TRC(w_{j+1})
  - Group SKUs that fall in common value ($D_i c_i$) buckets

Grouping Like Items Example

Selected $w_0 = 1$ week Number of weeks of supply (WOS) to order for item i ordering at time period j = $Q_{ij} = D_i(w_j/52)$ Selecting between options w_1 & w_2 (where $w_1 < w_2 < w_3$ etc.) becomes:

$$c_t D_i/Q_{i1} + (c_i h Q_{i1})/2 = c_t D_i/Q_{i2} + (c_i h Q_{i2})/2$$

$$52c_t D_i/D_i w_1 + c_i h D_i w_1/104 = 52c_t D_i/D_i w_2 + c_i h D_i w2/104$$

$$(c_i h D_i/104)(w_1 - w_2) = (52c_t)(1/w_2 – 1/w_1)$$

$$D_i c_i = [(104)(52c_t)/(h(w_1 - w_2))](1/w_2 – 1/w_1)$$

$$D_i c_i = 5408c_t/(hw_1 w_2)$$

Rule

If $D_i c_i \geq 5408c_t/(hw_1w_2)$ **then select w1**
Else: if $D_i c_i \geq 5408c_t/(hw_2w_3)$ **then select w2**
Else: if $D_i c_i \geq 5408ct/(hw_3w_4)$ **then select w3**
Else: . . . . . .

Grouping Like Items Example

Suppose you need to set up replenishment schedules for several hundred parts that have relatively stable (yet not necessarily the same) demand. They all have similar order costs ($c_t = \$5$) and holding charge (h = 0.20).You have the following potential ordering periods (in weeks): $w_1 = 1, w_2 = 2, w_3 = 4, w_4 = 13, w_5 = 26, w_6 = 52$.

What break-even ordering points should you establish?

Break-point for selecting between 1 week or 2 weeks is:
$D_i c_i = 5408c_t/(hw1w2)$ = 5408(5) / (.2)(1)(2) = \$67,600
If $D_i c_i \geq$\$67,600 then order 1 week's worth each week

Break-point for selecting between 2 weeks or 4 weeks is:
$D_i c_i = 5408c_t/(hw_2w_3)$ = 5408(5) / (.2)(2)(4) = \$16,900

If \$67,600 $> D_i c_i \geq$ \$16,900 then order 2 week's worth every 2 weeks

Final Ordering Break Points:
Order every 1 week if $D_i c_i \geq$ \$67,600
Order every 2 weeks if $D_i c_i \geq$ \$16,900
Order every 4 weeks if $D_i c_i \geq$ \$2,600
Order every 13 weeks if $D_i c_i \geq$ \$400
Order every 26 weeks if $D_i c_i \geq$ \$100
Order every 52 weeks otherwise

Power of Two Formula

- Order in time intervals of powers of two
- Select a realistic base period, T_base (day, week, month)

- Guarantees that TRC will be within 6% of optimal!

Managing Under Common Constraints

There is typically a budget or space constraint that limits the amount of inventory that you can actually keep on hand. Managing each inventory item separately could lead to violating this constraint. Exchange curves are a good way to use the managerial levers of holding charge, ordering cost, and safety factor to set inventory policies to meet a common constraint.

Exchange Curves: Cycle Stock

- Helps determine the best allocation of inventory budget across multiple SKUs
- Relevant Cost parameters
  - Holding Charge (h)
    - There is no single correct value
    - Cost allocations for time and systems differ between firms
    - Reflection of management's investment and risk profile
  - Order Cost (c_t)
    - Not know with precision
    - Cost allocations for time and systems differ between firms
- Exchange Curve
  - Depicts trade-off between total annual cycle stock (TACS) and number of replenishments (N)
  - Determines the c_t/h value that meets budget constraints

Exchange Curves: Safety Stock

- Need to trade-off cost of safety stock and level of service
- Key parameter is safety factor (k) – usually set by management
- Estimate the aggregate service level for different budgets
- The process is as follows:

  1. Select an inventory metric to target
  2. Starting with a high metric value calculate:
     (a) The required k_i to meet that target for each SKU
     (b) The resulting safety stock cost for each SKU and the total safety stock (TSS)
     (c) The other resulting inventory metrics of interest for each SKU and total
  3. Lower the metric value, go to step 2
  4. Chart resulting TSS versus Inventory Metrics

Managing Multiple Locations

Managing the same item in multiple locations will lead to a higher inventory level than managing them in a single location. Consolidating inventory locations to a single common location is known as inventory pooling. Pooling reduces the cycle stock needed by reducing the number of deliveries required and reduces the safety stock by risk pooling that reduces the CV of the demand. This is also called the square root "law" – which is insightful and powerful, but also makes some restrictive assumptions, such as uniformly distributed demand, use of EOQ ordering principles, and independence of demand in different locations.

Notation:

c_i : Purchase cost for item i (\$/unit)
c_t : Ordering Costs (\$/order)
c_e : Excess holding Costs (\$/unit/time); Equal to ch
c_s : Shortage costs (\$/unit)
D_i : Average Demand for item i (units/time)
h : Carrying or holding cost (\$/inventory \$/time)
Q : Replenishment Order Quantity (units/order)
T : Order Cycle Time (time/order)
$T_{Practical}$: Practical Order Cycle Time (time/order)
k : Safety Factor
w_0 : Base Time Period (time)
s : Reorder Point (units)
R : Review Period (time)
N : Number of Inventory Replenishment Cycles
TACS: Total Annual Cycle Stock
TSS: Total Value of Safety Stock
TVIS: Total Value of Items Short
G(k): Unit Normal Loss Function

Formulas:

Power of Two Policy

The process is as follows:

1. Create table of SKUs
2. Calculate T^* for each SKU
3. Calculate Tpractical for each SKU

$$T^* = \frac{Q^*}{D} = \sqrt{\frac{\frac{2c_tD}{c_e}}{D}} = \sqrt{\frac{2c_t}{Dc_e}}$$

In a spreadsheet this is: $T_{practical} = 2$ ˆ (ROUNDUP(LN($T_{optimal}$ / SQRT(2)) /LN(2),0))

Exchange Curves: Cycle Stock

$$TACS = \sum_{i=1}^{n} \frac{Q_i c_i}{2} = \sqrt{\frac{c_t}{h}} \frac{1}{\sqrt{2}} \sum_{i=1}^{n} \sqrt{D_i c_i}$$

$$N = \sum_{i=1}^{n} \frac{D_i}{Q_i} = \sqrt{\frac{h}{c_t}} \frac{1}{\sqrt{2}} \sum_{i=1}^{n} \sqrt{D_i c_i}$$

Process

- Create a table of SKUs with "Annual Value" $(D_i c_i)$ and $\sqrt{D_i c_i}$
- Find the sum of $\sqrt{D_i c_i}$ term for SKUs being analyzed
- Calculate TACS and N for range of (c_t/h) values
- Chart N vs TACS

Exchange Curves: Safety Stock

$$TSS = \sum_{i=1}^{n} k_i \sigma_{DLi} c_i$$

$$TVIS = \sum_{i=1}^{n} (\frac{D_i}{Q_i} c_i \sigma_{DLi} G(k_i))$$

Process:

1. Select an inventory metric to target
2. Starting with a high metric value calculate:
   - The required k_i to meet that target for each SKU
   - The resulting safety stock cost for each SKU and the total safety stock (TSS)
   - The other resulting inventory metrics of interest for each SKU and total
3. Lower the metric value, go to step 2
4. Chart resulting TSS versus Inventory Metrics

Pooled Inventories

Note that the safety stock formula for independent inventory in the figure should read: $\overline{SS}_{independent} = \sum_{i=1}^{n} k\sigma_{d_i} = k\sigma_D \sqrt{n}$

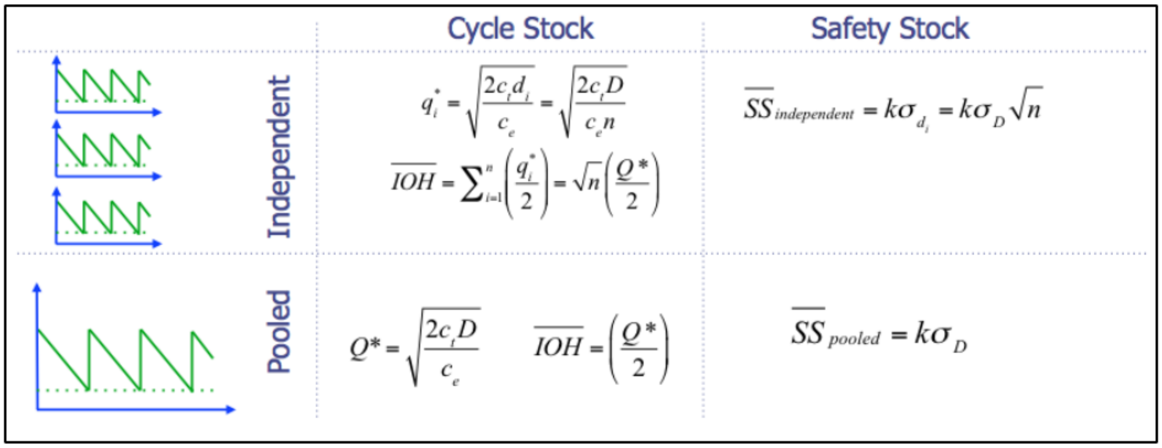

FIGURE 2.14: Comparison between independent and pooled inventories.

	A Items	B Items	C Items
Type of Records	Extensive, Transactional	Moderate	None-use a rule
Level of Management Reporting	Frequent (Monthly or More)	Infrequently—Aggregated	Only as Aggregate
Interaction with Demand	Direct Input, High Data Integrity, Manipulate (pricing, etc.)	Modified Forecast (promotions, etc.)	Simple Forecast at Best
Interaction with Supply	Actively Manage	Manage by Exception	Non
Initial Deployment	Minimize Exposure (high v)	Steady State	Steady State
Frequency of Policy Review	Very Frequent (Monthly or More)	Moderate (Annually/Event Based)	Very Infrequent
Importance of Parameter Precision	Very High—Accuracy Worthwhile	Moderate—Rounding and Approximation ok	Very Low
Shortage Strategy	Actively Manage (Confront)	Set Service Level & Manage by Exception	Set & Forget Service Levels
Demand Distribution	Consider Alternatives to Normal as Situation Fits	Normal	N/A
Management Strategy	Active	Automatic	Passive

FIGURE 2.15: Inventory Management by Segment

Type of Item	Continuous Review	Periodic Review
A Items	(s, S)	(R, s, S)
B Items	(s, Q)	(R, S)
C Items		Manual ~(R, S)

FIGURE 2.16: Inventory Policies (Rules of Thumb)

2.3.10 Inventory Models for Class A & C Items

Inventory Management by Segment

Inventory Policies (Rules of Thumb)

Managing Class A Items

There are two general ways that items can be considered Class A:

- Fast Moving but Cheap (Large D, Small $c \rightarrow Q > 1$)
- Slow Moving but Expensive (Large c, Small $D \rightarrow Q = 1$)

This dictates which Probability Distribution to use for modeling the demand

- Fast Movers
  - Normal or Lognormal Distribution
  - Good enough for B items
  - OK for A items if μ_{DL} or $\mu_{DL+R} \geq 10$
- Slow Movers
  - Poisson Distribution
  - More complicated to handle
  - OK for A items if μ_{DL} or μ_{DL} or $\mu_{DL+R} < 10$

Managing Class C Items

Class C items have low cD values but comprise the lion-share of the SKUs. When managing them we need to consider the implicit & explicit costs. The objective is to minimize management attention. Regardless of policy, savings will most likely not be significant, so try to design simple rules to follow and explore opportunities for disposing of inventory. Alternatively, try to set common reorder quantities. This can be done by assuming common ct and h values and then finding $D_i c_i$ values for ordering frequencies.

Disposing of Excess Inventory

- Why does excess inventory occur?
  - SKU portfolios tend to grow
  - Poor forecasts - Shorter lifecycles
- Which items to dispose?
  - Look at DOS (days of supply) for each item = IOH/D
  - Consider getting rid of items that have $DOS > x$ years
- What actions to take?
  - Convert to other uses
  - Ship to more desired location
  - Mark down price
  - Auction

Real World Inventory Challenges

While models are important, it is also important to understand where there are challenges implementing models in real life.

- Models are not used exactly as in textbooks
- Data is not always available or correct
- Technology matters
- Business processes matter even more
- Inventory policies try to answer three questions:
  - How often should I check my inventory?
  - How do I know if I should order more?
  - How much to order?
- All inventory models use two key numbers
  - Inventory Position
  - Order Point

Notation:

B_1 : Cost Associated with a Stock out Event
c : Purchase Cost ($/unit)
c_t : Ordering Costs ($/order)
c_e : Excess Holding Costs ($/unit/time); Equal to ch
c_s : Shortage Costs ($/unit)
c_g : One Time Good Deal Purchase Price ($/unit)
D : Average Demand (units/time)
h : Carrying or Holding Cost ($/inventory $/time)
L$[X_i]$: Discrete Unit Loss Function
Q : Replenishment Order Quantity (units/order)
T : Order Cycle Time (time/order)
μ_{DL} : Expected Demand over Lead Time (units/time)
σ_{DL} : Standard Deviation of Demand over Lead Time (units/time)
μ_{DL+R} : Expected Demand over Lead Time plus Review Period (units/time)
σ_{DL+R} : Standard Deviation of Demand over Lead Time plus Review Period (units/time)
k : Safety Factor
s : Reorder Point (units)
S : Order Up to Point (units)
R : Review Period (time)
N : Orders per Time or 1/T (order/time)
IP : Inventory Position = Inventory on Hand + Inventory on Order (IOO) – Backorders
IOH : Inventory on Hand (units)
IOO : Inventory on Order (units)
IFR : Item Fill Rate (%)
CSL : Cycle Service Level (%)
E[US] : Expected Units Short (units)
G(k) : Unit Normal Loss Function

Formulas

Fast Moving A Items

$$TRC = c_t(\frac{D}{Q}) + c_e(\frac{Q}{2} + k\sigma_{DL}) + B_1(\frac{D}{Q})P[x > k]$$

$$Q^* = EOQ\sqrt{(1 + \frac{B_1 P[x > k]}{c_t}}$$

$$k^* = \sqrt{2ln\frac{DB_1}{\sqrt{2\pi}Qc_e\sigma_{DL}}}$$

- Iteratively solve the two equations
- Stop when Q^* and k^* converge within acceptable range

Slow Moving A Items

Use a Poisson distribution to model sales

- Probability of x events occurring within a time period
- Mean = Variance = λ

$$p[x_0] = Prob[x = x_0] = \frac{e^{-\lambda}\lambda^{x_0}}{x_0!} \quad \text{for} \quad x_0$$

$$F[x_0] = Prob[x \leq x_0] = \sum_{x=0}^{x_0} \frac{e^{-\lambda}\lambda^{x}}{x!} \quad \text{for} \quad x_0$$

For a discrete function, the loss function L$[X_i]$ can be calculated as follows (Cachon & Terwiesch)

$$L[X_i] = L[X_{i-1}] - (X_i - X_{i-1})(1 - F[X_{i-1}])$$

Learning Objectives

- Understand the reasons for holding inventory and the different types of inventory.
- Understand the concepts of total cost and total relevant costs.
- Identify and quantify the four major cost components of total costs: Purchasing, Ordering, Holding, and Shortage.
- Able to estimate the Economic Order Quantity (EOQ) and to determine when it is appropriate to use.
- Able to estimate sensitivity of EOQ to underlying changes in the input data and understanding of its underlying robustness.
- Understand how to determine the EOQ with different volume discounting schemes.
- Understand how to determine the Economic Production Quantity (EPQ) when the inventory becomes available at a certain rate of time instead of all at once.
- Ability to use the Critical Ratio to determine the optimal order quantity to maximize expected profits.
- Ability to established inventory policies for EOQ with planned backorders as well as single period models.
- Ability to determine profitability, expected units short, expected units sold of a single period model.

- Understanding of safety stock and its role in protecting for excess demand over lead time.
- Ability to develop base stock and order-point, order-quantity continuous review policies.
- Ability to determine proper safety factor, k, given the desired CSL or IFR or the appropriate cost penalty for CSOE or CIS.
- Able to establish a periodic review, Order Up To (S,R) Replenishment Policy using any of the four performance metrics.
- Understand relationships between the performance metrics (CSL, IFR, CSOE, and CIS) and be able to calculate the implicit values.
- Able to use the inventory models to make trade-offs and estimate impacts of policy changes.
- Understand how to use different methods to aggregate SKUs for common inventory policies.
- Understand how to use Exchange Curves.
- Understand how inventory pooling impacts both cycle stock and safety stock.
- Understand how to use different inventory models for Class A and C items.

References

For General Inventory Management
There are more books that cover the basics of inventory management than there are grains of sand on the beach! Inventory management is also usually covered in Operations Management and Industrial Engineering texts as well. A word of warning, though. Every textbook uses different notation for the same concepts. Get used to it. Always be sure to understand what the nomenclature means so that you do not get confused.

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For Inventory Models for Class A & Class C Items

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2.4 Fundamentals of Transportation Management

2.4.1 Summary

This section provides an overview of transportation networks, explains how to analyze the trade-offs between cost and level of service to select the best mode of transportation, and provides guidance on how to address specific transportation problems.

Transportation networks can be broken down into three different layers: the physical network, the operational network, and the strategic or service network. The physical network represents how the product physically moves, the actual path that the shipment could take from origin to destination. The physical network is the basis for all the cost and distance calculations. The operational network represents the route the shipment takes in terms of decision points. Each node is a decision point, and each arc is a specific mode with an associated cost, distance, etc. The strategic or service network represents a series of paths through the network, from origin to destination. Each path represents a complete option that includes end-to-end cost, distance, and service characteristics. Decisions that tie into the inventory policies are made at the strategic or service network level.

Freight transportation analysis introduces the concept of the transportation product and economic transportation modes. The transportation product contains four fundamental operations: loading/unloading, vehicle routing, linehaul moves and facility sorting. There are three main transportation problems based on the number of origin points and the number of destination points: one to one, one to many, and many to many. Freight transportation is also broken down into economic modes, namely consolidated operations and direct operations. Each mode type has different economic characteristics, in terms of scale, scope and density.

2.4.2 Key Concepts

Packaging

An important component of freight transportation is packaging. Packaging can be classified in three categories. The Primary packaging, has direct contact with the product and is usually the smallest unit of distribution (e.g. a bottle of wine, a can, etc.). The Secondary packaging contains product and also a middle layer of packaging that is outside the primary packaging, mainly to group primary packages together (e.g. a box with 12 bottle of wines, cases, cartons, etc.). The Tertiary packaging is designed thinking more on transport shipping, warehouse storage and bulk handling (e.g. pallets, containers, etc.).

- Level of packaging mirrors handling needs.
- Pallets—standard size of 48 x 40 in in the USA (120 x 80 cm in Europe).
- Shipping Containers:
  - TEU (20 ft) 33 m3 volume with 24.8 kkg total payload
  - FEU (40 ft) 67 m3 volume with 28.8 kkg total payload
  - 53 ft long (Domestic US) 111 m3 volume with 20.5 kkg total payload

Transportation Networks

Transportation networks can be analyzed in three layers:

- Physical Network: The actual path that the product takes from origin to destination including guide ways, terminals and controls. Basis for all costs and distance calculations – typically only found once.
- Operational Network: The route the shipment takes in terms of decision points. Each arc is a specific mode with costs, distance, etc. Each node is a decision point. The four primary components are loading/unloading, local-routing, line-haul, and sorting.
- Strategic Network: A series of paths through the network from origin to destination. Each represents a complete option and has end-to-end cost, distance, and service characteristics.

The transportation product:

The transportation product consists of four fundamental operations and two types of economic modes.

The fundamental transportation operations are: loading & unloading, vehicle routing, linehaul moves, facility sorting. The way each of these operations is performed will determine the cost of the transportation product, as well as the service level.

There are two types of economic modes: direct, consolidated. Direct mode is when cargo moves directly from one origin to one destination (1:1). The direct mode brings strong economies of scope (balance). The consolidated mode tries to consolidate shipments ($1{:}\infty$, $\infty{:}1$, $\infty{:}\infty$). The consolidated mode brings strong economies of scale and density.

2.4.3 Mode Selection

Transportation modes have specific niches and perform better than other modes in certain situations. Also, in many cases, there are only one or two feasible options between modes.

Criteria for Feasibility

- Geography
  - Global: Air versus Ocean (trucks cannot cross oceans!)
  - Surface: Trucking (TL, LTL, parcel) vs. Rail vs. Intermodal vs. Barge
- Required speed
  - $>$ 500 miles in 1 day—Air
  - $<$ 500 miles in 1 day—Trucking
- Shipment size (weight/density/cube, etc.)
  - High weight, cube items cannot be moved by air.
  - Large oversized shipments might be restricted to rail or barge.
- Other restrictions
  - Nuclear or hazardous materials (HazMat)
  - Product characteristics

Trade-offs within the set of feasible choices
Once all feasible modes (or separate carrier firms) have been identified, the selection within this feasible set is made as a trade-off between costs and service level. It is important to translate the "non-cost" elements into costs via the total cost equation. The typical non-cost elements are:

- Time (mean transit time, variability of transit time, frequency)
- Capacity
- Loss and Damage

Transportation Cost Functions
Transportation costs can take many different forms, to include:

- Pure variable cost / unit
- Pure fixed cost / shipment
- Mixed variable & fixed cost
- Variable cost / unit with a minimum quantity
- Incremental discounts

Impact of Transportation on Inventory Cost
Transportation affects total inventory cost via:

- Cost of transportation (fixed, variable, or some combination)
- Lead time (expected value as well as variability)
- Capacity restrictions (as they limit optimal order size)
- Miscellaneous factors (such as material restrictions or perishability)

2.4.4 Lead Time Variability

Variability in lead time impacts the total cost equation for inventory. There are important linkages between lead time reliability, forecast accuracy, and inventory levels. Therefore, mode selection is heavily influenced not only by the value of the product being transported, but also the expected lead time and its variability.

Lead Time Reliability

There are two different dimensions of reliability that do not always match:

- Credibility (reserve slots are agreed, stop at all ports, load all containers, etc.)
- Schedule consistency (actual vs. quoted performance)

Contract reliability in procurement and operations do not always match as they are typically performed by different parts of an organization. Contract reliability differs dramatically across different route segments (origin port dwell time vs. port-to-port transit time vs. destination port dwell time for instance). For most shippers, the greatest transit variability occurs in the origin inland transportation legs and at the ports.

Demand Over Lead Time with Lead Time Variability

In order to account for lead time variability in the inventory cost equation, you should use the following formula to calculate the demand over lead time:

$$\mu_{DL} = \mu_L \mu_D$$
$$\sigma_{DL} = \sqrt{\mu_L \sigma_D^2 + (\mu_D)^2 \sigma_L^2}$$

This formula derives from a more general statistical formula that is applied to calculate random sums of random variables:

$$E[S] = E[\sum_{i=1}^{N} X_i] = E[N]E[X]$$
$$Var[S] = Var[\sum_{i=1}^{N} X_i] = E[N]Var[X] + (E[X])^2 Var[N]$$

2.4.5 Transportation Product: Main Operations

The transportation products consist of up to four fundamental operations: loading/unloading, linehaul moves, vehicle routing, and facility sorting. The cost and time associated with each one operation depends on a series of variables.

Loading/Unloading

- Cost is function of time and labor.
- Time is a function of quantity, ease and stowability.
- Fixed and variable costs with respect to quantity.

Linehaul moves

- Cost is function of time, distance and balance.

- Time is a function of distance.
- Fixed and variable costs with respect to distance and time.

Vehicle routing

- Cost is function of distance, number of stops and time.
- Time is a function of density of stops.
- Fixed and variable costs with respect to number of stops and distance.
- Different types of routing:
  - One-to-Many – single origin w/multiple stops
  - Many-to-One – multiple origins w/single stop
  - Interleavened – multiple pickups and deliveries enroute

Facility sorting

- Cost is function of quantity, ease and stowability.
- Fixed and variable costs with respect to quantity.

2.4.6 Transportation Product: Economic Modes

There are a series of underlying economic drivers that should be considered when making decisions about the transportation product. There economic drivers are: economies of scale, economies of scope, and economies of density.

The economies of scale refer to the decrease of unitary cost as the volume (scale) increases. This economies are present when there is a fixed cost of operation. They are found in loading/unloading, linehaul moves (in vehicle) and facility sorting.

The economies of scope refer to how a firm's average unitary cost decreases if another product is also produced or transported during the same operation. They are related to subadditivity and cost complementarity. They are found in linehaul moves.

Finally, the economies of density refer to how a firm's unitary cost is reduced as the density of customers or production in a region is increased. They are related to both shipment and location density. They are found in vehicle routing.

These underlying economic drivers are useful to analyze the economic modes and their options.

There are two types of economic modes in transportation: direct and consolidated. Direct modes refer to linehaul moves that transport goods from one location to another directly, without any stops or processing. Consolidated modes involve more complex transportation systems in which several linehaul moves, routing for deliveries or pick-ups, and sorting of goods at hubs or sorting centers take place.

	Direct Mode	**Consolidated Mode**
Economies of Scale	weak, lane based	very strong
Economies of Scope	very strong	weak
Economies of Density	moderate	strong
Number of Vendors	many (perfectly competitive)	fewer (big barriers)
Size of Carrier Base	dozens to hundred	few to one
Design	focus on linehauls and corridors	focus on aggregating low volume lanes
Procure	auction/bid	ad-hoc negotiation
Manage	tiered management by exception	deeper relations with few

2.4.7 Transportation Problems Examples

We can conclude that, according to the number of points of origin and destination, the transportation problems can be classified in four types: one to one, one to many, many to one, many to many.

The one to one (1:1) problem reflects the need to transport a certain cargo from a single origin point to a single destination point directly.

The one to many (1:∞) problem requires to transport certain products from a certain point A to several different destiny points.

The many to one (∞:1) problem represents the challenge of collecting items at several points of origin and delivering all of them at a certain specific destination point.

The many to many (∞:∞) problem represents the collection of products at several origin points to be delivered at several destination points.

Each of these transportation problems will include several basic operations, as illustrated in the table below:

	(1:1)	**(1:∞)**	**(∞:∞)**
Loading/unloading	X	X	X
Linehaul	X	X	X
Routing		X	X
Sorting			X

There are methods and techniques that can help us to solve each of these problem types in the most cost-effective way. In this section, a few of these techniques are explained to calculate linehaul cost, vehicle routing and cost of sorting, however this is not a comprehensive list of all the methods that can be used to solve transportation problems.

Costs for transportation:

- Loading: $c_{LF} + c_{LVu}(Q)$
- Unloading: $c_{UF} + c_{UVu}(Q)$
- Direct Linehaul: $c_{DF} + c_{DVd}(distance) + c_{DVu}(Q)$

Total cost per shipment : $(c_{LF} + c_{DF} + c_{UF}) + (c_{LVu} + c_{DVu} + c_{UVu})(Q) + c_{DVd}(distance)$

Total cost per year : $(c_{LVu} + c_{DVu} + c_{UVu})(Q)(S/Q) + [(c_{LF} + c_{DF} + c_{UF}) + c_{DVd}(distance)][S/Q]$ $+ (h)(Q/2)$

$Q* = \sqrt{(2S[FixedShipCosts]/h)}$

where c_{LF}, c_{UF} = fixed cost for (un)loading
c_{LVu}, c_{UVu} = variable cost for (un)loading by units
c_{DF} = fixed cost for direct linehaul
c_{DVd} = var cost for direct linehaul by distance
c_{DVu} = var cost for direct linehaul by units
Q = Quantity shipped (units/load)
S = Annual product supply (units/yr)
T = frequency of shipment (yr/lds)
h = holding/penalty cost ($ per year)
$distance$ = length of haul of shipment

Linehaul cost

The objective is to minimize the cost of point to point transportation. This cost will depend on the point to point distance. There are different ways to calculate the point to point distance, which depend on the topography of the underlying region.

- If Euclidean space, use Euclidean distance: dist((x, y), (a, b)) = $\sqrt{(x-a)^2+(y-b)^2}$
- If grid, use Manhattan distance: dist((x, y), (a, b)) =|x-a|+|y-b|
- If random network, use: D_{A-B} =k_{cf} × d_{A-B}, where k_{cf} is the circuity factor or k factor. For short distances, d_{A-B} can be considered to be the Euclidean distance. The k factor ranges from 1 to 2, or rarely a higher number, depending on the density or sparsity of the network.

Vehicle Routing

The objective is to find minimum cost tours from single origin to multiple destinations using multiple vehicles. There are many approaches available to solve this problem, and there is usually a trade-off between speed and solution quality.

Simple heuristics provide a solution fast, but quality of the solution can vary from problem to problem (e.g. Sweep Method). Optimization models find the optimal solution, but can take a long time to run (e.g. mixed integer linear programming). Advance heuristics are not as fast as simple heuristics, but they usually deliver a better quality solution (e.g. Clark-Wright Savings Algorithm).

- Sweep Method:
  This is a simple heuristic method. Please, be aware that different starting points and directions can yield different solutions.

  1. Draw a line from the starting node and select an angle and direction (clockwise or counter-clockwise).
  2. Select a new vehicle, j, that is empty, $w_j = 0$, and has capacity, c_j.
  3. Rotate the line in the selected direction until it hits a new node, i. (When the line reaches the starting point, go to step 5).
  4. If the demand at i (D_i) plus the current load already in the vehicle (w_j) is less than the vehicle capacity, add the new load to the vehicle, $w_j = D_i + w_j$ and go to step 3. Otherwise, close this vehicle, and go to step 2 to start a new tour.
  5. Solve the TSP for each independent vehicle tour.

- Optimization:
  Potential routes are inputs to the model and different types of cost can be included (not just distance-related). Mixed integer linear programming is used to select routes
  Min $\sum_j C_j Y_j$
  Such that:
  $\sum_{j=1}^{J} a_{ji} Y_j \geq D_i \quad \forall_i$
  $\sum_{j=1}^{J} Y_j \leq V$
  $\sum_i Y_{ij} = 1 \quad \forall_j = t\ Y_j = 0,1 \quad \forall_j$

  - Indices ::: Demand nodes i :::Vehicle routes j
  - Input Data : C_j = Total cost of route j ($) ::: Vehicle routes j ::: V = Maximum vehicles ::: a_{ij} =1 if node i is in route j; = 0 otherwise
  - Decision Variables: Y_j =1 if route j is used, =0 otherwise

- Clark-Wright Savings Algorithm:
  The Clarke and Wright savings algorithm is one of the best known heuristic for Vehicle Routing problems. It applies to problems for which the number of vehicles is not fixed (it is a decision variable).

  1. Start with a complete solution (out and back).
  2. Identify nodes to link to form a common tour by calculating savings
  3. Savings from connecting node i and j:
     $s_{i,j} = c_{o,i} + c_{o,j} - c_{i,j}$
  4. Calculate savings for every pair of nodes (all i and j).
  5. Sort savings $s_{i,j}$ by highest savings first.
  6. Obtain solution as long as vehicle capacity is not violated and interior tour nodes are not added.

  Example:

  – Current tour cost: $2c_{01} + 2c_{02}$
  – Joined tour cost: $c_{01} + c_{12} + c_{20}$
  – So if $2c_{01} + 2c_{02} > c_{01} + c_{12} + c_{20}$, then go for the joined tour. This will generate savings of: $c_{01} + c_{20} - c_{12}$

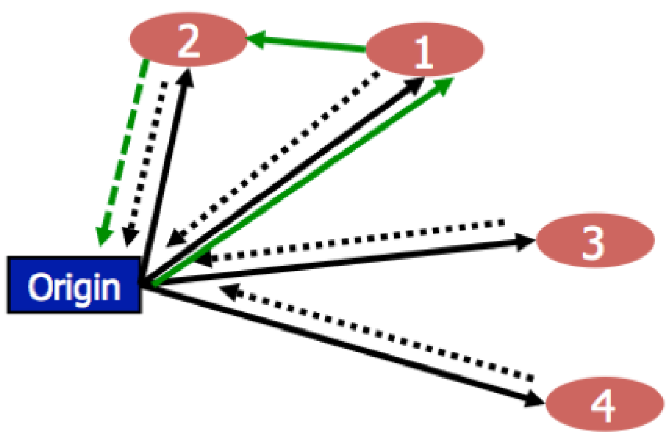

FIGURE 2.17: Savings algorithm

Sorting The impact on using sorting facilities can be evaluated via network design problems (optimization models). Below, we list some of the advantages and disadvantages of using sorting facilites or hubs.

- Advantages:
  – Lower costs through fewer conveyances needed with greater utilization
  – Higher LOS with similar or even fewer resources
  – Extended reach into smaller and more locations
- Disadvantages:
  – Costs of operating the hub

  - Higher level on circuity
  - Impact on service levels
  - Productivity/utilization loss
- Economic Factors:
  - Relative O-D versus Hub distances
  - Conveyance and shipment sizes
  - Demand patterns
  - Location of freight versus pax hubs

Learning Objectives

- Understand the common terminology and key concepts of global freight transportation.
- Understand the physical, operational, and strategic networks of a transportation system.
- Acquire the ability to select transportation mode by considering the trade-offs between Level of Service (LOS) and cost.
- Understand the impact of transportation on inventory costs.
- Understand the different transportation products and their characteristics.
- Understand the main operations in transportation problems, and know some of the approaches that can be used to find a cost-effective solution.

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2.4.8 Appendix A & B Unit Normal Distribution, Poisson Distribution Tables

Poisson distribution

Columns are means (λ) while rows are cumulative probabilibites (F(x). For example, the P[x≤2] for ~P(λ=0.5) = 0.98561

F(x)	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	F(x)
0	0.99005	0.98020	0.97045	0.96079	0.95123	0.94176	0.93239	0.92312	0.91393	0.90484	0.86071	0.81873	0.77880	0.74082	0.70469	0.67032	0.63763	0.60653	0
1	0.99995	0.99980	0.99956	0.99922	0.99879	0.99827	0.99766	0.99697	0.99618	0.99532	0.98981	0.98248	0.97350	0.96306	0.95133	0.93845	0.92456	0.90980	1
2	1.00000	1.00000	1.00000	0.99999	0.99998	0.99997	0.99995	0.99992	0.99989	0.99985	0.99950	0.99885	0.99784	0.99640	0.99449	0.99207	0.98912	0.98561	2
3	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99994	0.99987	0.99973	0.99953	0.99922	0.99880	0.99825	3
4	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99998	0.99997	0.99994	0.99989	0.99983	4
5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	5
6	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	6

F(x)	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	F(x)
0	0.47237	0.36788	0.28650	0.22313	0.17377	0.13534	0.10540	0.08208	0.06393	0.04979	0.03877	0.03020	0.02352	0.01832	0.01426	0.01111	0.00865	0.00674	0
1	0.82664	0.73576	0.64464	0.55783	0.47788	0.40601	0.34255	0.28730	0.23973	0.19915	0.16479	0.13589	0.11171	0.09158	0.07489	0.06110	0.04975	0.04043	1
2	0.95949	0.91970	0.86847	0.80885	0.74397	0.67668	0.60934	0.54381	0.48146	0.42319	0.36957	0.32085	0.27707	0.23810	0.20371	0.17358	0.14735	0.12465	2
3	0.99271	0.98101	0.96173	0.93436	0.89919	0.85712	0.80943	0.75758	0.70304	0.64723	0.59141	0.53663	0.48377	0.43347	0.38621	0.34230	0.30189	0.26503	3
4	0.99894	0.99634	0.99088	0.98142	0.96710	0.94735	0.92199	0.89118	0.85538	0.81526	0.77165	0.72544	0.67755	0.62884	0.58012	0.53210	0.48540	0.44049	4
5	0.99987	0.99941	0.99816	0.99554	0.99087	0.98344	0.97263	0.95798	0.93916	0.91608	0.88881	0.85761	0.82288	0.78513	0.74494	0.70293	0.65973	0.61596	5
6	0.99999	0.99992	0.99968	0.99907	0.99780	0.99547	0.99163	0.98581	0.97757	0.96649	0.95227	0.93471	0.91372	0.88933	0.86169	0.83105	0.79775	0.76218	6
7	1.00000	0.99999	0.99995	0.99983	0.99953	0.99890	0.99773	0.99575	0.99265	0.98810	0.98174	0.97326	0.96238	0.94887	0.93257	0.91341	0.89140	0.86663	7
8	1.00000	1.00000	0.99999	0.99997	0.99991	0.99976	0.99945	0.99886	0.99784	0.99620	0.99371	0.99013	0.98519	0.97864	0.97023	0.95974	0.94701	0.93191	8
9	1.00000	1.00000	1.00000	1.00000	0.99998	0.99995	0.99988	0.99972	0.99942	0.99890	0.99803	0.99669	0.99469	0.99187	0.98801	0.98291	0.97636	0.96817	9
10	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99998	0.99994	0.99986	0.99971	0.99944	0.99898	0.99826	0.99716	0.99557	0.99333	0.99030	0.98630	10
11	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99997	0.99993	0.99985	0.99971	0.99947	0.99908	0.99849	0.99760	0.99632	0.99455	11
12	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99998	0.99996	0.99992	0.99985	0.99973	0.99952	0.99919	0.99870	0.99798	12
13	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99998	0.99996	0.99992	0.99986	0.99975	0.99957	0.99930	13
14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99998	0.99996	0.99993	0.99987	0.99977	14
15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99998	0.99996	0.99993	15
16	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	0.99998	16
17	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	17
18	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	18

F(x)	5.25	5.50	5.75	6.00	6.25	6.50	6.75	7.00	7.25	7.50	7.75	8.00	8.25	8.50	8.75	9.00	9.25	9.50	F(x)
0	0.00525	0.00409	0.00318	0.00248	0.00193	0.00150	0.00117	0.00091	0.00071	0.00055	0.00043	0.00034	0.00026	0.00020	0.00016	0.00012	0.00010	0.00007	0
1	0.03280	0.02656	0.02148	0.01735	0.01400	0.01128	0.00907	0.00730	0.00586	0.00470	0.00377	0.00302	0.00242	0.00193	0.00154	0.00123	0.00099	0.00079	1
2	0.10511	0.08838	0.07410	0.06197	0.05170	0.04304	0.03575	0.02964	0.02452	0.02026	0.01670	0.01375	0.01131	0.00928	0.00761	0.00623	0.00510	0.00416	2
3	0.23167	0.20170	0.17495	0.15120	0.13025	0.11185	0.09577	0.08177	0.06963	0.05915	0.05012	0.04238	0.03576	0.03011	0.02530	0.02123	0.01777	0.01486	3
4	0.39777	0.35752	0.31991	0.28506	0.25299	0.22367	0.19704	0.17299	0.15138	0.13206	0.11487	0.09963	0.08619	0.07436	0.06401	0.05496	0.04709	0.04026	4
5	0.57218	0.52892	0.48662	0.44568	0.40640	0.36904	0.33377	0.30071	0.26992	0.24144	0.21522	0.19124	0.16939	0.14960	0.13174	0.11569	0.10133	0.08853	5
6	0.72479	0.68604	0.64639	0.60630	0.56622	0.52652	0.48759	0.44971	0.41316	0.37815	0.34485	0.31337	0.28380	0.25618	0.23051	0.20678	0.18495	0.16495	6
7	0.83925	0.80949	0.77762	0.74398	0.70890	0.67276	0.63591	0.59871	0.56152	0.52464	0.48837	0.45296	0.41864	0.38560	0.35398	0.32390	0.29544	0.26866	7
8	0.91436	0.89436	0.87195	0.84724	0.82038	0.79157	0.76106	0.72909	0.69596	0.66197	0.62740	0.59255	0.55770	0.52311	0.48902	0.45565	0.42320	0.39182	8
9	0.95817	0.94622	0.93221	0.91608	0.89779	0.87738	0.85492	0.83050	0.80427	0.77641	0.74712	0.71662	0.68516	0.65297	0.62031	0.58741	0.55451	0.52183	9
10	0.98118	0.97475	0.96686	0.95738	0.94618	0.93316	0.91827	0.90148	0.88279	0.86224	0.83990	0.81589	0.79032	0.76336	0.73519	0.70599	0.67597	0.64533	10
11	0.99216	0.98901	0.98498	0.97991	0.97367	0.96612	0.95715	0.94665	0.93454	0.92076	0.90527	0.88808	0.86919	0.84866	0.82657	0.80301	0.77810	0.75199	11
12	0.99696	0.99555	0.99366	0.99117	0.98798	0.98397	0.97902	0.97300	0.96581	0.95733	0.94749	0.93620	0.92341	0.90908	0.89320	0.87577	0.85683	0.83643	12
13	0.99890	0.99831	0.99749	0.99637	0.99487	0.99290	0.99037	0.98719	0.98324	0.97844	0.97266	0.96582	0.95782	0.94859	0.93805	0.92615	0.91285	0.89814	13
14	0.99963	0.99940	0.99907	0.99860	0.99794	0.99704	0.99585	0.99428	0.99227	0.98974	0.98659	0.98274	0.97810	0.97257	0.96608	0.95853	0.94986	0.94001	14
15	0.99988	0.99980	0.99968	0.99949	0.99922	0.99884	0.99831	0.99759	0.99664	0.99539	0.99379	0.99177	0.98925	0.98617	0.98243	0.97796	0.97269	0.96653	15
16	0.99996	0.99994	0.99989	0.99983	0.99972	0.99957	0.99935	0.99904	0.99862	0.99804	0.99728	0.99628	0.99500	0.99339	0.99137	0.98889	0.98588	0.98227	16
17	0.99999	0.99998	0.99997	0.99994	0.99991	0.99985	0.99976	0.99964	0.99946	0.99921	0.99887	0.99841	0.99779	0.99700	0.99597	0.99468	0.99306	0.99107	17
18	1.00000	0.99999	0.99999	0.99998	0.99997	0.99995	0.99992	0.99987	0.99980	0.99970	0.99955	0.99935	0.99907	0.99870	0.99821	0.99757	0.99675	0.99572	18
19	1.00000	1.00000	1.00000	0.99999	0.99999	0.99998	0.99997	0.99996	0.99993	0.99989	0.99983	0.99975	0.99963	0.99947	0.99924	0.99894	0.99855	0.99804	19
20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	0.99998	0.99996	0.99994	0.99991	0.99986	0.99979	0.99969	0.99956	0.99938	0.99914	20
21	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	0.99998	0.99997	0.99995	0.99992	0.99988	0.99983	0.99975	0.99964	21
22	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	0.99998	0.99997	0.99996	0.99993	0.99990	0.99985	22
23	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	0.99998	0.99998	0.99996	0.99994	23
24	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	0.99999	0.99998	24
25	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	25
26	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	26

FIGURE 2.18: Poisson Distribution

Unit Normal distribution

Example, for k=1.67, the Probability that u<k = 0.9525 and the Expected Unit Normal Loss is 0.0197

k	P[u<k]	G(k)	k	P[u<k]	G(k)	k	P[u<k]	G(k)	k	P[u<k]	G(k)	k	P[u<k]	G(k)	k	P[x≤k]	G(k)	k	P[x≤k]	G(k)	k	P[x≤k]	G(k)
0.00	0.5000	0.3989	0.50	0.6915	0.1978	1.00	0.8413	0.0833	1.50	0.9332	0.0293	2.00	0.9772	0.0085	2.50	0.9938	0.00200	3.00	0.9987	0.000382	3.50	0.9998	0.000058
0.01	0.5040	0.3940	0.51	0.6950	0.1947	1.01	0.8438	0.0817	1.51	0.9345	0.0286	2.01	0.9778	0.0083	2.51	0.9940	0.00194	3.01	0.9987	0.000369	3.51	0.9998	0.000056
0.02	0.5080	0.3890	0.52	0.6985	0.1917	1.02	0.8461	0.0802	1.52	0.9357	0.0280	2.02	0.9783	0.0080	2.52	0.9941	0.00188	3.02	0.9987	0.000356	3.52	0.9998	0.000054
0.03	0.5120	0.3841	0.53	0.7019	0.1887	1.03	0.8485	0.0787	1.53	0.9370	0.0274	2.03	0.9788	0.0078	2.53	0.9943	0.00183	3.03	0.9988	0.000344	3.53	0.9998	0.000052
0.04	0.5160	0.3793	0.54	0.7054	0.1857	1.04	0.8508	0.0772	1.54	0.9382	0.0267	2.04	0.9793	0.0076	2.54	0.9945	0.00177	3.04	0.9988	0.000332	3.54	0.9998	0.000050
0.05	0.5199	0.3744	0.55	0.7088	0.1828	1.05	0.8531	0.0757	1.55	0.9394	0.0261	2.05	0.9798	0.0074	2.55	0.9946	0.00171	3.05	0.9989	0.000320	3.55	0.9998	0.000048
0.06	0.5239	0.3697	0.56	0.7123	0.1799	1.06	0.8554	0.0742	1.56	0.9406	0.0255	2.06	0.9803	0.0072	2.56	0.9948	0.00166	3.06	0.9989	0.000309	3.56	0.9998	0.000046
0.07	0.5279	0.3649	0.57	0.7157	0.1771	1.07	0.8577	0.0728	1.57	0.9418	0.0249	2.07	0.9808	0.0070	2.57	0.9949	0.00161	3.07	0.9989	0.000298	3.57	0.9998	0.000044
0.08	0.5319	0.3602	0.58	0.7190	0.1742	1.08	0.8599	0.0714	1.58	0.9429	0.0244	2.08	0.9812	0.0068	2.58	0.9951	0.00156	3.08	0.9990	0.000287	3.58	0.9998	0.000042
0.09	0.5359	0.3556	0.59	0.7224	0.1714	1.09	0.8621	0.0700	1.59	0.9441	0.0238	2.09	0.9817	0.0066	2.59	0.9952	0.00151	3.09	0.9990	0.000277	3.59	0.9998	0.000041
0.10	0.5398	0.3509	0.60	0.7257	0.1687	1.10	0.8643	0.0686	1.60	0.9452	0.0232	2.10	0.9821	0.0065	2.60	0.9953	0.00146	3.10	0.9990	0.000267	3.60	0.9998	0.000039
0.11	0.5438	0.3464	0.61	0.7291	0.1659	1.11	0.8665	0.0673	1.61	0.9463	0.0227	2.11	0.9826	0.0063	2.61	0.9955	0.00142	3.11	0.9991	0.000258	3.61	0.9998	0.000038
0.12	0.5478	0.3418	0.62	0.7324	0.1633	1.12	0.8686	0.0659	1.62	0.9474	0.0222	2.12	0.9830	0.0061	2.62	0.9956	0.00137	3.12	0.9991	0.000249	3.62	0.9999	0.000036
0.13	0.5517	0.3373	0.63	0.7357	0.1606	1.13	0.8708	0.0646	1.63	0.9484	0.0216	2.13	0.9834	0.0060	2.63	0.9957	0.00133	3.13	0.9991	0.000240	3.63	0.9999	0.000035
0.14	0.5557	0.3328	0.64	0.7389	0.1580	1.14	0.8729	0.0634	1.64	0.9495	0.0211	2.14	0.9838	0.0058	2.64	0.9959	0.00129	3.14	0.9992	0.000231	3.64	0.9999	0.000033
0.15	0.5596	0.3284	0.65	0.7422	0.1554	1.15	0.8749	0.0621	1.65	0.9505	0.0206	2.15	0.9842	0.0056	2.65	0.9960	0.00125	3.15	0.9992	0.000223	3.65	0.9999	0.000032
0.16	0.5636	0.3240	0.66	0.7454	0.1528	1.16	0.8770	0.0609	1.66	0.9515	0.0201	2.16	0.9846	0.0055	2.66	0.9961	0.00121	3.16	0.9992	0.000215	3.66	0.9999	0.000031
0.17	0.5675	0.3197	0.67	0.7486	0.1503	1.17	0.8790	0.0596	1.67	0.9525	0.0197	2.17	0.9850	0.0053	2.67	0.9962	0.00117	3.17	0.9992	0.000207	3.67	0.9999	0.000029
0.18	0.5714	0.3154	0.68	0.7517	0.1478	1.18	0.8810	0.0584	1.68	0.9535	0.0192	2.18	0.9854	0.0052	2.68	0.9963	0.00113	3.18	0.9993	0.000199	3.68	0.9999	0.000028
0.19	0.5753	0.3111	0.69	0.7549	0.1453	1.19	0.8830	0.0573	1.69	0.9545	0.0187	2.19	0.9857	0.0050	2.69	0.9964	0.00110	3.19	0.9993	0.000192	3.69	0.9999	0.000027
0.20	0.5793	0.3069	0.70	0.7580	0.1429	1.20	0.8849	0.0561	1.70	0.9554	0.0183	2.20	0.9861	0.0049	2.70	0.9965	0.00106	3.20	0.9993	0.000185	3.70	0.9999	0.000026
0.21	0.5832	0.3027	0.71	0.7611	0.1405	1.21	0.8869	0.0550	1.71	0.9564	0.0178	2.21	0.9864	0.0047	2.71	0.9966	0.00103	3.21	0.9993	0.000178	3.71	0.9999	0.000025
0.22	0.5871	0.2986	0.72	0.7642	0.1381	1.22	0.8888	0.0538	1.72	0.9573	0.0174	2.22	0.9868	0.0046	2.72	0.9967	0.00099	3.22	0.9994	0.000172	3.72	0.9999	0.000024
0.23	0.5910	0.2944	0.73	0.7673	0.1358	1.23	0.8907	0.0527	1.73	0.9582	0.0170	2.23	0.9871	0.0045	2.73	0.9968	0.00096	3.23	0.9994	0.000166	3.73	0.9999	0.000023
0.24	0.5948	0.2904	0.74	0.7704	0.1334	1.24	0.8925	0.0517	1.74	0.9591	0.0166	2.24	0.9875	0.0044	2.74	0.9969	0.00093	3.24	0.9994	0.000160	3.74	0.9999	0.000022
0.25	0.5987	0.2863	0.75	0.7734	0.1312	1.25	0.8944	0.0506	1.75	0.9599	0.0162	2.25	0.9878	0.0042	2.75	0.9970	0.00090	3.25	0.9994	0.000154	3.75	0.9999	0.000021
0.26	0.6026	0.2824	0.76	0.7764	0.1289	1.26	0.8962	0.0495	1.76	0.9608	0.0158	2.26	0.9881	0.0041	2.76	0.9971	0.00087	3.26	0.9994	0.000148	3.76	0.9999	0.000020
0.27	0.6064	0.2784	0.77	0.7794	0.1267	1.27	0.8980	0.0485	1.77	0.9616	0.0154	2.27	0.9884	0.0040	2.77	0.9972	0.00084	3.27	0.9995	0.000143	3.77	0.9999	0.000019
0.28	0.6103	0.2745	0.78	0.7823	0.1245	1.28	0.8997	0.0475	1.78	0.9625	0.0150	2.28	0.9887	0.0039	2.78	0.9973	0.00081	3.28	0.9995	0.000137	3.78	0.9999	0.000019
0.29	0.6141	0.2706	0.79	0.7852	0.1223	1.29	0.9015	0.0465	1.79	0.9633	0.0146	2.29	0.9890	0.0038	2.79	0.9974	0.00079	3.29	0.9995	0.000132	3.79	0.9999	0.000018
0.30	0.6179	0.2668	0.80	0.7881	0.1202	1.30	0.9032	0.0455	1.80	0.9641	0.0143	2.30	0.9893	0.0037	2.80	0.9974	0.00076	3.30	0.9995	0.000127	3.80	0.9999	0.000017
0.31	0.6217	0.2630	0.81	0.7910	0.1181	1.31	0.9049	0.0446	1.81	0.9649	0.0139	2.31	0.9896	0.0036	2.81	0.9975	0.00074	3.31	0.9995	0.000123	3.81	0.9999	0.000016
0.32	0.6255	0.2592	0.82	0.7939	0.1160	1.32	0.9066	0.0436	1.82	0.9656	0.0136	2.32	0.9898	0.0035	2.82	0.9976	0.00071	3.32	0.9995	0.000118	3.82	0.9999	0.000016
0.33	0.6293	0.2555	0.83	0.7967	0.1140	1.33	0.9082	0.0427	1.83	0.9664	0.0132	2.33	0.9901	0.0034	2.83	0.9977	0.00069	3.33	0.9996	0.000114	3.83	0.9999	0.000015
0.34	0.6331	0.2518	0.84	0.7995	0.1120	1.34	0.9099	0.0418	1.84	0.9671	0.0129	2.34	0.9904	0.0033	2.84	0.9977	0.00066	3.34	0.9996	0.000109	3.84	0.9999	0.000014
0.35	0.6368	0.2481	0.85	0.8023	0.1100	1.35	0.9115	0.0409	1.85	0.9678	0.0126	2.35	0.9906	0.0032	2.85	0.9978	0.00064	3.35	0.9996	0.000105	3.85	0.9999	0.000014
0.36	0.6406	0.2445	0.86	0.8051	0.1080	1.36	0.9131	0.0400	1.86	0.9686	0.0123	2.36	0.9909	0.0031	2.86	0.9979	0.00062	3.36	0.9996	0.000101	3.86	0.9999	0.000013
0.37	0.6443	0.2409	0.87	0.8078	0.1061	1.37	0.9147	0.0392	1.87	0.9693	0.0119	2.37	0.9911	0.0030	2.87	0.9979	0.00060	3.37	0.9996	0.000097	3.87	0.9999	0.000013
0.38	0.6480	0.2374	0.88	0.8106	0.1042	1.38	0.9162	0.0383	1.88	0.9699	0.0116	2.38	0.9913	0.0029	2.88	0.9980	0.00058	3.38	0.9996	0.000094	3.88	0.9999	0.000012
0.39	0.6517	0.2339	0.89	0.8133	0.1023	1.39	0.9177	0.0375	1.89	0.9706	0.0113	2.39	0.9916	0.0028	2.89	0.9981	0.00056	3.39	0.9997	0.000090	3.89	0.9999	0.000012
0.40	0.6554	0.2304	0.90	0.8159	0.1004	1.40	0.9192	0.0367	1.90	0.9713	0.0111	2.40	0.9918	0.0027	2.90	0.9981	0.00054	3.40	0.9997	0.000087	3.90	1.0000	0.000011
0.41	0.6591	0.2270	0.91	0.8186	0.0986	1.41	0.9207	0.0359	1.91	0.9719	0.0108	2.41	0.9920	0.0026	2.91	0.9982	0.00052	3.41	0.9997	0.000083	3.91	1.0000	0.000011
0.42	0.6628	0.2236	0.92	0.8212	0.0968	1.42	0.9222	0.0351	1.92	0.9726	0.0105	2.42	0.9922	0.0026	2.92	0.9982	0.00051	3.42	0.9997	0.000080	3.92	1.0000	0.000010
0.43	0.6664	0.2203	0.93	0.8238	0.0950	1.43	0.9236	0.0343	1.93	0.9732	0.0102	2.43	0.9925	0.0025	2.93	0.9983	0.00049	3.43	0.9997	0.000077	3.93	1.0000	0.000010
0.44	0.6700	0.2169	0.94	0.8264	0.0933	1.44	0.9251	0.0336	1.94	0.9738	0.0100	2.44	0.9927	0.0024	2.94	0.9984	0.00047	3.44	0.9997	0.000074	3.94	1.0000	0.000009
0.45	0.6736	0.2137	0.95	0.8289	0.0916	1.45	0.9265	0.0328	1.95	0.9744	0.0097	2.45	0.9929	0.0023	2.95	0.9984	0.00046	3.45	0.9997	0.000071	3.95	1.0000	0.000009
0.46	0.6772	0.2104	0.96	0.8315	0.0899	1.46	0.9279	0.0321	1.96	0.9750	0.0094	2.46	0.9931	0.0023	2.96	0.9985	0.00044	3.46	0.9997	0.000069	3.96	1.0000	0.000009
0.47	0.6808	0.2072	0.97	0.8340	0.0882	1.47	0.9292	0.0314	1.97	0.9756	0.0092	2.47	0.9932	0.0022	2.97	0.9985	0.00042	3.47	0.9997	0.000066	3.97	1.0000	0.000008
0.48	0.6844	0.2040	0.98	0.8365	0.0865	1.48	0.9306	0.0307	1.98	0.9761	0.0090	2.48	0.9934	0.0021	2.98	0.9986	0.00041	3.48	0.9997	0.000063	3.98	1.0000	0.000008
0.49	0.6879	0.2009	0.99	0.8389	0.0849	1.49	0.9319	0.0300	1.99	0.9767	0.0087	2.49	0.9936	0.0021	2.99	0.9986	0.00040	3.49	0.9998	0.000061	3.99	1.0000	0.000007

FIGURE 2.19: Normal Distribution

k	P[u<k]	G(k)	k	P[u<k]	G(k)	k	P[u<k]	G(k)	k	P[u<k]	G(k)	k	P[u<k]	G(k)	k	P[x≤k]	G(k)	k	P[x≤k]	G(k)	k	P[x≤k]	G(k)
0.00	0.5000	0.3989	-0.50	0.3085	0.6978	-1.00	0.1587	1.0833	-1.50	0.0668	1.5293	-2.00	0.0228	2.0085	-2.50	0.0062	2.50200	-3.00	0.0013	3.000382	-3.50	0.0002	3.500058
-0.01	0.4960	0.4040	-0.51	0.3050	0.7047	-1.01	0.1562	1.0917	-1.51	0.0655	1.5386	-2.01	0.0222	2.0183	-2.51	0.0060	2.51194	-3.01	0.0013	3.010369	-3.51	0.0002	3.510056
-0.02	0.4920	0.4090	-0.52	0.3015	0.7117	-1.02	0.1539	1.1002	-1.52	0.0643	1.5480	-2.02	0.0217	2.0280	-2.52	0.0059	2.52188	-3.02	0.0013	3.020356	-3.52	0.0002	3.520054
-0.03	0.4880	0.4141	-0.53	0.2981	0.7187	-1.03	0.1515	1.1087	-1.53	0.0630	1.5574	-2.03	0.0212	2.0378	-2.53	0.0057	2.53183	-3.03	0.0012	3.030344	-3.53	0.0002	3.530052
-0.04	0.4840	0.4193	-0.54	0.2946	0.7257	-1.04	0.1492	1.1172	-1.54	0.0618	1.5667	-2.04	0.0207	2.0476	-2.54	0.0055	2.54177	-3.04	0.0012	3.040332	-3.54	0.0002	3.540050
-0.05	0.4801	0.4244	-0.55	0.2912	0.7328	-1.05	0.1469	1.1257	-1.55	0.0606	1.5761	-2.05	0.0202	2.0574	-2.55	0.0054	2.55171	-3.05	0.0011	3.050320	-3.55	0.0002	3.550048
-0.06	0.4761	0.4297	-0.56	0.2877	0.7399	-1.06	0.1446	1.1342	-1.56	0.0594	1.5855	-2.06	0.0197	2.0672	-2.56	0.0052	2.56166	-3.06	0.0011	3.060309	-3.56	0.0002	3.560046
-0.07	0.4721	0.4349	-0.57	0.2843	0.7471	-1.07	0.1423	1.1428	-1.57	0.0582	1.5949	-2.07	0.0192	2.0770	-2.57	0.0051	2.57161	-3.07	0.0011	3.070298	-3.57	0.0002	3.570044
-0.08	0.4681	0.4402	-0.58	0.2810	0.7542	-1.08	0.1401	1.1514	-1.58	0.0571	1.6044	-2.08	0.0188	2.0868	-2.58	0.0049	2.58156	-3.08	0.0010	3.080287	-3.58	0.0002	3.580042
-0.09	0.4641	0.4456	-0.59	0.2776	0.7614	-1.09	0.1379	1.1600	-1.59	0.0559	1.6138	-2.09	0.0183	2.0966	-2.59	0.0048	2.59151	-3.09	0.0010	3.090277	-3.59	0.0002	3.590041
-0.10	0.4602	0.4509	-0.60	0.2743	0.7687	-1.10	0.1357	1.1686	-1.60	0.0548	1.6232	-2.10	0.0179	2.1065	-2.60	0.0047	2.60146	-3.10	0.0010	3.100267	-3.60	0.0002	3.600039
-0.11	0.4562	0.4564	-0.61	0.2709	0.7759	-1.11	0.1335	1.1773	-1.61	0.0537	1.6327	-2.11	0.0174	2.1163	-2.61	0.0045	2.61142	-3.11	0.0009	3.110258	-3.61	0.0002	3.610038
-0.12	0.4522	0.4618	-0.62	0.2676	0.7833	-1.12	0.1314	1.1859	-1.62	0.0526	1.6422	-2.12	0.0170	2.1261	-2.62	0.0044	2.62137	-3.12	0.0009	3.120249	-3.62	0.0001	3.620036
-0.13	0.4483	0.4673	-0.63	0.2643	0.7906	-1.13	0.1292	1.1946	-1.63	0.0516	1.6516	-2.13	0.0166	2.1360	-2.63	0.0043	2.63133	-3.13	0.0009	3.130240	-3.63	0.0001	3.630035
-0.14	0.4443	0.4728	-0.64	0.2611	0.7980	-1.14	0.1271	1.2034	-1.64	0.0505	1.6611	-2.14	0.0162	2.1458	-2.64	0.0041	2.64129	-3.14	0.0008	3.140231	-3.64	0.0001	3.640033
-0.15	0.4404	0.4784	-0.65	0.2578	0.8054	-1.15	0.1251	1.2121	-1.65	0.0495	1.6706	-2.15	0.0158	2.1556	-2.65	0.0040	2.65125	-3.15	0.0008	3.150223	-3.65	0.0001	3.650032
-0.16	0.4364	0.4840	-0.66	0.2546	0.8128	-1.16	0.1230	1.2209	-1.66	0.0485	1.6801	-2.16	0.0154	2.1655	-2.66	0.0039	2.66121	-3.16	0.0008	3.160215	-3.66	0.0001	3.660031
-0.17	0.4325	0.4897	-0.67	0.2514	0.8203	-1.17	0.1210	1.2296	-1.67	0.0475	1.6897	-2.17	0.0150	2.1753	-2.67	0.0038	2.67117	-3.17	0.0008	3.170207	-3.67	0.0001	3.670029
-0.18	0.4286	0.4954	-0.68	0.2483	0.8278	-1.18	0.1190	1.2384	-1.68	0.0465	1.6992	-2.18	0.0146	2.1852	-2.68	0.0037	2.68113	-3.18	0.0007	3.180199	-3.68	0.0001	3.680028
-0.19	0.4247	0.5011	-0.69	0.2451	0.8353	-1.19	0.1170	1.2473	-1.69	0.0455	1.7087	-2.19	0.0143	2.1950	-2.69	0.0036	2.69110	-3.19	0.0007	3.190192	-3.69	0.0001	3.690027
-0.20	0.4207	0.5069	-0.70	0.2420	0.8429	-1.20	0.1151	1.2561	-1.70	0.0446	1.7183	-2.20	0.0139	2.2049	-2.70	0.0035	2.70106	-3.20	0.0007	3.200185	-3.70	0.0001	3.700026
-0.21	0.4168	0.5127	-0.71	0.2389	0.8505	-1.21	0.1131	1.2650	-1.71	0.0436	1.7278	-2.21	0.0136	2.2147	-2.71	0.0034	2.71103	-3.21	0.0007	3.210178	-3.71	0.0001	3.710025
-0.22	0.4129	0.5186	-0.72	0.2358	0.8581	-1.22	0.1112	1.2738	-1.72	0.0427	1.7374	-2.22	0.0132	2.2246	-2.72	0.0033	2.72099	-3.22	0.0006	3.220172	-3.72	0.0001	3.720024
-0.23	0.4090	0.5244	-0.73	0.2327	0.8658	-1.23	0.1093	1.2827	-1.73	0.0418	1.7470	-2.23	0.0129	2.2345	-2.73	0.0032	2.73096	-3.23	0.0006	3.230166	-3.73	0.0001	3.730023
-0.24	0.4052	0.5304	-0.74	0.2296	0.8734	-1.24	0.1075	1.2917	-1.74	0.0409	1.7566	-2.24	0.0125	2.2444	-2.74	0.0031	2.74093	-3.24	0.0006	3.240160	-3.74	0.0001	3.740022
-0.25	0.4013	0.5363	-0.75	0.2266	0.8812	-1.25	0.1056	1.3006	-1.75	0.0401	1.7662	-2.25	0.0122	2.2542	-2.75	0.0030	2.75090	-3.25	0.0006	3.250154	-3.75	0.0001	3.750021
-0.26	0.3974	0.5424	-0.76	0.2236	0.8889	-1.26	0.1038	1.3095	-1.76	0.0392	1.7758	-2.26	0.0119	2.2641	-2.76	0.0029	2.76087	-3.26	0.0006	3.260148	-3.76	0.0001	3.760020
-0.27	0.3936	0.5484	-0.77	0.2206	0.8967	-1.27	0.1020	1.3185	-1.77	0.0384	1.7854	-2.27	0.0116	2.2740	-2.77	0.0028	2.77084	-3.27	0.0005	3.270143	-3.77	0.0001	3.770019
-0.28	0.3897	0.5545	-0.78	0.2177	0.9045	-1.28	0.1003	1.3275	-1.78	0.0375	1.7950	-2.28	0.0113	2.2839	-2.78	0.0027	2.78081	-3.28	0.0005	3.280137	-3.78	0.0001	3.780019
-0.29	0.3859	0.5606	-0.79	0.2148	0.9123	-1.29	0.0985	1.3365	-1.79	0.0367	1.8046	-2.29	0.0110	2.2938	-2.79	0.0026	2.79079	-3.29	0.0005	3.290132	-3.79	0.0001	3.790018
-0.30	0.3821	0.5668	-0.80	0.2119	0.9202	-1.30	0.0968	1.3455	-1.80	0.0359	1.8143	-2.30	0.0107	2.3037	-2.80	0.0026	2.80076	-3.30	0.0005	3.300127	-3.80	0.0001	3.800017
-0.31	0.3783	0.5730	-0.81	0.2090	0.9281	-1.31	0.0951	1.3546	-1.81	0.0351	1.8239	-2.31	0.0104	2.3136	-2.81	0.0025	2.81074	-3.31	0.0005	3.310123	-3.81	0.0001	3.810016
-0.32	0.3745	0.5792	-0.82	0.2061	0.9360	-1.32	0.0934	1.3636	-1.82	0.0344	1.8336	-2.32	0.0102	2.3235	-2.82	0.0024	2.82071	-3.32	0.0005	3.320118	-3.82	0.0001	3.820016
-0.33	0.3707	0.5855	-0.83	0.2033	0.9440	-1.33	0.0918	1.3727	-1.83	0.0336	1.8432	-2.33	0.0099	2.3334	-2.83	0.0023	2.83069	-3.33	0.0004	3.330114	-3.83	0.0001	3.830015
-0.34	0.3669	0.5918	-0.84	0.2005	0.9520	-1.34	0.0901	1.3818	-1.84	0.0329	1.8529	-2.34	0.0096	2.3433	-2.84	0.0023	2.84066	-3.34	0.0004	3.340109	-3.84	0.0001	3.840014
-0.35	0.3632	0.5981	-0.85	0.1977	0.9600	-1.35	0.0885	1.3909	-1.85	0.0322	1.8626	-2.35	0.0094	2.3532	-2.85	0.0022	2.85064	-3.35	0.0004	3.350105	-3.85	0.0001	3.850014
-0.36	0.3594	0.6045	-0.86	0.1949	0.9680	-1.36	0.0869	1.4000	-1.86	0.0314	1.8723	-2.36	0.0091	2.3631	-2.86	0.0021	2.86062	-3.36	0.0004	3.360101	-3.86	0.0001	3.860013
-0.37	0.3557	0.6109	-0.87	0.1922	0.9761	-1.37	0.0853	1.4092	-1.87	0.0307	1.8819	-2.37	0.0089	2.3730	-2.87	0.0021	2.87060	-3.37	0.0004	3.370097	-3.87	0.0001	3.870013
-0.38	0.3520	0.6174	-0.88	0.1894	0.9842	-1.38	0.0838	1.4183	-1.88	0.0301	1.8916	-2.38	0.0087	2.3829	-2.88	0.0020	2.88058	-3.38	0.0004	3.380094	-3.88	0.0001	3.880012
-0.39	0.3483	0.6239	-0.89	0.1867	0.9923	-1.39	0.0823	1.4275	-1.89	0.0294	1.9013	-2.39	0.0084	2.3928	-2.89	0.0019	2.89056	-3.39	0.0003	3.390090	-3.89	0.0001	3.890012
-0.40	0.3446	0.6304	-0.90	0.1841	1.0004	-1.40	0.0808	1.4367	-1.90	0.0287	1.9111	-2.40	0.0082	2.4027	-2.90	0.0019	2.90054	-3.40	0.0003	3.400087	-3.90	0.0000	3.900011
-0.41	0.3409	0.6370	-0.91	0.1814	1.0086	-1.41	0.0793	1.4459	-1.91	0.0281	1.9208	-2.41	0.0080	2.4126	-2.91	0.0018	2.91052	-3.41	0.0003	3.410083	-3.91	0.0000	3.910011
-0.42	0.3372	0.6436	-0.92	0.1788	1.0168	-1.42	0.0778	1.4551	-1.92	0.0274	1.9305	-2.42	0.0078	2.4226	-2.92	0.0018	2.92051	-3.42	0.0003	3.420080	-3.92	0.0000	3.920010
-0.43	0.3336	0.6503	-0.93	0.1762	1.0250	-1.43	0.0764	1.4643	-1.93	0.0268	1.9402	-2.43	0.0075	2.4325	-2.93	0.0017	2.93049	-3.43	0.0003	3.430077	-3.93	0.0000	3.930010
-0.44	0.3300	0.6569	-0.94	0.1736	1.0333	-1.44	0.0749	1.4736	-1.94	0.0262	1.9500	-2.44	0.0073	2.4424	-2.94	0.0016	2.94047	-3.44	0.0003	3.440074	-3.94	0.0000	3.940009
-0.45	0.3264	0.6637	-0.95	0.1711	1.0416	-1.45	0.0735	1.4828	-1.95	0.0256	1.9597	-2.45	0.0071	2.4523	-2.95	0.0016	2.95046	-3.45	0.0003	3.450071	-3.95	0.0000	3.950009
-0.46	0.3228	0.6704	-0.96	0.1685	1.0499	-1.46	0.0721	1.4921	-1.96	0.0250	1.9694	-2.46	0.0069	2.4623	-2.96	0.0015	2.96044	-3.46	0.0003	3.460069	-3.96	0.0000	3.960009
-0.47	0.3192	0.6772	-0.97	0.1660	1.0582	-1.47	0.0708	1.5014	-1.97	0.0244	1.9792	-2.47	0.0068	2.4722	-2.97	0.0015	2.97042	-3.47	0.0003	3.470066	-3.97	0.0000	3.970008
-0.48	0.3156	0.6840	-0.98	0.1635	1.0665	-1.48	0.0694	1.5107	-1.98	0.0239	1.9890	-2.48	0.0066	2.4821	-2.98	0.0014	2.98041	-3.48	0.0003	3.480063	-3.98	0.0000	3.980008
-0.49	0.3121	0.6909	-0.99	0.1611	1.0749	-1.49	0.0681	1.5200	-1.99	0.0233	1.9987	-2.49	0.0064	2.4921	-2.99	0.0014	2.99040	-3.49	0.0002	3.490061	-3.99	0.0000	3.990007

FIGURE 2.20: Normal Distribution

2.5 Warehousing

2.5.1 Summary

We now move into an important, yet often under-explored component of the supply chain, warehouses. Warehouses as are often overlooked because they generally do not add value to a product, but are intermediary stations in the supply chain. Warehouses store, handle and/or flow product. Their primary operation functions are to receive; put-away; store; pick, pack and ship product. In some cases they play a role Value-Add Services such as labeling, tagging, special packaging, minor assembling, kitting, re-pricing, etc. In addition, sometime they play a role in returns. The main approach to assessing warehouse performance is to profile its activity and benchmark. With continuous assessment and feedback, efficiencies at a warehouse can be improved.

Businesses typically have warehouses to better match supply and demand. Supply is not always in sync with what is demanded at the store. Having warehouses serves as a buffer for unexpected shortages and demands. In this lesson we will cover warehouse basics of the different types of warehouses, their core operational functions, and common flow patterns. We then review each of the major functions, what is entailed, and how best to optimize practices. We conclude with different ways of assessing performance for best performance of warehouses.

2.5.2 Warehousing Basics

Based on needs, companies select different warehouses. The warehouses can simply be a place to store additional product or can go all the way to serving a partial assembly and finishing stage. Within the warehouse there are two competing priorities: space and time. This means that they want to maximize their utilization of space and optimize throughput. These are the types of warehouses and their function:

- **Raw Material Storage** – close to a source or manufacturing points
- **WIP Warehouses** – partially completed assemblies and components
- **Finished Goods warehouses** – buffers located near point of manufacture
- **Local Warehouses** – in the field near customer locations to provide rapid response to customers
- **Fulfillment Centers** – holds product and ships small orders to individual consumers (cases or eaches) – predominately for e-commerce
- **Distribution Centers** – accumulate and consolidate products from multiple sources for common shipment to common destination/customer
- **Mixing Centers** – receives material from multiple sources for cross-docking and shipment of mixed materials (pallets to pallets)

Package Size

Because warehouses are constantly concerned about saving space, this means that the package size is of great concern. There are several principles in package sizing. The general **Handling Rule** is: The smaller the handling unit, the greater the handling cost. As well as that in general, the unit of storage for a product gets smaller as it moves downstream from container to pallet to case to eaches. Size impacts design and operations with an inbound and outbound flow from pallets to pallets, pallets to cases and pallets to eaches.

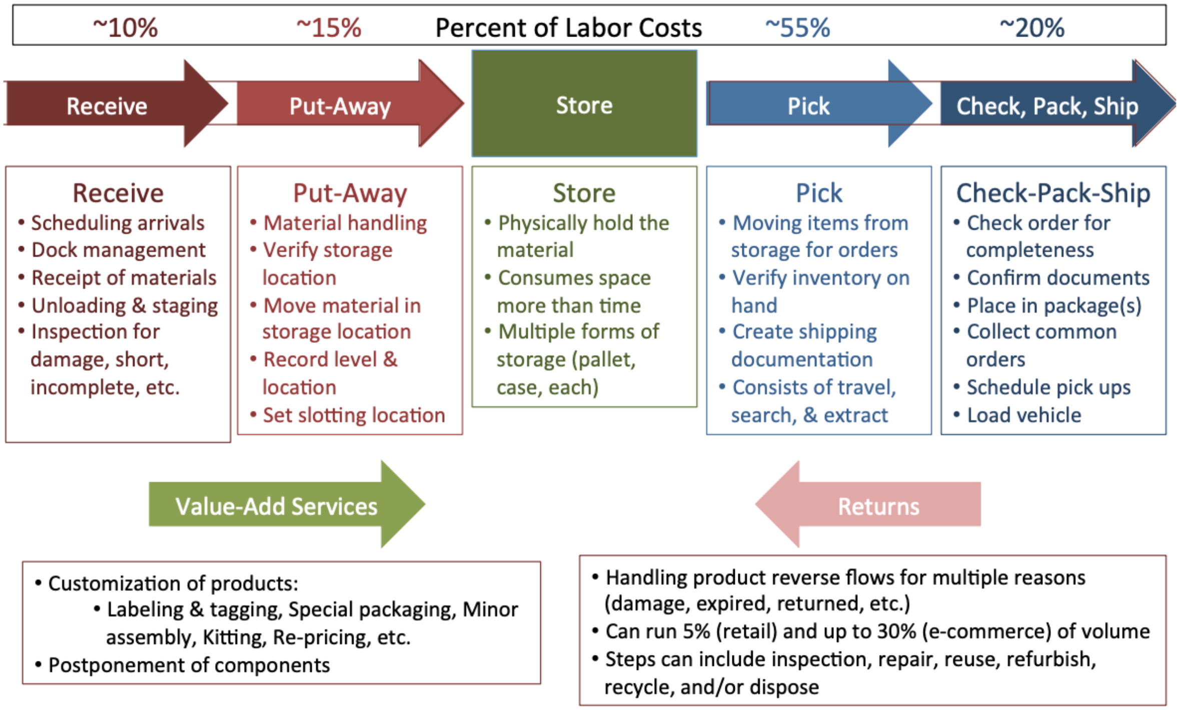

FIGURE 2.21: Core Operational Functions

2.5.3 Core Operational Functions

There are several core operational functions in warehouses beyond storage. These include receiving; putaway; pick; check, pack; ship; as well as additional but not always included steps such as value-added services and returns. See figure 5.14.

Receiving

The receiving function of the warehouse is one of the most important because it sets up the interaction at the warehouse as well as next steps. There are some generally agreed upon best practices which include:

- Use ASNs (advanced shipping notice)
- Integrate yard and dock scheduling
- Prepare for shipment at receiving
- The best option is to minimize receiving activity
- Pursue drop shipping whenever possible
- IF drop ship is not possible, explore cross-docking

Putaway

The putaway function is essentially order picking in reverse. The Warehouse Management System (WMS) plays a significant role in this step by determining storage location for received items (slotting). It also directs staff where to place product and records inventory level. Required data for WMS include: size, weight, cube, height, segmentation status, current orders, current status of pick face as well as identification of products and locations.

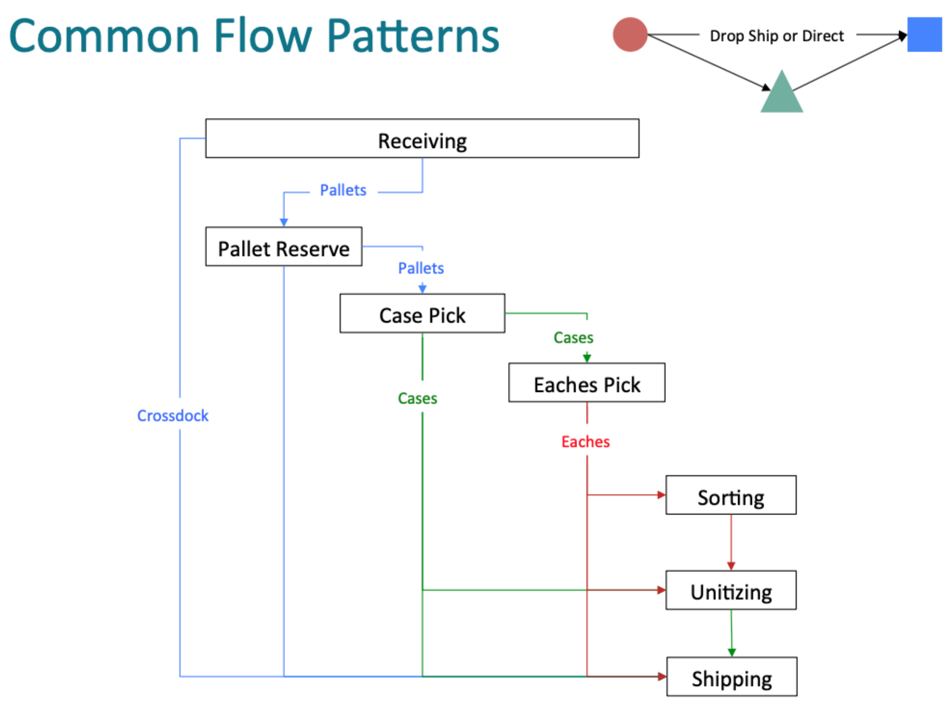

FIGURE 2.22: Common Flow Patterns

There are different approaches to the putaway function, which can be directed (specific location ahead of time). It can be batched & sequenced which means there is a pre-sort at staging for commonly located items. Or it can be chaotic, where the user picks any location and records item-location.

Order Picking

Order Picking is the most labor-intensive task 50-60%. Picking strategies change based on the size of the object being picked. For instance, full pallet retrieval is the easiest and fastest. Case picking being the next in ease and small item picking being the most expensive and time consuming. The break down of order picking effort is:

- Traveling 55%
- Searching 15%
- Extracting 10%
- Other tasks 20%

Layout When selecting a placement for items, the warehouse is typically set up with a flow between receiving and shipping. The Flow-Through Design places the most convenient items directly in line with receiving and shipping. A convenient location is one that minimizes total labor time (distance) to putaway and retrieve.

$$\text{Min} \quad c\sum_i (d_i n_i)$$

where:
c = labor cost per distance
d_i = distance for pallet location i from receiving to location to shipping

n_i = average number of times location i is visited per year
$\cong$ # pallets sold / # pallets in order

Simple Heuristic:

1. Rank all positions from low to high d_i
2. Rank all SKUs from high to low n_j
3. Assign next highest SKU (n_j) to next lowest location (d_i)

Aisle Layout
In terms of aisle layout, it is typically best to have aisles parallel to the flow to avoid inconveniences in flow. Cross aisles in a warehouse can shorten distances, but they take up significantly more space and also increase the amount of aisle crossing. Angled or fishbone aisle can increase efficiency especially when there is a central dispatch point. In addition, fast moving items should be put in convenience locations.

Picking Strategies

Single Picker – Single Order: good for low number of lines/order; suitable for short pick paths; no need to marry the orders afterwards; travel time can be high

Single Picker – Multiple Orders: expected travel time (distance) per item is reduced; requires sorting and "marrying" items; can sort "on-cart" or after tour; works for both picker-to-stock and stock-to-picker.

Multiple Picker - Multiple Orders: well suited for orders with high line count; expected travel time per item is reduced; minimizes congestion & socializing in pick aisles; pickers can become "experts" in a zone but lose order completion accountability; requires sorting and consolidation of items; allows for simultaneous filling of orders; difficult to balance workload across zones.

Check, Pack & Ship

The final function of standard warehouse functions is check, pack & ship. Checking includes creating and verifying shipping labels and confirming weight and cube. Packing consists of ensure damage protection and unitize pallets. Shipping is the final step; it is essentially the reverse of receiving. Shipping activities include dock door and yard management; minimizing staging requirements; and container/trailer loading optimization.

2.5.4 Profiling & Assessing Performance

Warehouse Activity Profile

When organizations are either designing a new warehouse/DC or revamping an existing one, there needs to be some advance critical thinking. For instance, a few data points that are worth looking into include:

- Number of SKUs in the warehouse
- Number of pick-lines per day & number of units per pick-line
- Number & size of customer orders shipped and shipments received per day
- Rate of new SKU introductions and respective lifecycle

When evaluating these data points, we need to make sure to look at the distribution not just the averages to understand peaks and dips over time. The data sources are typically in the Master SKU data, order history, and warehouse location.

Segmentation Analysis

We were first introduced to segmentation analysis for demand planning, however, it can be applied to warehousing as well. Segmentation can provide some important insights for warehouse design. Different segmentation views give different insights:

Frequency of SKUs sold: Top selling SKUs influence retail operations – not necessarily warehouse operations.

Frequency of pallets/cases/cartons by SKU: Will not necessarily follow SKU frequency; provides insights into receiving, putaway, and restocking.

Frequency of picks by SKU: Order picking drives most labor costs; determines slotting and forward pick locations.

It is also important to note that there is common variability of demand that is affected by seasonality - such as by year, quarter, day or week, time of day. There is also correlation to other products (affinity between items and families). These can all have an influence on how a warehouse/DC should be designed.

Measuring and Benchmarking

To best understand the effective operation of a warehouse, there are a few ways to measure and benchmark activity. First, it is important to understand where the major cost drivers are: labor, space, and equipment. Regular assessment of these can provide feedback on surges and dips of spend. Next there are key performance measures that provide information about the activity in the warehouse and can be used to make operational decisions.

Major Warehousing Cost Drivers
Labor = (person-hours/year) x (labor rate)
Space = (area occupied) x (cost of space)
Equipment = (money invested) x (amortization rate)

Performance Measures
Productivity/Efficiency: Ratio of output to the inputs required; e.g., labor = (units, cases, or pallets) / (labor hours expended).

Utilization: Percentage of an asset being actively used; e.g., storage density = (storage capacity in WH) / (total area of WH)

Quality / Effectiveness: Accuracy in putaway, inventory, picking, shipping, etc.

Cycle Time: Dock-to-Stock time – time from receipt to being ready to be picked; Order Cycle Time – time from when order is dropped until it is ready to ship.

Learning Objectives

- Review types of warehouse and recognize their primary use.
- Understand the core functions of the warehouse.
- Become familiar with common flow patterns of a warehouse.
- Review the activities within each of the functions and how to optimize them.
- Recognize how to assess and benchmark warehouse activity.

References

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Part III

SC2x - Supply Chain Design

3

Supply Chain Design

CONTENTS

3.1 Introduction to Supply Chain Design

3.1.1 Summary

Supply Chain Design is as much an art as it is science. Science because we can quantify the impact of different choices and find the optimal trade-offs between different choice parameters. But also, an art because the assumptions we make are never going to completely match reality, and because the data we use for our models is never going to be completely accurate. There is no single right way of making these assumptions – doing this in a "good" way for a particular problem is more art than science!

Still, we learned that there are some frameworks, tools, and methods to aid the design process. The course examines the flows in a supply chain, with each design issue discussed in a separate module:

- **Design of Physical Flows.** How should materials flow through the supply chain? We will model physical flows taking into account the costs of transportation and facilities. We will balance costs and service using Mixed Integer Programs. However, key is in not just solving the models, but in interpreting the results. Remember, the tools are decision-*support* tools, they are intended to support decisions, not make them for us!
- **Design of Financial Flows.** How to translate supply chain concepts and actions into the language of the CFO (Chief Financial Officer)? We will go through activity-based costing, cash flow analysis and capital budgeting to better understand how supply chain design decisions translate to changes in the income statement and the balance sheet.
- **Design of Information Flows.** For this module, we will follow the SCOR-model's three phases of Source, Make, and Deliver. How should you work with suppliers? How should information be coordinated between different manufacturers, internal and external? And how should you coordinate and collaborate with customers? In this module strategies and procedures for this is discussed.
- **Designing the organization.** How should a supply chain be organized? We will investigate supply chain processes and how the measuring of the performance of the processes create different incentives. We will also discuss organizational structure.

3.1.2 Key Concepts

- **Supply Chains** – Two or more parties linked by a flow of resources that ultimately fulfill a customer request.
- **Supply Chain Flows** – physical flows, financial flows, and information flows. All are important to consider when designing a supply chain.

3.1.3 Review of SC1x Concepts

Demand forecasting

Forecasting truisms:

- Forecast are always wrong – use ranges and track forecast error
- Aggregate forecasts are more accurate – risk pooling reduces coefficient of variation
- Shorter time forecasts are more accurate – postpone customization as late as possible

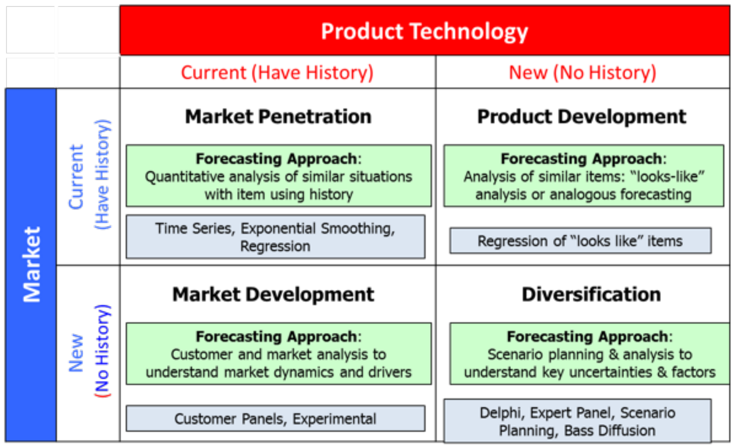

FIGURE 3.1: Product forecasting
Adapted from Kahn & Kennet (2006) New Product Forecasting

Inventory management

Inventory is generally managed using one of two key types of policies: continuous review or periodic review.

- *Continuous review* means that the inventory position is continuously supervised (presumably by software). As soon as the inventory position reaches a pre-determined level s a replenishment order of Q units is placed. Consequently, the time between orders is uncertain.
- *Periodic review* means that the inventory position is reviewed at certain reoccurring points in time, such as every evening after a retailer closes the store. Based on the current inventory position, a replenishment order is placed to bring the inventory position up to a pre-determined level S. Consequently, the order quantity is uncertain.

Both policies are used when decisions are made for a long horizon, where the items can be stored between periods of replenishment, but demand in each replenishment period is uncertain.

Transportation

There are several ways of organizing the transportation in a supply chain. Among them:

- One-to-one: direct point-to-point movements from origin to destination, e.g. daily full van loads to each customer
- One-to-many: multi-stop moves from a single origin to many destinations
- Many-to-many: moving from multiple origins to multiple destinations usually with a hub or terminal – this decouples line haul (from e.g. a supplier to a terminal) from local delivery operations (from e.g. a terminal to stores)

Total cost equation

The total cost equation specifies the total logistics cost for a system for an arbitrary time period:

$$\text{Total cost} = \text{Purchase Cost} + \text{Order (Set Up) Cost} + \text{Holding (Carrying) Cost} + \text{Shortage Cost}$$

- Purchase: Cost per item or total landed cost for acquiring product.

- Ordering: It is a fixed cost and contains cost to place, receive and process a batch of goods including processing invoicing, auditing, labor, etc. In manufacturing this is the set up cost for a run.
- Holding: Costs required to hold inventory such as storage cost (warehouse space), service costs (insurance, taxes), risk costs (lost, stolen, damaged, obsolete), and capital costs, both for units in-transit (pipeline inventory) and in warehouse (cycle stock + safety stock)
- Shortage: Costs of not having an item in stock including backorder, lost sales, lost customers, and disruption costs.

Notation

- D: Demand rate (units/time)
- Q: Order quantity (units)
- L: Lead time (time)
- σ_{DL}: Standard deviation of demand during the lead time
- k: Safety stock factor
- c: Purchase cost ($/unit)
- c_t: Ordering Costs ($/order)
- h: Holding rate – usually expressed as a percentage ($/$ value/time)
- c_e: Excess holding Costs ($/unit-time); also equal to c_h
- c_s: Shortage costs ($/unit)
- TC: Total Costs – the sum of all four cost elements

We get that

$$TC = cD + c_t\left(\frac{D}{Q}\right) + c_e\left(\frac{Q}{2} + k\sigma_{DL} + DL\right) + c_s P[StockOutType]$$

From the formula we see that transport speed as well as forecast accuracy has an impact on total cost through the inventory costs.

Fisher's model: Innovative versus functional products

When designing a supply chain, the design needs to be adapted to the type of products it is intended for. In Fisher's model, products can be classified as being either more Functional or more Innovative.

- *Functional:* predictable demand, long life cycle, low margin, low error in production, low stock-out rates
- *Innovative:* unpredictable demand, short life cycle, high margin, high error in production, high stock-out rates

As a rule of thumb, functional products should have a design focusing more on efficiency, whereas innovative products should have a supply chain design focusing more on matching supply with demand.

Remember that these are not hard and fast rules. In practice there are, of course, many products that share characteristics with both segments. Most firms are going to have a portfolio of supply chains. Further, innovative products often move into becoming more functional as their markets become more mature. This requires adaptation of the supply chain design. For instance, when the patent protection for the cholesterol lowering drug Lipitor went out, Pfizer had to reduce the price to protect the drug from generic competition. With a lower margin, the supply chain must be design with a higher focus on efficiency.

SCOR-model

The Source-Make-Deliver process of the SCOR-model describes the business processes required to satisfy customers' demand.

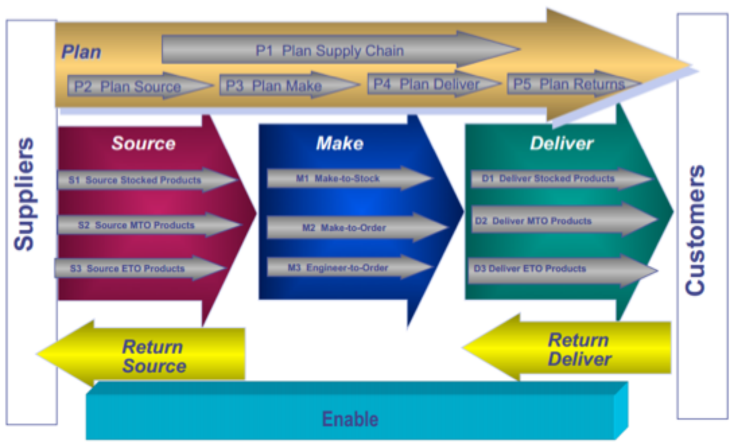

FIGURE 3.2: Supply Chain Operations Reference (SCOR) model
Source: Supply Chain Council

Learning Objectives

- Identify physical, financial, and information flows inherent to supply chains
- Recap SC1x: demand forecasting, inventory management, transportation
- How to approach Supply Chain Design for different products and companies

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3.2 Supply Chain Network Design Models

3.2.1 Summary

We review supply chain network design. Sometimes it is referred to as network modeling because you need to build a mathematical model of the supply chain. This model is then solved using optimization techniques and then analyzed to pick the best solution. Specifically, we will focus on modeling the supply chain to determine the optimal location of facilities (plants, warehouses, etc.) and the best flow of products through this facility network structure.

3.2.2 Key Concepts

3.2.3 Network Models

Network models are a useful class of models that can be utilized to aid many types of supply chain decisions. We review the basic notations of these models and show how to set up a simple spreadsheet model in Excel or Libre Office to solve the model. We focus on two types of models that are relatively simple yet powerful in deriving insights to support supply chain design decisions: the Transportation Problem and the Transshipment Problem.

Both the transportation problem and the transshipment problem consider an underlying network of nodes (facilities) and arcs (transport flows between facilities). The objective is to minimize costs for a given period (day, week, year) by choosing the number of units to be shipped/transported on different arcs during the period to fulfill demand, while meeting the capacity constraints. Both problems can be formulated as linear programs (see below) and solved using your spreadsheet software of choice (e.g. Excel or Libre Office). For details on how to run the solver, please refer to the separate guide for how to setup and run the solver, which is found in the course material.

We discuss some of the limitations to these network models. First, in the models review, we limited ourselves to *variable costs for the arcs* (i.e. transport costs per unit). Clearly, the cost structure for a network is, in practice, often more complex, involving costs that are both fixed or vary over different parameters. We also considered a single *commodity* – all plants produced perfectly substitutable goods. The demand was *deterministic*, that is, we assumed that demand was perfectly known for the entire period. Finally, we also assumed there was no capacity limits on the arcs, so that any amount could be transported. Clearly, all of these assumptions limit our results, so we have to be careful what inferences we make from the models. In the coming lessons we will relax some of these and also discuss how to make inference while still using a relatively simple model.

Also – we noted that transportation and transshipment problems may have several optima (the same value of the objective function is found by different value on the decision variables).

Depending on the algorithm and software, different optima may be the "first choice" of the algorithm.

Network terminology

- **Node or vertices** – a point (facility, DC, plant, region)
- **Arc or edge** – link between two nodes (roads, flows, etc.)
- **Network or graph** – a collection of nodes and arcs

The transportation problem

The transportation problem considers transports from i supply nodes to j demand nodes over arcs ij. With each arc is associated a cost c_{ij}. The number of units transported on each arc ij is denoted x_{ij}. These x_{ij}'s are our decision variables – we want to find the amounts for each arc that minimizes total cost.

Let z be the objective function (i.e., the function expressing the total cost we want to minimize).

Notation

- Indices
  - Supply nodes i
  - Demand nodes j
- Input data
  - S_i = Available supply from supply node i (units) $\quad \forall i \in S$
  - D_j = Demand in demand node j (units) $\quad \forall j \in D$
  - c_{ij} = Cost for sending one unit from supply node i to demand node j (\$/unit) $\quad \forall i, j$

- Decision variables
  - x_{ij} = Flow on arc from supply node i to demand node j $\quad \forall i, j$

Mathematical model

$$Min \quad z = \sum_i \sum_j c_{ij} x_{ij} \tag{3.1}$$

s.t.

$$\sum_j x_{ij} \leq S_i \quad \forall i \in S \tag{3.2}$$

$$\sum_i x_{ij} \geq D_j \quad \forall j \in D \tag{3.3}$$

$$x_{ij} \geq 0 \quad \forall ij \tag{3.4}$$

Equations 3.2 are the supply constraints - the total number of shipped units from a supply node i to **all** demand nodes j must be less than (or equal to) the supply capacity of node i. Equations 3.3 are the demand constraints - the number of shipped units to a demand node j from **all** supply nodes i must be at least the demand at node j. Equations 3.4 are the non-negativity constraints - we do not allow negative volumes on any arc.

The transshipment problem

The transshipment problem is similar to the transportation problem. The difference is that we now introduce a set of nodes that are neither supply nodes nor demand nodes, but transshipment nodes, meaning that over a period, anything that goes in to the transshipment node needs to come out.

Notation

- Indices
  - Supply nodes i
  - Demand nodes j
- Input data
  - S_i = Available supply from supply node i (units) $\quad \forall i \in S$
  - D_j = Demand in demand node j (units) $\quad \forall j \in D$
  - c_{ij} = Cost for sending one unit from supply node i to demand node j (\$/unit) $\quad \forall i, j$
- Decision variables
  - x_{ij} = Flow on arc from supply node i to demand node j $\quad \forall i, j$

Mathematical model

$$Min \quad z = \sum_i \sum_j c_{ij} x_{ij} \tag{3.5}$$

s.t.

$$\sum_j x_{ij} \leq S_i \quad \forall i \in S \tag{3.6}$$

$$\sum_i x_{ij} \geq D_j \quad \forall j \in D \tag{3.7}$$

$$\sum_i x_{ij} - \sum_i x_{ji} = 0 \quad \forall j \notin D, \notin S \tag{3.8}$$

$$x_{ij} \geq 0 \quad \forall ij \tag{3.9}$$

Note that we have the same formulation as for the transportation problem **but** with one difference: Equations 3.8 have been added. This constraint is the constraint forcing the transshipment node to be "empty" – the *conservation of flow* constraint. Consider transshipment node j. The first sum in the constraint is then the total number of units shipped to node j from all other nodes i. The second sum is the total number of units shipped from node j to all other nodes i. The constraint says that this difference must be zero.

3.2.4 Facility Location Models

In the previously introduce models we assumed that every facility in the network was used. We now relax that assumption. First, we review where to locate a facility, given that we need a *single facility*. We then investigate how many (and which) facilities to use, given a set of candidates. Lastly, we review how to incorporate explicit Level Of Service (LOS) constraints in our models, to identify certain cost and service trade-offs in more detail.

For the *single facility* case we look at two fundamentally different ways of approaching the decision. The first was by considering all points in the Euclidian space as potential candidates, and search for an optimal location for a facility anywhere in this space. For this we use both the center-of-gravity- method and the Weber method. While the former has an intuitive appeal, the Weber method is more appropriate as it minimizes the actual transportation costs. The fundamental problem of relying on either of these methods, though, is that the optimal solution may end up in places that are not feasible for a wide number of reasons: lack of infrastructure (we could end up in a lake!), high construction costs, difficulty of getting permits etc. So while these methods are useful to get a ball park figure of where to locate the facility, they provide only a region to target.

We review another approach where, instead of considering the full Euclidian space, there is only a finite set of candidate locations. This problem can be approached using a Mixed Integer Linear Program (MILP), for which we can use a spreadsheet solver to find the solution.

MILP used to solve the single facility problem could be easily extended to investigate the case of multiple facilities. The model could then be used to answer both how many facilities to use, and which facilities to use.

Finally, we incorporate explicit LOS constraints in our models. Two types of service performance were considered: the average weighted distance to customers, and the amount of demand within a certain distance from a DC. We can specify bounds for these performance measures and include them in the MILP, to ensure that the optimal solution met the LOS requirements.

Continuous single facility problem

With a continuous single facility model, the aim is to find one optimal point in the Euclidian space.

Center of gravity. With a center of gravity model, we let the optimal point's coordinates be given by weighted x and y coordinates, where the weight given to each node k is given by the demand in that node. For instance, if we want to find the optimal location for a DC that will support 3 stores, the optimal location will be given by the weighted coordinates of the stores, where each store's coordinates are weighted by the demand at the store.

$$x = \sum_{k \in K} w_k x_k \tag{3.10}$$

$$y = \sum_{k \in K} w_k y_k \tag{3.11}$$

Weber method. With the Weber method, instead of taking the weighted coordinates of the nodes, we try to minimize the weighted Euclidian distances between nodes k and location (x, y). The weights are still given by the demand at the different nodes.

$$Min \quad z = \sum_{k \in K} w_k d_k(x, y) = \sum_{k \in K} w_k \sqrt{(x - x_k)^2 + (y - y_k)^2} \tag{3.12}$$

Where z is optimized by changing the decision variables x and y.

Network facility location problem

When we have a number of candidate locations to choose between, we can create a network flow model to find which of the candidate locations provide the lowest cost. To do this, we formulate a Mixed Integer Linear Program (MILP). Our problem is MILP because it has a linear objective function and linear constraints, but with some variables being integers instead of continuous. The integer values are the Y_i's that describe whether facility i is used ($Y_i = 1$) or not ($Y_i = 0$). Associated with each facility is a fixed cost for the time period, f_i. As with the transportation problem, the number of units shipped between two nodes are given by the x_{ij}'s.

3.2.4.1 Notation

- M_{ij} : An arbitrary large number, specific to each arc (could be the same for all arcs)
- P_{min} : Minimum number of facilities
- P_{max} : Maximum number of facilities

With z being the objective function, the problem is formulated as follows:

$$Min \quad z = \sum_i \sum_j c_{ij} x_{ij} + \sum_i f_i Y_i \tag{3.13}$$

s.t.

$$\sum_j x_{ij} \le S_i \quad \forall i \in S \tag{3.14}$$

$$\sum_i x_{ij} \ge D_j \quad \forall j \in D \tag{3.15}$$

$$x_{ij} - M_{ij} Y_i \le 0 \quad \forall ij \tag{3.16}$$

$$\sum_i Y_i \ge P_{MIN} \tag{3.17}$$

$$\sum_i Y_i \le P_{MAX} \tag{3.18}$$

$$x_{ij} \ge 0 \quad \forall ij \tag{3.19}$$

$$Y_i = \{0, 1\} \quad \forall i \tag{3.20}$$

Equations 3.14 and 3.15 are the supply and demand constraints.

Equations 3.16 are the *linking constraints* - to ensure that we do not allocate shipments to a location that is not used, the units shipped on an arc must be less than (or equal to) a large number of times the Y associated with the location node.

Equations 3.17 and 3.18 are constraints on the number of the facilities to use - the sum of the Y variables will be the total number of facilities in use.

Equations 3.19 and 3.20 are the non-negativity constraints for (x's) and the binary constraints (for the Y's).

Note that for a single facility location problem, we let $P_{MIN} = P_{MAX} = 1$.

Multiple location selection model

For this we use the same MILP as for the network facility location problem above. If we have a given number of locations we want to choose, we let $P_{MIN} = P_{MAX} = k$, where k is the number of locations. If we instead want to find the optimal number of locations, we remove the constraints on the sum of Y.

Enforcing LOS

To enforce LOS, we use the same basic setup as before, with the highlighted addition:

- $S_i =$ Available supply at DC i (units) $\quad \forall i \in S$
- $D_j =$ Demand by Customer j (units) $\quad \forall j \in D$
- $c_{ij} =$ Cost to serve Customer j from DC i (\$/unit) $\quad \forall ij$
- $f_i =$ Fixed cost for opening DC i (\$) $\quad \forall i \in S$
- $P_{MIN} =$ Minimum number of DCs required to open
- $P_{MAX} =$ Maximum number of DCs allowed to open
- $M =$ A really big number, but not too big!
- $d_{ij} =$ Distance to Customer j from DC i (miles) $\quad \forall ij$
- $a_{ij} = 1$ if Customer j to DC $i \leq 50$ miles, $= 0$ otherwise $\quad \forall ij$
- $MaxAvgDist =$ Max allowable average DCs to Customers
- $MinPctIn50 =$ Min allowable demand within 50 miles of a DC

$$Min \quad z = \sum_i \sum_j c_{ij} x_{ij} + \sum_i f_i Y_i \tag{3.21}$$

s.t.

$$\sum_j x_{ij} \leq S_i \quad \forall i \in S \tag{3.22}$$

$$\sum_i x_{ij} \geq D_j \quad \forall j \in D \tag{3.23}$$

$$x_{ij} - M_{ij} Y_i \leq 0 \quad \forall ij \tag{3.24}$$

$$\sum_i Y_i \geq P_{MIN} \tag{3.25}$$

$$\sum_i Y_i \leq P_{MAX} \tag{3.26}$$

$$\sum_{ij} \left(\frac{d_{ij} x_{ij}}{\sum_j D_j} \right) \leq MaxAvgDist \tag{3.27}$$

$$\sum_{ij} \left(\frac{a_{ij} x_{ij}}{\sum_j D_j} \right) \geq MinPctIn50 \tag{3.28}$$

$$x_{ij} \geq 0 \quad \forall ij \tag{3.29}$$

$$Y_i = \{0, 1\} \quad \forall i \tag{3.30}$$

Note that two constraints were added 3.27 and 3.28.

- *A constraint on the average weighted distance:* This constraint takes the (demand)-weighted average distance and ensures that it is less than or equal to some critical level *MaxAvgDist*, the maximum allowed average weighted distance. Consequently, the model will make sure that the average customer is not "too far away".

- *A constraint on the amount of demand within a certain distance:* The previous constraint considers only the average, which means that we do not know how much of our demand has an LOS below a certain threshold. For instance, even if we know that the average distance is less than 50 miles, we may be interested in ensuring that at least 75% of our demand is less than 50 miles from a DC. This constraint ensures that. For this we need to specify a distance (say 50 miles). The binary variable a_{ij} then denotes whether or not a certain link is shorter ($a_{ij} = 1$) than 50 miles or not ($a_{ij} = 0$). That is, the constraint ensures that not "too many" customers are far away.

It is also important to note that when we introduce service constraints, we may need to introduce binding constraints on the demand. If not, the model may try to enforce the service constraint by delivering more than demanded to certain demand nodes. This will however be artificial (and unsold) demand. To ensure this is not the case, let the demand constraint be given by equality, instead of the computationally more efficient inequality.

3.2.5 Supply Chain Network Design

We now combine the different techniques learned into one model. We take network optimization and combine it with the facility location models together into what's known as a supply chain network design model.

Data collection: Transportation Data

The first step in building the model is to collect data. For transports, we want to know the costs and capacities associated with both inbound and outbound transportation.

With the Supply Chain Network Design Model, transportation costs are assumed linear in transport volumes. Clearly, this is not always the case: there may be minimum charges, fixed costs, or many other types of fixed and variable cost components. However, in order to build the model, a linear approximation must be found. This could be done in several different ways:

- Take average transport cost from historical data
- Use list prices
- Use regression analysis to find the costs
- Use benchmark rates from other sources

Whichever way you use to uncover these costs, you need to be aware of how your linear approximation affects the reliability of the model.

Data collection: Facility Data

The Supply Chain Network Design Model allows facility costs to be both fixed and variable in volume. There will always be people arguing that "all costs are fixed" or "all costs are variable", but what you need to consider is how fixed and variable cost components *affect the solution your model searches for.*

Fixed costs will make it expensive to have many facilities, so the model will try to reduce the number of facilities. Variable costs will make it costly to have long distances, and the model will therefore try to increase the number of facilities (or at least reduce average distances).

If facilities are owned and/or operated by a third-party, finding the fixed and variable costs is normally straight-forward – they are given in the contract. If a facility is operated in-house, other techniques have to be used. Among them:

- Activity-based costing to find variable costs
- Regression analysis over volume (note that the intercept is the fixed cost component)

For facilities we also need to understand the capacity. The capacity is the maximum throughput over a specified unit of time (e.g. a week). Note that this can be very difficult to measure in practice, since shifts can be added, while capacity also depends on planning and scheduling.

Network Design Baselines

You should be careful in which baseline you use for comparing your results. Three baselines are important to consider:

- *Baseline 1 – Actual costs:* what cost does the model give us if we use the actual, current flows? We want to know how well the model matches reality. You use this baseline to calibrate the model.
- *Baseline 2 – Adhere to policy:* there may be a policy in place that you are simply not adhering to in your operations. With this baseline you want to know how other solutions compare to what you ought to be doing according to the policy in place.
- *Baseline 3 – Optimal DC assignment:* if you were to use the optimal assignment for the DC – what would the solution look like? With this baseline you can isolate the effect of number of DCs since you optimize allocation.

It is important to keep in mind that you compare design changes to the right baseline. For instance, if you use the model to figure out how much you could reduce costs by reducing the number of DCs, you need to be aware the model will tell you this while, at the same time, optimizing the allocation given the number of DCs. Hence you should compare your solution - the "optimal number of DCs under optimal allocation" - to Baseline 3, where you have the "current number of DCs under optimal allocation". Why? Because otherwise you compare solutions that have optimal allocations to solutions that do not have optimal allocations – you do not isolate the effect of changing the number of DCs.

Running Scenarios

One of the key benefits of the Supply Chain Network Design Model is that it is easy and fast to run different scenarios. Scenarios can investigate uncertain parameters (sensitivity analysis) or explore how different constraints affect the optimal solution.

While this is a great benefit, it is important not to get "analysis paralysis" – just because it is possible to run many scenarios does not mean it is the way to go. It is important to understand which scenarios are relevant and, most importantly, how to interpret the results. For instance, which baseline should be used for comparing the results of the scenario?

3.2.6 Advanced Supply Chain Network Design Topics

There are many advanced supply chain network design topics and we introduce several here. The first is robust optimization using simulation along with optimization. This is a relatively simple and quick way to get a handle on the impact of the variability of demand (or other factors) on facility selection. We also cover multi-commodity flow problems, which significantly increases both the complexity and size of the models. While more complicated, multi-commodity flow models are still very similar to the transshipment and facility location models explored previously. The concept of Flexibility was introduced following the approach developed by Jordan and Graves (1995). Forming chains between plants increases the overall flexibility at a much lower cost than if we provided full flexibility. The final expansion of the model covers multiple time periods. This is meant for more tactical planning periods – but again, the models were very similar to the simpler single commodity transshipment models with the addition of inventory balance equations. The final section reviews how pipeline inventory, safety stock, and cycle stock can be included in strategic network design models.

Robustness – demand uncertainty

Supply Chain Network Design (SCND) models are deterministic. They assume that each input value is known with certainty and exhibits no variability. We know this is not the case in reality. So, while there are many more sophisticated mathematical techniques that address this shortcoming, we can use simple Monte-Carlo simulation to try to understand how robust our solutions are.

Essentially, the method involves re-solving the model with new input information each time. The new input data is randomly selected using estimates of the distribution of the variables. The easiest way to simulate random variables in spreadsheets is to use the RAND() function. It returns a number between 0 and 1.00 following a uniform distribution.

- To simulate a uniform distribution with mean of X, plus or minus Y%, we would set the value to

$$= X * (1 + (RAND() - 0.50) * (2 * Y))$$

  So that if we wanted 250 +/- 20%, we'd use =250*(1+(RAND()-0.50)*0.40) returning values from ∼ 200 to 300.

- To simulate a Normal Distribution with a mean of X and a standard deviation of Y, we would use:

$$= NORM.INV(RAND(), X, Y)$$

So for mean of 300 with standard deviation of 45 we'd use =NORM.INV(RAND(),300,45) returning values ∼ N(300,45).

After each new simulation of input values, the model is run and the results are stored. The analysis of the resulting runs can help determine which facilities, for example, are more likely to stay open under a variety of demand outcomes. This is not an exhaustive method – simply one approach to gauge the robustness of the design.

Multi-commodity flow model

Multi-commodity flow (MCF) models introduce multiple products. Each product will have its own demand, supply and other characteristics. If each product is independent and there are no interactions, then we can simply model each commodity individually. Whenever there is a shared resource (common capacity constraint, for example), then a multi-commodity flow model is needed.

The formulation is shown below. The primary addition to the original formulation is the additional subscript of k for the different commodities for the costs, decision variables, demand, and supply. We added, in this formulation, two types of supply capacity constraints: one that is location-commodity specific and one that is location specific for all commodities there. These are quite common in practice.

$$Min \quad z = \sum_i \sum_j \sum_k c_{ijk} x_{ijk} \tag{3.31}$$

s.t.

$$\sum_j x_{ijk} \leq S_{ik} \quad \forall i \in S, k \in K \tag{3.32}$$

$$\sum_j \sum_k x_{ijk} \leq S_i \quad \forall i \in S \tag{3.33}$$

$$\sum_i x_{ijk} \geq D_{jk} \quad \forall j \in D, k \in K \tag{3.34}$$

$$\sum_i x_{ijk} - \sum_i x_{jik} = 0 \quad \forall j \notin D, \notin S, k \in K \tag{3.35}$$

$$\sum_i \sum_k x_{ijk} \leq CAP_j \quad \forall j \notin D, \notin S \tag{3.36}$$

$$x_{ijk} \geq 0 \quad \forall ijk \tag{3.37}$$

Solving the MCF models are the same as the single commodity models – they are simply larger and more difficult to interpret. The multiple layers of constraints make the calculation of individual costs difficult – this is covered in activity based costing and managerial accounting.

Chain approach to flexibility

Supply chain network design (SCND) models do not consider variability of demand. A MILP solution, while optimal, can often be quite fragile. This means, that if any of the input values change, the total costs could change dramatically. We might want to find a SCND that is more flexible in its ability to handle changes in demand – in particular sudden surges or peaks in demand for certain products or commodities.

Full flexibility can be achieved by forcing each plant to manufacture every product. This provides complete demand flexibility as capacity can be diverted from any plant to handle surges of a certain product. This is very expensive, however. The opposite extreme is to create dedicated plants that only manufacture a single product. Dedicated plants are able to leverage economies of scale and tend to increase the level of expertise, however, they also increase risk exposure and limit flexibility to respond to surges. A clever design called chaining allows manufacturing networks to achieve most of the flexibility allowed in fully flexible designs, but at a fraction of the cost.

Multiple time period models

Up to this point, each model assumed a single bucket of time. We have expanded this now to consider multiple time periods. This is more common with tactical time frame (weeks to months) models.

To model multiple time periods, we need to introduce the idea of the inventory level at a location, j, at time t, I_{jt}. This is the quantity available at location j at the end of time t. This allows us to charge an inventory holding charge (h) for each time period.

We also need to include an inventory balance constraint – similar to the conservation of flow constraints in transshipment models. This is the 3rd constraint in the formulation below, which states, the sum of all flow into node j during time t, minus the sum of all flow out of node j during time t, plus the inventory available at node j at the end of time t-1, minus the inventory available at node j at the end of time t is zero.

$$Min \quad z = \sum_i \sum_j \sum_t c_{ijt} x_{ijt} + \sum_j \sum_t h I_{jt} \tag{3.38}$$

s.t.

$$\sum_j x_{ijt} \leq S_{it} \quad \forall i \in S, t \in T \tag{3.39}$$

$$\sum_i x_{ijt} \geq D_{jt} \quad \forall j \in D, t \in T \tag{3.40}$$

$$\sum_i x_{ijt} - \sum_i x_{jit} + I_{j,t-1} - I_{j,t} = 0 \quad \forall j \notin S, t \in T \tag{3.41}$$

$$I_{jt} = I_{initial} \quad \forall j, t = 0 \tag{3.42}$$

$$x_{ijt} \geq 0 \quad \forall ijt \tag{3.43}$$

Inventory considerations

Thus far, inventory has not been considered in the design of the supply chains. Surprisingly, it is very common to ignore inventory considerations when designing a supply chain. This is due to a couple different reasons. First, inventory policies are based on probabilistic models that do not combine well with deterministic MILP optimization models. Second, most of the key determinants of inventory levels (demand variability, lead time, service level, order size, etc.) are only known tactically not strategically. Finally, inventory balance equations, which would be needed to model inventory costs, cannot be used to track and optimize strategic inventory decisions that are made at yearlong increments.

Instead, we offer an approach that leverages the non-linear empirical relationship between inventory and throughput. It has been found that average inventory for a specified time period is equal to: αT^{β} where T is the throughput in items or value, and α and β are estimated coefficients. The β parameter is typically between 0.5 and 0.8. One can simply add this quantity to different runs as a post-hoc analysis or incorporate it into the model with binary variables.

3.2.7 Practical Considerations in Supply Chain Network Design

While SCND problems are highly mathematical and depend on optimization, the majority of the problems that occur in practice have nothing to do with the actual models. It is usually a problem with the people involved: the objectives were not agreed upon or clearly stated, the scope kept changing, the wrong problem was solved, etc.

It is important to realize that SCND is just one problem in a larger set of supply chain problems that need to be solved. As shown in the figure below, SCND is a strategic decision that is conducted with multiple year time horizons and has a very high potential impact on Return on Assets or Investment. The transactional and tactical/operational problems are conducted more frequently but have a lower potential impact. All of these lower level solutions should align with the overall supply chain strategy and network design.

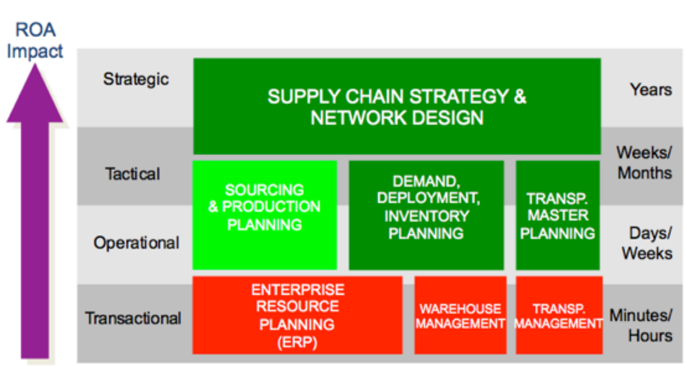

FIGURE 3.3: Fitting SCND into the bigger picture
Source: Chainalytics

Know Thy Project – It is important that you have a solid understanding of not only the physical supply chain (sources, facilities, products, flows, etc.), but also the people involved. You need to know the stakeholders but also the ultimate decision makers. There is an excellent article in the Harvard Business Review called "Who has the D?" that discusses how the decision making process in a firm can be improved. It is important that you know "who has the D" – that is, the ultimate decision – for your project. Scoping meetings with all stakeholders are very critical here to establish the boundaries, objectives, and other rules of the road for a large project like a SCND.

Focus on the Problem – There is always tremendous pressure in a SCND engagement from the different stakeholders to explore every exception and essentially boil the ocean to explore every possible solution. This is impossible to do in reality. So, you need to know how to separate the important aspects from the trivial using segmentation and data aggregation as much as possible. Identifying the fewest possible number of products to model is critical – be sure to think of supply chain distinctions for products – not just marketing distinctions. Many managers (and your future stakeholders) will not understand that the model is a caricature of the supply chain, not a high-definition photograph! You have a variety of different analysis tools and techniques to address questions of varying importance: from large-scale MILP models to Side Scenario Runs to Quick Heuristic Analysis. Use them appropriately – if everything is equally important, then nothing is important!

Be Experimental –There are two aspects to this suggestion: building the model and using the model. When building the SCND model itself, we recommend using a spiral method (start small, test it, evaluate, adapt it, and repeat) versus the traditional waterfall method (collect all input, develop complete requirements, build the full model, test the complete model, release to users). By creating the model iteratively, you build confidence in it, identify potential outliers, keep stakeholders engaged, and might end up with an early and unexpected solution. When running the model, we recommend you try as many different scenarios and options as possible. This is why you go through the pain of creating these complex models! Running multiple scenarios allows you to: test and cost out competing ideas and strategies, understand internal trade-offs within the network, uncover opportunities for serendipitous discovery, and better communicate with stakeholders.

Separate the Math from the Decision – The final suggestion deals with understanding the virtues of a mathematical model versus humans. Mathematical models are exceptionally good at making trade-offs between accurately quantified options. However, models will NEVER consider all factors and cannot fully represent reality. Humans are needed to determine what aspects are important and to provide the options and input for the model to consider. Also, because optimization models will do anything for a dollar (penny, euro, ruble, peso, or any other currency) the results should be scrutinized. The absolute lowest cost solution is rarely the right business solution. The key point is that mathematical models should be used for Decision Support not for the Decision itself. Executives and managers have additional experience and insights into the larger environment that need to be considered when making large and important decisions, like supply chain network design.

Learning Objectives

- Introduce network models
- Review how formulate the transportation problem and the transshipment problem
- Learn how to formulate the continuous single facility problem
- Formulate and solve network facility location problems (discrete candidate selection)
- Use network flow models to evaluate the number of facilities in a network
- Incorporate Level-Of-Service (LOS) constraints in the network models
- Identify how to combine previous models to a Supply Chain Network Design Model
- Construct relevant baselines for the analysis
- Perform robust optimization using supply network models

- Construct multi-commodity flow models
- Review how to expand the model into several time periods
- Understand when to incorporate inventory costs into the network models
- Gain an appreciation and understanding of practical considerations when conducting SCND engagements

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3.3 Production Planning

3.3.1 Summary

We now review the information flow within a firm, and the coordination and planning of production. What is being coordinated? Well, what we want to achieve is to match the demand of our customers with the supply from our firm, which requires components from our suppliers. For this match to be possible, information about future demand should be used as an input of the master production schedule, which is then used to determine the materials requirements.

3.3.2 Key Concepts

3.3.3 Fixed Planning Horizon

Master Production Schedule (MPS)

The MPS is the basic communication between the market and manufacturing. It is a statement of production – not demand. It is a plan to meet the forecasted demand, not a forecast of demand. There are several manufacturing or production strategies that determine how the MPS can look like:

- **Level production strategy.** This is a strategy aiming at even production levels over time. The major benefit is a smooth and stable operations which reduces switching costs and minimizes the need for outsourcing, overtime, or other flexibility measures. The downside is that it leads to heavy inventory build-up when demand is low, and possible shortage when demand is high.

$$t = P_t = (\sum F_t - I_0)/n$$

- **Chase demand strategy.** With this strategy the aim is to let production quantities follow demand as closely as possible, so that more is produced in times of high demand and vice versa. The benefit of this is that inventory is kept at a minimum, and so is obsolescence. On the other hand, it tends to lead to large swings in production quantities and labor needs.

$$t = P_t = F_t$$

- **Hybrid strategy.** A hybrid strategy tries to combine the benefits of the level production strategy and the chase demand strategy, by balancing the costs associated with the strategies. This can be done in several ways. Below we report in more detail how this is handled when there is a fixed planning horizon.

Fixed planning horizon (FPH) problem

The FPH problem considers the problem of finding the lowest cost production plan over a fixed horizon with multiple time periods. Demand is considered deterministic but can vary between periods.

- Demand
  - Constant vs **Variable**
  - **Known** vs Random
  - **Continuous** vs Discrete
- Lead time
  - **Instantaneous**
  - Constant or Variable (deterministic/ stochastic)
- Dependence of items
  - **Independent**
  - Correlated
  - Indentured
- Review Time
  - **Continuous**
  - Periodic
- Number of Echelons
  - **One**
  - Multi (>1)
- Capacity / Resources
  - **Unlimited**
  - Limited / Constrained
- Discounts
  - **None**
  - All Units or Incremental
- Excess Demand
  - **None**
  - All orders are backordered
  - Lost orders
  - Substitution
- Perishability
  - **None**
  - Uniform with time
- Planning Horizon
  - Single Period
  - **Finite Period**
  - Infinite
- Number of Items
  - **One**
  - Many
- Form of Product
  - **Single Stage**
  - Multi-Stage

FIGURE 3.4: Information given to solve a production plan

There are several ways to solve the FPH problem:

Simple heuristics.

These are simple decision rules, for instance that you run only one production run (One time run heuristic), that you produce in every period exactly what is demanded (lot for lot), or that you produce either a fixed “optimal” quantity or according to fixed “optimal” intervals. These heuristics are simple making decision-making fast, although the results are not necessarily particularly good.

Specialized heuristics.

There are several more sophisticated heuristic developed for the FHP, including the Silver-Meal (SM) heuristic, the least unit cost heuristic and the part-period balancing heuristic. In this course we focus on the SM heuristic. This heuristic searches through the periods to find the lowest cost per period. It first tests if it is less costly (per period) to produce next period’s demand in this period. If it is, it sees also if it is less costly to include the period after next period in this period's production. It continues this process until the cost per period increases. When it does, say in period k, it starts over by considering production in period k, and sees if the cost per period is reduced by including also the demand from $k+1$ in the production of period k. It continues like this until the end of the horizon. While more sophisticated, it is not guaranteed to find a very good solution.

Optimal methods.

The Wagner-Whitin algorithm provides the optimal solution by relying on dynamic programming. This is an efficient method for large problems. For smaller problems, a spreadsheet MILP model is a quick way to find the optimal solution.

In summary - there are many methods available. Heuristics are fast and easy to implement, but are not always good in the sense that they provide a near-optimal solution. Specialty heuristics are more sophisticated, a bit harder to set up, but tend to provide better "real-world' results. Optimal methods require more time and data, allow for several constraints and give the optimal solutions. However, keep in mind that the information fed into the model is often not exact.

Approaches for Solving the FPH Problem

Simple Heuristics

One Time Run – Manufacture all in one run for that month
Example: See Table 3.1

Month	Forecast	Production	IOH	Holding Cost	Set-Up Cost	Total Cost
1	200	2,000	1,800	$ 1,800	$ 500	$2,300
2	150	0	1,650	$ 1,650	$ -	$1,650
3	100	0	1,550	$ 1,550	$ -	$1,550
4	50	0	1,500	$ 1,500	$ -	$1,500
5	50	0	1,450	$ 1,450	$ -	$1,450
6	100	0	1,350	$ 1,350	$ -	$1,350
7	150	0	1,200	$ 1,200	$ -	$1,200
8	200	0	1,000	$ 1,000	$ -	$1,000
9	200	0	800	$ 800	$ -	$800
10	250	0	550	$ 550	$ -	$550
11	300	0	250	$ 250	$ -	$250
12	250	0	0	$ -	$ -	$ -
Total	**2,000**	**2,000**	**13,100**	$ **13,100**	$ **500**	$ **13,600**

TABLE 3.1: One time run production plan

Lot for Lot (Chase) – Each month manufacture product forecasted for that month
Example: See Table 3.2

Fixed order quantity (FOQ) – Order

$$Q^* = \text{EOQ} \quad \text{if} \quad F_t > IOH \quad (Q \text{ is Stable, } T \text{ varies})$$

Example: See Table 3.3

Month	Forecast	Production	IOH	Holding Cost	Set-Up Cost	Total Cost
1	200	200	0	$ -	$ 500	$500
2	150	150	0	$ -	$ 500	$500
3	100	100	0	$ -	$ 500	$500
4	50	50	0	$ -	$ 500	$500
5	50	50	0	$ -	$ 500	$500
6	100	100	0	$ -	$ 500	$500
7	150	150	0	$ -	$ 500	$500
8	200	200	0	$ -	$ 500	$500
9	200	200	0	$ -	$ 500	$500
10	250	250	0	$ -	$ 500	$500
11	300	300	0	$ -	$ 500	$500
12	250	250	0	$ -	$ 500	$500
Total	**2,000**	**2,000**	**0**	**$ -**	**$ 6,000**	**$ 6,000**

TABLE 3.2: Lot for lot production plan

Month	Forecast	Production	IOH	Holding Cost	Set-Up Cost	Total Cost
1	200	400	200	$ 200	$ 500	$700
2	150	0	50	$ 50	$ -	$50
3	100	400	350	$ 350	$ 500	$850
4	50	0	300	$ 300	$ -	$300
5	50	0	250	$ 250	$ -	$250
6	100	0	150	$ 150	$ -	$150
7	150	0	0	$ -	$ -	$ -
8	200	400	200	$ 200	$ 500	$700
9	200	0	0	$ -	$ -	$ -
10	250	400	150	$ 150	$ 500	$650
11	300	400	250	$ 250	$ 500	$750
12	250	0	0	$ -	$ -	$ -
Total	**2,000**	**2,000**	**1,900**	**$ 1,900**	**$ 2,500**	**$ 4,400**

TABLE 3.3: Fixed order quantity production plan

Review of EOQ

The Economic Order Quantity or EOQ is the most influential and widely used inventory model in existence. Essentially, the EOQ is a trade-off between fixed (ordering) and variable (holding) costs. The minimum of the Total Cost equation is the EOQ or Q^*. The Inventory Replenishment Policy becomes "Order Q^* every T^* time periods" which under our assumptions is the same as "Order Q^* when Inventory Position (IP)=0".

EOQ Model

- Assumptions
  - Demand is uniform and deterministic.
  - Lead time is instantaneous (0) – although this is not restrictive at all since the lead time, L, does not influence the Order Size, Q.
  - Total amount ordered is received.
- Inventory Replenishment Policy
  - Order Q^* units every T^* time periods.

– Order Q^* units when inventory on hand (IOH) is zero.

- Essentially, the Q^* is the Cycle Stock for each replenishment cycle. It is the expected demand for that amount of time between order deliveries.

Notation

- c_t : Ordering Costs ($/order)
- c_e : Holding Costs ($/order/time)
- D : Demand (units/time)
- Q^* : Optimal Order Quantity under EOQ (units/order)
- T : Order Cycle Time (time/order)
- T^* : Optimal Time between Replenishments (time/order)

Optimal Order Quantity (Q^*)
Recall that this is the first order condition of the TRC equation – where it is a global minimum.

$$Q^* = \sqrt{\frac{2c_t D}{c_e}}$$

Optimal Time between Replenishments
Recall that $T^* = Q^*/D$. That is, the time between orders is the optimal order size divided by the annual demand. Similarly, the number of replenishments per year is simply $N^* = 1/T^* = D/Q^*$. Plugging in the actual Q^* gives you the formula below.

$$T^* = \sqrt{\frac{2c_t}{Dc_e}}$$

Periodic Order Quantity (POQ) – order sum of forecasts every $\sim T^*$ periods (T is stable, Q varies)
Example: See Table 3.4

Month	Forecast	Production	IOH	Holding Cost	Set-Up Cost	Total Cost
1	200	450	250	$ 250	$ 500	$750
2	150	0	100	$ 100	$ -	$100
3	100	0	0	$ -	$ -	$ -
4	50	200	150	$ 150	$ 500	$ 650
5	50	0	100	$ 100	$ -	$100
6	100	0	0	$ -	$ -	$ -
7	150	550	400	$ 400	$ 500	$ 900
8	200	0	200	$ 200	$ -	$ 200
9	200	0	0	$ -	$ -	$ -
10	250	800	550	$ 550	$ 500	$ 1,050
11	300	0	250	$ 250	$ -	$250
12	250	0	0	$ -	$ -	$ -
Total	**2,000**	**2,000**	**2,000**	**$ 2,000**	**$ 2,000**	**$ 4,000**

TABLE 3.4: Periodic order quantity production plan

Optimal method

The FPH can be set up as a MILP problem, similar to the problems discussed previously. With a MILP formulation, the objective is to minimize the total costs, which consist of setup costs for every production run/batch and holding costs from finished goods inventory. Index denotes time period.

We let Z_t be a binary decision variable indicating production in period t, and Q_t be the decision variable determining production quantity in period t. We thus have that:

$$Min \quad z = \sum_i c_{setup} Z_t + \sum_t h I_t \tag{3.44}$$

$s.t.$
$I_0 = 0$
$Q_t - D_t + I_{t-1} - I_t = 0 \quad \forall t \in T$
$MZ_t - Q_t \geq 0 \quad \forall t \in T$
$Q_t \leq CAP_t \quad \forall t \in T$
$I_t, Q_t \geq 0 \quad \forall t \in T$
$Z_t = \{0,1\} \quad \forall t \in T$

where c_{setup} is the setup cost, h is the holding cost per unit and time period, I_t is inventory level at the end of period t, D_t is the (forecasted) demand in period t, M is a large number, and CAP_t is the production capacity in period t.

In the above formulation, the initial inventory is zero. As before, we have a conservation of flow constraint as well as a linking constraint. The conservation of flow states that the difference in inventory between the end and the start of the period must be equal to the difference between demand and production quantity in that period. The linking constraint forces Q to be zero if Z is zero in the same period.

3.3.4 Material and Distribution Requirements Planning

Available to Promise (ATP)

The portion of existing inventory and/or planned production that is not committed or consumed is considered to be Available to Promise. This means that these units can still be sold to customers.
There are two slightly different ways to calculate this:

- **Discrete** – meaning that each production cycle is planning independently. Inventory from one production run is not carried over to the next run.
- **Cumulative** – meaning that inventory is carried over between periods. That is, inventory from the first production run which is unsold at the start of the second production run can still be sold during and after the second production run.

Time Fencing is often used to stabilize production planning (demand and planning)

- if $t <$ Demand Fence then use only committed orders for MPS
- if Demand Fence $< t <$ Planning Fence then limited MPS overrides
- if $t >$ Planning Fence then all MPS changes allowed (within limits)

Materials Requirements Planning (MRP)

Material Requirements Planning (MRP) systems ensure that all components and parts required for an end item are on hand when needed. These systems are intended to answer questions about what should be ordered or manufactured, how much, and when. These questions are answered by feeding the MRP with the MPS, inventory records, and the Bill of Materials (see below).

- Benefits of MRP
  - Leads to lower inventory levels
  - Fewer stock outs
  - Less expediting
  - Fewer production disruptions
- Limitations of MRP
  - Scheduling, not a stockage, algorithm
  - Does not address how to determine lot size
  - Does not inherently deal with uncertainty
  - Assumes constant, known lead times
  - Does not provide incentives for improvement

Bill of Materials (BOM)

The Bill of materials (BOM) shows which parts, subassemblies, and components that make up a given product to be produced. For illustrative purposes, the BOM can be shown as a tree-diagram, with the completed product at the top, and all parts and components being the roots. One example for a bicycle is shown below:

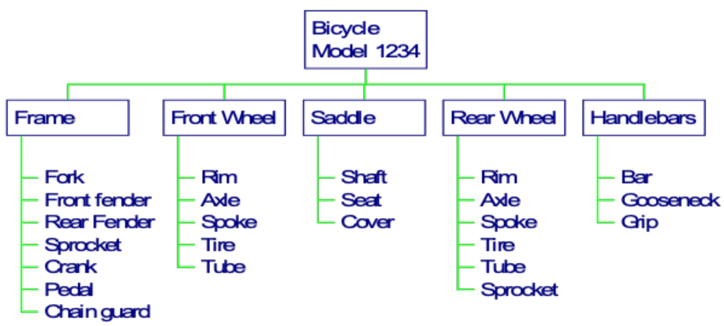

FIGURE 3.5: Bill of materials

MRP coordination

There are different ways to coordinate the MRP between a firm and its (independent) suppliers. In this lesson we went through three different approaches:

- **Simple MRP rules.** This means using simple rules also for coordination with the supplier. For instance, if there is a lead time of x weeks to get a component from the supplier, order release is scheduled x weeks before the units are needed for the assembly at the firm. The supplier uses this as the demand input for its own production planning.
- **Sequential optimization.** With this approach, the firm optimizes its production first, and then feeds the requirements to the supplier who optimizes its production.
- **Simultaneous optimization.** With this approach, the firm and the supplier's production are optimized simultaneously. This means that firm's or the suppliers' costs may increase, while total costs will decrease.

Distribution Requirements Planning (DRP)

DRP can be thought of as the application of the MRP principles to distribution inventories. It is used for inventory control in a distribution environment with many products, many stockage locations, and multiple echelons. In practice, it is an algorithm for scheduling and stocking, however, it does not determine lot size or safety stock.

Learning Objectives

- Identify how coordinate the internal information flow of a firm
- Learn how to use demand information to create the master production schedule (MPS)
- How to solve the fixed planning horizon problem (FPH) using several different algorithms
- Determine available to promise (ATP)
- Understand how material requirements planning (MRP) and distribution requirements planning (DRP) work
- Understand the bill of materials (BOM)
- Understand benefits and limitations of MRP and DRP

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3.4 Connecting Sales to Operations

3.4.1 Summary

We review how the Sales & Operations Planning (S&OP) process within a firm can be used to find the right balance between operational capacity and demand. The aggregate planning model allows a firm to set various operational and sales levers in order to determine the best trade-offs to make to maximize profitability. The primary levers on the operational side are: workforce, outsourced production, inventory, internal production levels, backlogs, and overtime hours. There might be more levers for specific situations, of course. On the demand side, we discussed how sales changes and promotions can be used to shape demand and how the aggregate planning model can help determine the operational impact. The optimization model can also be used to test out different strategies and potential policies.

3.4.2 Key Concepts

3.4.3 Aggregate Planning Model

The aggregate planning model is used to plan for the mid-term 3-18 month time frame. It takes as input the proposed demand as well as starting inventory, workforce, and costs for different options. The longer horizon planning involves more strategic decisions – such as network design etc. – while the closer time frame is planned using the Master Production Schedule. The planning unit for the aggregate planning model is typically not the individual SKU, but rather a family or related SKUs or general flow unit – such as tons, or composite units.

The model is used to find the best mix of Production, Inventory, Workforce, Outsourcing, and Capacity Levels in order to maximize the firm's profit over the 3-18 month planning horizon given the forecasted demand over the planning horizon by changing:

- Production Rate – how much to produce each month
- Workforce – how many employees to hire or lay off each month
- Overtime – how many overtime hours to plan for each month
- Machine capacity – how much capacity should be allocated to production lines each month
- Outsourcing – how many items to produce using contract or outsourced manufacturing each month
- Backlog – how many units each month should we allow to be backlogged
- Inventory on hand – how much inventory should we plan on holding each month
- Pricing – how should the product pricing be changed (promotions, discounts, etc.) each month

The output of the aggregate planning model is essentially the production (and pricing) plan. Two basic strategies that can be implemented are: Chase and Level. The chase strategy implies that production is made as close to the point of demand as possible. This leads to low inventory costs but generally higher production (overtime, workforce, outsourcing) costs needed to adjust the production to meet demand peaks. The level strategy attempts to keep the production level for the entire time period and let the inventory build up to cover any demand peaks. The hybrid strategy that combines these two extreme solutions is generally used.

Mathematical Formulation

$$Min \quad z = \sum_t (c_W W_t + c_H H_t + c_F F_t + c_o O_t + c_I I_t + c_B B_t + c_M P_t + c_C C_t) \tag{3.45}$$

$s.t.$
$P_t - (\frac{HW_t + O_t}{L}) \leq 0 \quad \forall t \in T$
$W_t - W_{t-1} - H_t + F_t = 0 \quad \forall t \in T$
$I_t - I_{t-1} - P_t - C_t + D_t + B_{t-1} - B_t = 0 \quad \forall t \in T$
$O_t - MW_t \leq 0 \quad \forall t \in T$
$W_t, H_t, F_t, O_t, I_t, B_t, P_t, C_t \geq 0 \quad \forall t$

Decision variables

$$H_t = \text{ Number of employees to hire at start of month } t \quad \forall t$$
$$F_t = \text{ Number of employees to fire at start of month } t \quad \forall t$$
$$W_t = \text{ Number of employees in month } t \quad \forall t$$
$$I_t = \text{Number of units of inventory on hand in month } t \quad \forall t$$
$$O_t = \text{Number of overtime hours to work in month } t \quad \forall t$$
$$B_t = \text{Number of units to put on backorder in month } t \quad \forall t$$
$$P_t = \text{Number of units to manufacture internally month } t \quad \forall t$$
$$C_t = \text{Number of units to outsource to contract manufacturing in month } t \quad \forall t$$

Input Data

$$W_0 = \text{Size of workforce at start of planning period (month 0)}$$
$$W_T = \text{Size of workforce at end of planning period (month T)}$$
$$I_0 = \text{Amount of inventory at start of planning period (month 0)}$$

$$I_T = \text{Amount of inventory at end of planning period (month T)}$$
$$B_0 = \text{Amount of backlog at start of planning period (month 0)}$$
$$B_t = \text{Amount of backlog at end of planning period (month T)}$$
$$D_t = \text{Demand in month t (units)} \forall T$$
$$c_t = \text{Cost of raw material per (\$/unit)}$$
$$c_I = \text{Cost of holding inventory (\$/unit/month)}$$
$$C_B = \text{Cost of a backorder (\$/unit/month)}$$
$$c_F = \text{Cost of firing one employee (\$/person)}$$
$$c_H = \text{Cost of hiring one employee (\$/person)}$$
$$c_W = \text{Cost of employee (\$/person/month)}$$
$$c_0 = \text{Cost of overtime (\$/hour)}$$
$$c_C = \text{Cost of outsourcing to a contract manufacturer (\$/unit)}$$
$$L = \text{Number of hours required to make each item (hours/unit)}$$
$$H = \text{Number of hours each employee can work per month (hours/person)}$$
$$M = \text{Maximum hours of overtime allowed per employee per month (hours/person)}$$

Aggregate Planning Model Levers

The aggregate planning model is mainly used to determine the best mix or both operational and sales based strategies. The operational levers control the supply while the sales levers influence the demand. On the operational side, we can manage supply by controlling inventory (common components and pre-build Inventory) and/or capacity (workforce flexibility, seasonal workforce, outsourcing production, or product flexibility).

Managing demand is a little less direct. The objectives on the demand side are to (1) Grow market, (2) Steal market share from competitors, or (3) Shift buying patterns of the customers. The levers available are:

- Pricing – incentives (discounts, MOQ's, etc.),
- Advertising – increasing brand awareness, and
- Promotions – price reduction over short period of time.

Demand Elasticity

The concept of demand elasticity with respect to price is useful in determining how a change in price will impact the demand.

$$e = \frac{\text{Percent Change In Demand}}{\text{Percent Change In Price}}$$

$$= \frac{\frac{D_{New}-D_{Old}}{D_{Old}}}{\frac{P_{New}-P_{Old}}{P_{Old}}} = \left(\frac{P_{Old}}{D_{Old}}\right)\left(\frac{D_{New}-D_{Old}}{P_{New}-P_{Old}}\right)$$

Recall that the elasticity will almost always be negative. That is, a negative change in price (a discount) will lead to higher demand. The elasticity of demand for many goods can range from -0.01 to as high as -10 or more. The way to interpret an elasticity of, say, -2.5 is that for every 1% change in price, the demand will increase by 2.5%.

3.4.4 Monthly Sales & Operations Planning Process

There are many flavors of S&OP out there. Most feature a set of structured meetings where specific tasks are performed by different functions within the firm and eventually are presented with recommendations to senior leadership to make a final decision. We described a five-step process as follows:
Step 1: **Data Gathering** – the core data on the most recent month's demand, inventory, production levels, sales, etc. is gathered, cleaned, and made ready to distribute. This should be accomplished in the first few days of a new month.
Step 2: **Demand Planning** – The demand planning team from the sales/marketing organization develops the initial set of forecasts. These include their expert opinions or modifications to any numeric forecasts that were made.
Step 3: **Supply Planning** – The supply planning/operations team receives the initial forecast and the most recent set of data and develop a resource plan to meet the proposed demand forecast. Any conflicts or restrictions are noted.
Step 4: **Pre-Meeting** – The supply and demand plans are brought together and compared. This larger group makes suggestions and recommendations. Any conflicts that cannot be solved at this level are identified.
Step 5: **Executive Meeting** – The senior team makes decisions on any outstanding conflicts – they have a go no go decision authority.

3.4.5 Distribution and Channel Strategies

In this lesson we cover distribution and channel strategies. A distribution channel is the path by which all goods and services travel from the original vendor to the end consumer and the pathway that payments make from the end consumer to the original vendor. We dive deeper into this topic by exploring e-commerce and omni-channel strategies and omni-channel network design. We conclude with a discussion of reverse logistics and reverse logistics network design.

There are four types of players in distribution channels that include manufacturer, wholesaler/ distributor, retailer, and consumer:

- Manufacturer: Makes products
- Wholesaler/Distributor: Traditional middleman. Purchases products in large quantities, consolidates, assembles, and resells to retailers and others.
- Retailer: Sells good directly to the consumer
- Consumer: Person who buys the product

The different types of distribution channels include wholesaler channel, direct to retailer, and direct to consumer (see figure 3.6).

Distribution Channel Strategies

To address the different distribution channel types, there are various strategies that address needs of the business and the actors present in the channel to ensure an efficient flow. Types of strategies include the traditional multi-tiered, mixing/consolidation center, distributor/ wholesaler, direct to store delivery, and drop shipping from manufacturer (see figure 3.7). We further articulate these strategies below by describing the product flow and the pros/cons of each.

Traditional Multi-Tiered Distribution
Product Flow

- Retailer places orders to each manufacturer
- Product flows from manufacturer to retailer DCs
- Retailer DCs receive product from multiple vendors

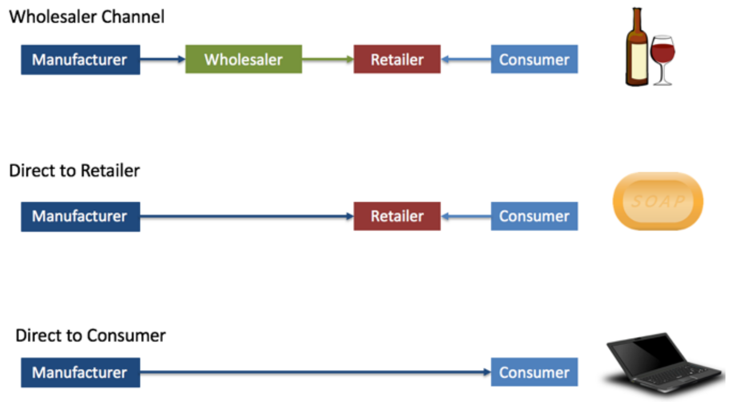


FIGURE 3.6: Channel type

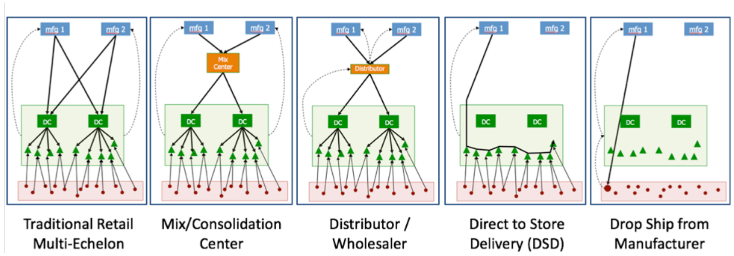


FIGURE 3.7: Distribution design options

- DCs send full truckloads of mixed goods to stores, and
- Consumers/customers view, select, receive, and pay for goods at store

Pros/Cons

- Flow leverages retailer economies of scale but with lots of handling
- On the inbound, DCs need to be able to justify full loads from vendors
- On the outbound, DCs are able to mix product from multiple sources
- High level of inventory stored at retail locations
- Lower transportation costs due to economies of scale
- Good for fast moving items
- Shorter transit times from vendors to stores

Mixing Centers / Hubs

Product Flow

- Retailer places orders to each manufacturer
- Product flows from manufacturer to Mixing Center
- No (or very little) inventory is held at the Mixing Centers – In-transit Merge
- Retailer DCs receive full truckloads of goods from multiple manufacturers from the Mixing Center

- DCs send full truckloads of mixed goods to stores, and
- Consumers/customers view, select, receive, and pay for goods at store

Pros/Cons

- Flow leverages retailer economies of scale with lots of handling
- On both inbound and outbound, DCs are able to mix product from multiple sources
- High level of inventory stored at retail locations
- Lower transportation costs due to economies of scale
- Good for slower moving items
- Longer transit times from vendors to stores

Distributors

Product Flow

- Retailer places orders to distributor
- Product flows from manufacturer to distributor
- Inventory is held by the distributor
- Retailer DCs receive full truckloads of goods from multiple manufacturers from the distributor
- DCs send full truckloads of mixed goods to stores, and
- Consumers/customers view, select, receive, and pay for goods at store

Pros/Cons

- Flow leverages the distributor's economics with lots of handling
- On both inbound and outbound, DCs are able to mix product from multiple vendors
- High level of inventory stored at retail locations
- Lower transportation costs due to economies of scale
- Good for much slower moving items
- Much longer transit times from vendors to stores
- Higher cost due to third party involvement

Direct to Store Delivery

Product Flow

- Retailer places orders to each manufacturer
- Product flows from manufacturer directly to the retail stores bypassing the DCs
- Vendor often merchandises, orders, and manages the retailer's shelf
- Consumers/customers view, select, receive, and pay for goods at store

Pros/Cons

- Flow leverages the vendor's economics

- Bypassing the retail DC reduces congestion
- Vendor can take over ordering, merchandising, etc.
- Can lead to congestion and added costs at the retail stores
- Loss of control of product flow and availability by the retailer
- Used for selected items (beverages, newspapers, snack foods)

Drop Ship to Customer

Product Flow

- Consumer orders from retailer
- Retailer passes order to manufacturer
- Manufacturer sends product directly to consumer
- Manufacturer and retailer could be same firm

Pros/Cons

- Very slow response time to consumer
- Very strong product availability while keeping inventory levels low
- Bypassing retailer DC and store reduces congestion
- Transportation cost can be high – small shipment size to consumer
- Allows for high level of customization
- Consumer places order – triggers pull operation

The distribution channel strategies are not mutually exclusive. Most firms use multiple channels for products. Selection of the strategy is based on several needs including: velocity (demand) of the product; desired response time to consumers; physical and other characteristics of products, supplier, and customers; and value of products.

Omni-Channel Distribution

From around 1995 to 2000 at its earliest stage, e-commerce was largely made up of "pure players". These made up of firms that allowed customers to buy goods and services over the internet using a website (ebay or Amazon). These firms also did not have a physical presence. This began early discussions that maybe traditional stores might be unnecessary in the future. Around 2000 – 2010, multi-channel became the next wave. In this case, firms use multiple online and traditional distribution channels. In most cases, traditional and online channel operations were kept separate and distinct; referred to as "Bricks and Clicks". This caused inefficiencies in traditional distribution channel strategies.

With the release of the smartphone around 2007 and consumer ability to use the internet on the phone, e-commerce grew at a faster rate than ever before, and with that a desire for ease of access across online and brick and mortar purchase and returns. As of 2010 on, the new path forward for many firms has been Omni-Channel. This is multi-channel retailing strategy where firms provide a seamless retailing experience through all available shopping channels (online & offline). Operations are mixed between challenges and consumers have multiple options for researching, ordering, receiving, paying, and/or returning products.

Retail Perspective:

Where to prepare the orders?

- Distribution Center vs. Fulfillment Center vs. Store
- Trading off distance vs efficiency vs. effectiveness
- Changes in packaging requirements and usability
- Adapting workforce, facilities, and systems to new tasks
- Maintaining inventory visibility across the channels and facilities

Where and how does the order meet the customer?

- Stores vs. Home vs. Third party locations (work, lockers, gas station)
- Attended vs. Unattended Delivery
- Own fleet vs. Parcel vs. Crowdsourcing vs. Mobile lockers

How to handle returns?

- Product returns ∼ 30% for online vs. 8% for traditional
- At location of purchase vs. Drop points vs. Everywhere
- Allowable reasons: Damaged, Wrong size/color/style, etc.

Omni-Channel Network Design

An omni-channel retailing experience requires flexible delivery networks. Omni, means in "all places", "all ways", "without limits". Omni-channel strategies affect ordering, fulfillment and returns. An Omni-Channel Network Design model formulation is illustrated in the figure below.

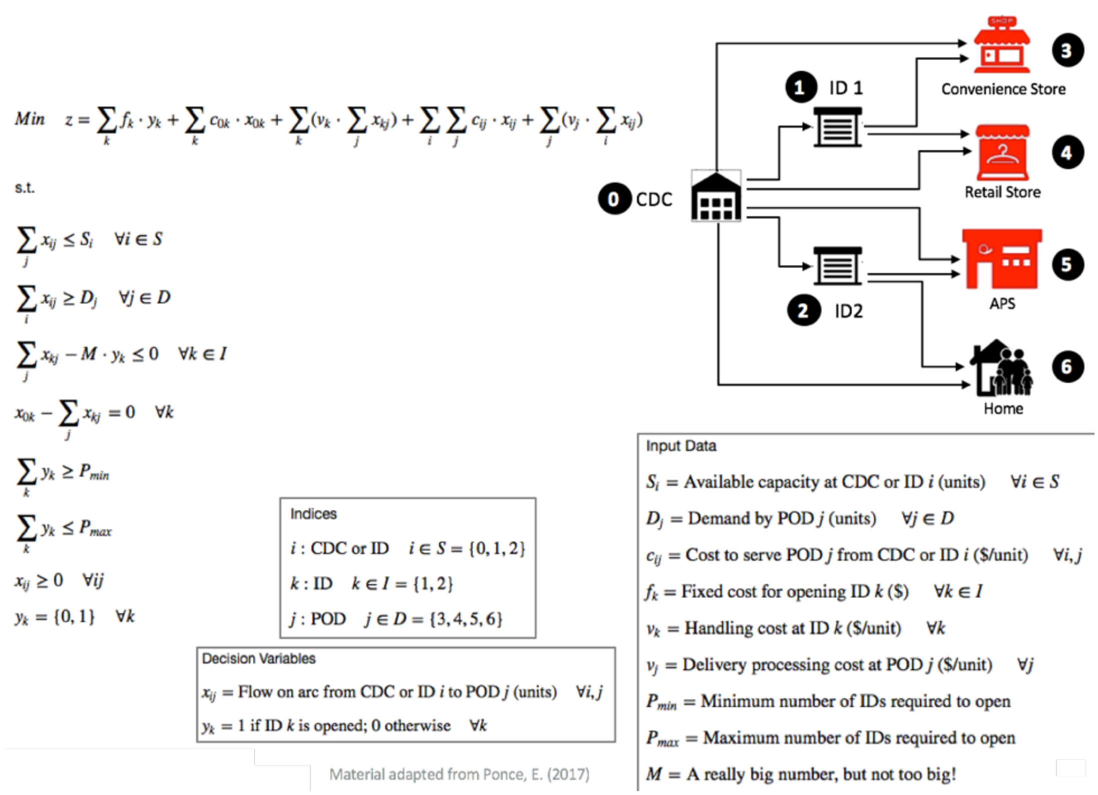


FIGURE 3.8: Omni-Channel network design model

Distribution Channel Strategies

Reverse Supply Chains (Reverse Logistics)

Reverse supply chain is all operations related to the reuse of products and materials. It involves collecting items at their "end of life" for capturing value or proper disposal. Reverse supply chains are most effective for reusable items.

Reusable items: Durable products intended to be used multiple times by different users in different locations of a supply chain network.

Reverse supply chains are becoming an increasing focus for consumers and companies alike because:

Increasing amount of waste generated:

- Over 300,000 cell phones disposed daily in United States
- Each EU citizen produces 17-20 kg of technological waste per year

Many modern products containing hazardous materials:

- Electronic devices contain lead, cadmium, mercury, etc.

Many products have shrinking product life cycles:

- Lifecycle of cell phones is 18 months, laptops 24 months, etc.

There are emerging regulations:

- In some regions, producers are responsible for the environmentally safe disposal of products. Extended Producer Responsibility (EPR)
- End of life waste framework directives in the EU for batteries, vehicles, tires, etc.

Product Recovery Options

There are different types of product recover options. The figure below builds out the traditional supply chain with recovery options at each phase of the supply chain.

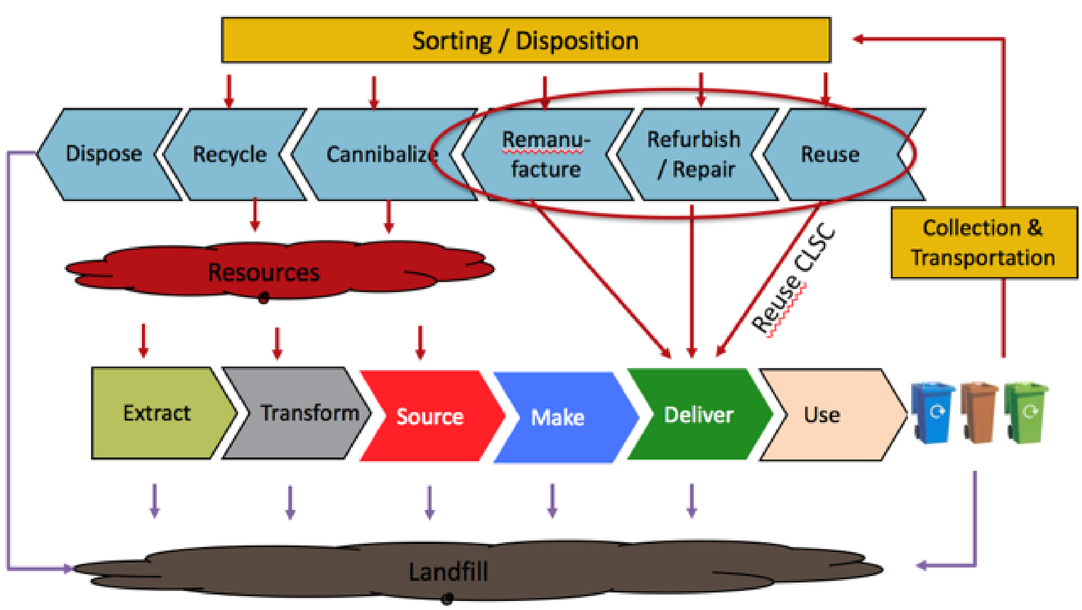


FIGURE 3.9: Product recovery options

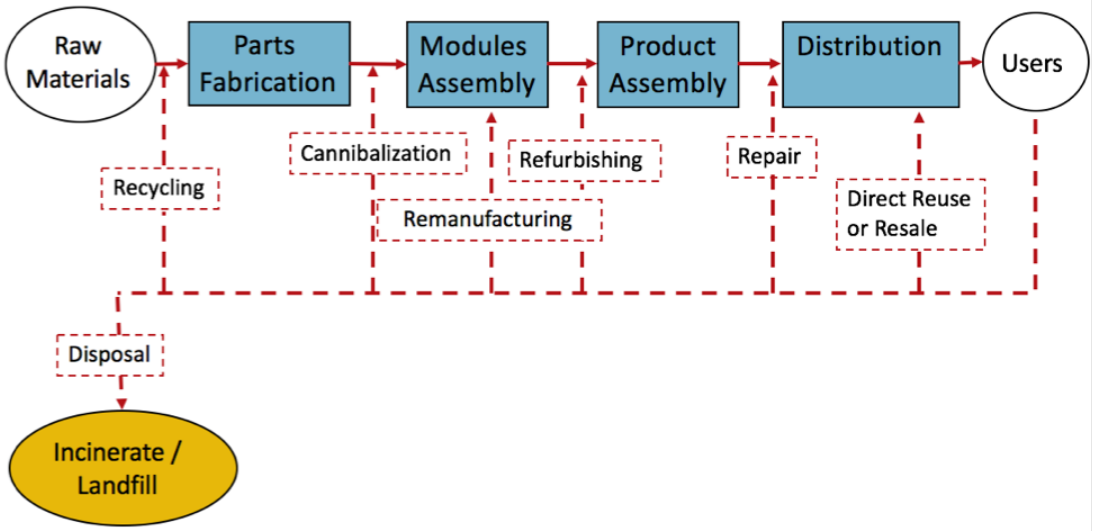


FIGURE 3.10: Reverse flow recovery option

There are different forms of reverse supply chains based on condition of the material:

Direct Reuse or Resale: This includes both Reusable Articles (containers, totes, pallets, books, etc.) and normal (non-reusable) products.

- <u>Reusable Articles</u> are inspected and, if needed, sterilized/cleaned/prepared for redeployment within firm or across network.
- <u>Non-reusable Products</u> are inspected and deemed ready for re-use or resale to other customers without any repair or refurbishing (e.g. wrong size/color/style apparel)

Repair: Used products returned to working order. Typically involves fixing and/or replacing broken parts (e.g. computers, phones, electronic devices).

Refurbish: Used products brought up to specified quality level. Critical modules are inspected and fixed or replaced. Sometimes includes technology upgrading (e.g. aircraft).

Remanufacture: Used products brought up to quality standards as rigorous as new products. Complete disassembly, inspection, and part/module replacement as needed (e.g. automotive engines).

Cannibalization: The returned product is used to recover a limited set of reusable parts and components for use in other products. Only a small proportion of the used product is typically reused. The remaining parts are recycled or disposed of (e.g. integrated circuits from computers).

Recycling: Materials from used products and components are processed and can be reused in production of original or other parts (e.g. metals, paper and cardboards, plastics, etc.).

Differences Between Forward and Reverse Logistics

The following figure illustrates the differences between forward and reverse logistics. It illustrates that reverse logistics can be considerably changed due to inconsistency, non-uniformity, and uncertainty embedded in reverse supply chains.

Network Design for Reverse Logistics

The following network design example illustrates a MILP for reverse logistics.

	Forward Logistics	Reverse Logistics
Forecasting	Hard, but relatively straightforward	More difficult due to uncertainty of end of life
Transportation	One to many	Many to one
Product Quality	Uniform	Not uniform
Product Packaging	Uniform	Not uniform
Destination/Routing	Clear	Unclear
Pricing	Relatively uniform	Dependent on many factors
Delivery speed	Importance recognized	Often not considering a priority
Distribution costs	Accounting systems	Less directly visible
Inventory Management	Consistent	Not consistent
Real-time information	Available	Less transparent

FIGURE 3.11: Differences between forward and reverse logistics

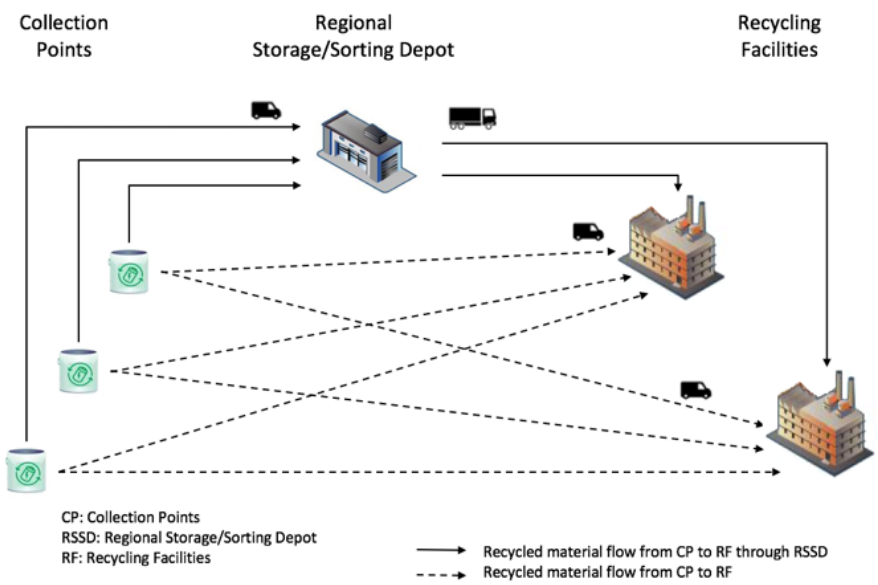


FIGURE 3.12: Battery recycling network

Learning Objectives

- Understand the fundamental concepts of Sales & Operations Planning in balancing supply and demand
- Formulate and solve an aggregate planning model
- Understand trade-offs and levers available for making aggregate planning decisions
- Appreciation for the trade-offs that can be made between sales and operations in terms of supply versus demand and volume versus product mix
- Understand what distributions channels are and who the players are
- Review flow paths for products that are a function of suppliers, customer and product
- Introduction to Omni-Channel and how it is impacting the supply chain
- Understand the important of reverse logistics

$$Min \quad z = \sum_j f_j \cdot y_j + \sum_j (v_j \cdot \sum_k x_{kj}) + \sum_k \sum_j c_{kj} \cdot x_{kj}$$

s.t.

$$\sum_j x_{ij} = S_i \quad \forall i \in S$$

$$\sum_k x_{kj} \leq K_j \quad \forall j \in P$$

$$\sum_k x_{kj} - M \cdot y_j \leq 0 \quad \forall j$$

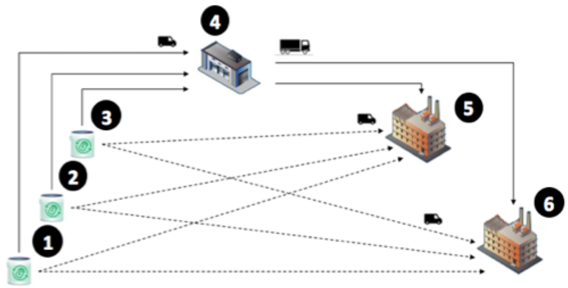
$$\sum_j x_{4j} - \sum_k x_{k4} = 0$$

$$\sum_{j=5}^{6} y_j \geq P_{min}$$

$$\sum_{j=5}^{6} y_j \leq P_{max}$$

$$x_{kj} \geq 0 \quad \forall k, j$$

$$y_j = \{0, 1\} \quad \forall j$$



Indices

i : Collection points (CP) $\quad i \in S = \{1, 2, 3\}$

k : Collection points (CP) or Storage/Sorting depot (RSSD) $\quad k \in O = \{1, 2, 3, 4\}$

j : Storage/Sorting depot (RSSD) or Recycling facility (RC) $\quad j \in P = \{4, 5, 6\}$

Decision Variables

x_{kj} = Flow on arc from facility k to facility j (tons) $\quad \forall k, j$

y_j = 1 if facility j is opened; 0 otherwise $\quad \forall j$

Input Data

S_i = Available supply at collection point i (tons) $\quad \forall i \in S$

K_j = Available capacity at facility j (tons) $\quad \forall j \in P$

c_{kj} = Transportation cost from facility k to facility j ($/ton) $\quad \forall k, j$

f_j = Fixed cost for opening facility j ($) $\quad \forall j$

v_j = Handling cost (RSSD) or processing cost (RF) at facility j ($/ton) $\quad \forall j$

P_{min} = Minimum number of recycling facilities required to open

P_{max} = Maximum number of recycling facilities required to open

M = A really big number, but not too big!

FIGURE 3.13: Battery recycling mathematical formulation

- Recognize the variety of recovery options
- Review network design for reverse logistics

References

- ”Sales & operations planning: the how-to handbook” Thomas F. Wallace and Robert A. Stahl., 2008, T.F. Wallace & Co., Cincinnati, Ohio.
- ”Sales and operations planning- best practices: lessons learned from worldwide companies” John R. Dougherty and Christopher D. Gray., 2006, Partners for Excellence, Belmont, N.H.
- ”Manufacturing planning and control for supply chain management” F. Robert Jacobs, William Berry, D. Clay Whybark, Thomas Vollmann 2011, McGraw-Hill, New York.

3.5 Supply Chain Sourcing

3.5.1 Summary

Sourcing is an important part of a successful supply chain; it often helps a business maintain a competitive edge. Effective sourcing keeps the costs down, while providing quality goods and services on time and efficiently. Sourcing is driven largely by the relationship between the buyer and supplier. We review some of the main components of supply chain sourcing.

3.5.2 Key Concepts

3.5.3 Procurement Strategy

To further our discussion on Supply Chain Sourcing, we review the basics of procurement and sourcing - which are the processes for buying the materials and services needed for the company to conduct its business.

The Price Iceberg

While many purchasers focus on the price of a component or raw material, this is only part of the total cost of the purchase. This is illustrated by the "price Iceberg" – while we tend to only see the price paid upfront, there are many aspects that determine the total cost to our business of making the purchase.

Figure 13. The price iceberg

FIGURE 3.14: The price iceberg

Value and Risk Mapping

One way to differentiate the purchasing portfolio is by considering where an item falls along two dimensions: 1) the supply risk and, 2) the annual spend (or profit impact on business). Based on this, a two-by-two matrix can be created to describe all items (see figure 3.15).

For each quadrant, different types of procurement strategies are suitable.

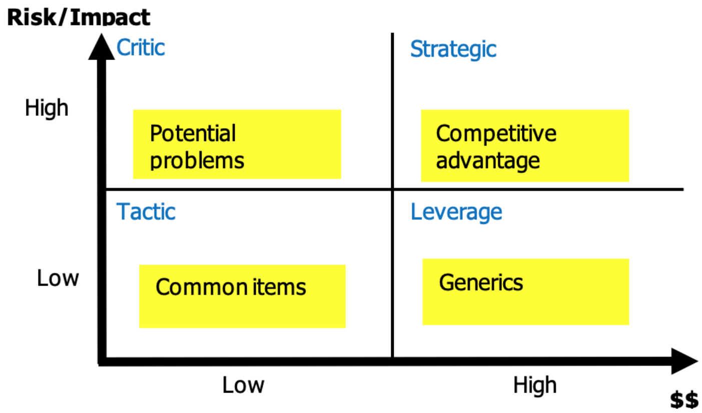


FIGURE 3.15: Risk-spend framework

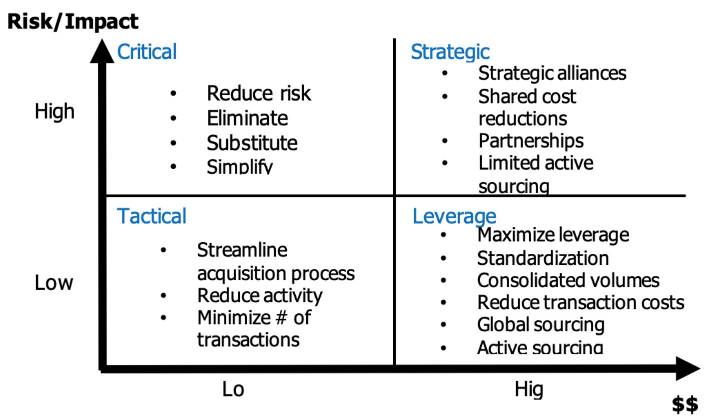


FIGURE 3.16: Procurement strategies

The Sourcing Process

On a general level, the sourcing process consist of the following steps:

- **Internal assessment.** This is to confirm category definition, validate baseline information and understand the key constituents. That is, focus is on the own business.
- **Market assessment.** This is to analyze market dynamics and identify which potential suppliers that may be of interest. It also includes understanding what competitors do.
- **Collect supplier information.** In this step information is collected to understand which criteria to use in the process as well as to understand current spend.
- **Sourcing strategy.** After the initial assessments, the sourcing strategy is developed. This specifies the approach, the specifications, and how to approach the subsequent steps.
- **Bidding process.** In the bidding process, request for proposals (RFPs) are developed and sent out. The buying firm decides on a bidding format and short-list suppliers.
- **Negotiate, select.** After the bidding process is over, the buying firm negotiates with selected short-listed suppliers. This step ends with a selection.
- **Contract Implement.** This involves developing category implementation plan, communications plan as well as measurements and audit plans.

There is also a feedback loop, consisting of

- Measure and report
- Capture lessons
- Ensure compliance

Value-Based Sourcing

Value based sourcing is the name given to the practice of going beyond the purchase price in a procurement event. For instance, instead of focusing on price, focus is on value; instead of focusing on total cost of ownership, focus is on total contribution.

Handling Volatility

Financial Hedging
There are primarily three ways to hedge financially when buying materials with high price volatility:

- **Simple hedge:** Negotiate long term, fixed price contract in the company's preferred currency
- **Forward contract:** buy/sell the commodity or a related one for future delivery on a given date at a given price
- **Option:** a call/put option is the right to buy/sell at a certain price at a certain future date

Physical Hedging Apart from financial hedging, a firm can use physical hedging to handle volatility. Create conditions in which the fluctuations are mitigated "naturally." (used mainly for currency hedging) Examples:

- Build a plant in countries where labor rates and currency are not expected to appreciate
- Manufacture and sell in the same country
- Actually buy a commodity when the price is low

Capital Goods

Sourcing of capital goods goes beyond standard purchasing in several ways. First, one must focus much more on the total life cycle of the good. One must also more explicitly consider the expected value of the good at the end of its useful life. Further, aspects such as training, trials, and subsidies may have a large impact on the total cost of the good. The lifetime cost of capital items are summarized in figure 3.17.

Outsourcing

There are several reasons why firms outsource. These include:

- External supplier has better capability
- External supplier has greater or more appropriate capacity

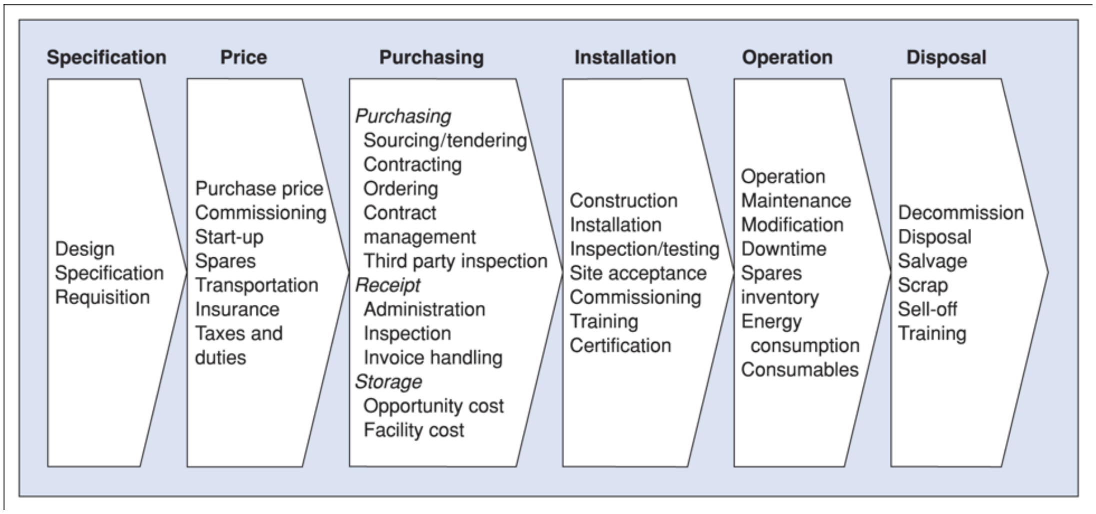


FIGURE 3.17: Lifetime cost of capital items
Source: AT Kearney

- Freeing resources for other purposes
- Reduction in operating costs
- Infusion of cash by selling asset to provider
- Reducing or spreading risk
- Lack of internal resource
- Desire to focus more tightly on core business
- Economies of scale of supplier

When outsourcing, the contract is extremely important. A well written contract makes sure to not just have clear decisions about how to handle disputes and how to measure and reward, but also how to handle intellectual property, how to define the service to be performed, and how to handle the evolution of the relationship. Outsourcing is subject to considerable risk. Some of the key risks are:

- Creating a competitor
- Losing control of the channel to a supplier
- Losing control of the channel to a customer

Purchasing Social Responsibility

Corporate social responsibility (CSR)has a wide range of underlying objectives. These include philanthropy, brand image, human resources attraction, mission support, social business and profitable business. Based on these, companies work in different ways, including using extensive codes of conduct, audit suppliers, train suppliers or work hard to ensure supply chain transparency.

Independently of activity, there is a hierarchy for the different responsibilities within CSR. The top responsibility for all firms is the economic responsibility – a firm cannot survive in the long run if it does not meet this responsibility. After the economic responsibility follows, in the following order: Legal responsibilities, ethical responsibilities, and discretionary responsibilities.

3.5.4 Procurement Optimization

In a way, each procurement is an optimization process in the sense that the procurement department of the company tries to get the best set of suppliers to serve you.

Item-supplier (lane-carrier) specific constraints

Item-supplier specific constraints are constraints that apply to one of the products at one of the suppliers. For instance, when procuring transport services for a number of lanes in a network, each carrier may have capacity restrictions for a given lane. In the MILP-formulation of the problem, these constraints are easily implemented as a separate matrix.

Optimizing with service attributes

Service attributes are often associated with an individual item at a given supplier. For instance, "on time delivery performance" may vary between different suppliers for a given item as well as between different items from the same supplier.

There are many types of service attributes that may be important evaluation criteria in the procurement process. These can be incorporated into the MILP-problem, either as constraints or by attaching a dollar value to the service level. For instance, if a certain service measure is believed to represent $10 per %-unit of service, a LOS-adjusted cost can be minimized.

System attributes/constraints

System attributes are not tied to a specific item. Such constraints are usually concerned with constraints on the suppliers as a group. These are not as straight-forward to implement in the MILP-formulation as the item-supplier specific constraints. One has to investigate each case individually and may need to introduce binary variables for different suppliers and/or items. Implementing certain system constraints in the model is also useful for making sensitivity analyses. For instance, it enables an analysis of how reducing the supplier base would affect total cost.

Economies of scope

In many cases the bid one receives for an item depends on whether or not the supplier believes it will be supplying another item. For instance, when buying transport services, a supplier may be willing to offer a lower price for a lane if he also gets the return transport on the lane. Similar economies of scope may appear in all procurement situations.

Combinatorial auctions

Combinatorial auctions is a way to capture the potential economies of scope. In essence, the idea is to let all suppliers bid for all possible combinations of related products.

Learning Objectives

- Review how to map items based on risk and spend
- Differentiate procurement strategy based on the mapping
- Understand the structured sourcing process
- Review capital goods purchasing

- Highlight importance of CSR in procurement
- Introduce procurement optimization when one evaluates bids for many items from several potential suppliers
- Identify how to take service criteria and systems attributes into account
- Introduce combinatorial auctions in presence of economies of scope

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- Cachon, G. P. (2003). Supply chain coordination with contracts. Handbooks in operations research and management science, 11, 227-339.

3.6 Supply Chain Finance

3.6.1 Summary

This review covers some of the most important concepts of accounting and finance from a supply chain manager's perspective including a short discussion of the role of accounting, the choices and accountant can make, the role and difficulties of depreciation, and an overview of basic accounting concepts.

3.6.2 Key Concepts

3.6.3 Accounting Fundamentals

The Income Statement

The Income Statement provides a summary of the flows in (revenue) and out (expenses) of the firm over a period of time; the net difference between the revenue and expense being the profit or loss of the firm. Put in another way, the income statement describes how the assets and liabilities are used during a particular period. You could also think of it as the sum of income-generating transactions, financial transactions over a stated period of time.

The three main components are:

- Revenues (turnover, sales, proceeds, 'top line') – the incoming flow
- Expenses (costs) – the outgoing flow
- Profit (income, earnings, 'bottom line') – the difference between incoming and outgoing flow

For a supply chain manager, there are several components to Expenses to be aware of:

- **COGS** (Cost of Goods Sold, Cost of products sold) – these are the direct costs of producing the goods/services that are sold to generate revenues
- **Cost of Revenue** (cost of sales) – similar to COGS, but includes also costs outside of production (if any) that can be assigned to the goods sold, e.g. expenses incurred in closing sales, marketing expenses
- **Depreciation** – these are non-cash expenses associated with the use of capital equipment (for more about this, see below). Related is amortization – this a reduction in goodwill (goodwill is an intangible asset that arises when the firm acquires another firm at a price higher than the book value of the acquired firm)
- **SG&A** (Sales, General, and Administration) – overhead costs associated with generating revenue, including the sales force
- **Operating Expenses** – the sum of Depreciation and SG&A.

The Balance Sheet

The Balance Sheet presents the financial condition of the firm at one point in time. It lists the assets, which is something that is owned by the firm and has measured value. It also lists the liabilities, which are claims against the assets by other parties. You could also think of these as debts or obligations that the firm has to pay other people. The Balance Sheet gives a snapshot of the assets and obligations of the firm at a single moment in time. Assets always equal liabilities. Remember, the balance sheet report Book value, not Market value. These could be equal, but as we will see in the next lesson they may not be.

- **Assets.** You want to separate Current Assets from Long Term Assets. Current assets can be used on short-term to pay for obligations (e.g. cash, account receivables, inventories), whereas long-term assets are, as the name implies, long term (e.g. facilities, equipment – sometimes abbreviated PPE for Property, Plant and Equipment)
- **Liabilities.** Normally liabilities are divided into Current Liabilities, Long-term Liabilities (Debt), and Equity. Current liabilities generally have to be paid in the next accounting period and include, e.g., accounts payables. Long-term liabilities include e.g. loans, bonds, or mortgages. Equity is the capital that owners have put into the firm.

The Role of Accounting

The role of accounting is to record the transactions of a business. The accountant classifies the sources and use of funds. There are primarily three types of reports prepared:

- Financial reports. These are intended for the firm's investors, includes the income statement and the balance sheet.
- Tax accounting reports. These are provided for the government, in the US for the IRS.
- Product costing reports. These are used for management decision making.

There are certain choices that can be made, as long as the account follow generally accepted accounting practice.

Note that the financial reports provided to investors can differ from the tax reports. They serve different purposes. Financial accounting's objective is to present fairly the results of operations and the financial condition of the company to its stockholders, whereas corporate tax accounting's objective is to minimize current tax liability and defer payment of liability as long as possible. This means that the tax code allows companies to deviate from financial accounting in specific areas when calculating taxable income without having to change the corporate financial reports issued to stockholders.

Depreciation

Depreciation is a non-cash expense to the business. It can be thought of as the estimated cost of using an asset. For instance, if a machine is purchased to be used in a manufacturing setting, the depreciation of the machine represents the reduction in the value of the machine over the accounting period. This means that while depreciation is an expense, the value of the corresponding asset reduces by the same amount. So if the depreciation of a machine is $10,000 in a year, this will impact the income statement ($10,000 expense) as well as the balance sheet (-$10,000 in long-term assets).

There are different ways to construct the rate of depreciation. Most common is straight-line depreciation, where the asset is reduced with the same value each accounting period until it reaches its residual value (its scrap value).

Since depreciation is an expense it will affect the taxable income of the firm. This will affect cash-flows, even though the depreciation itself is non-cash expense.

Basic Accounting Concepts

- **Dual Aspect Concept** – assets always equal liabilities
- **Accounting Period Concept** – Income Statement over a period of time
- **Conservatism Concept** – choose recording method that results in the lowest asset or highest liability figures
- **Accrual Concept** – record revenues when they are earned (e.g. when the product ships) and record expenses when they are incurred.
- **Cost Concept** – record costs, not market value
- **Materiality Concept** – disregard immaterial transactions
- **Realization Concept** – recognize revenue when goods are delivered
- **Consistency Concept** – use the same method for recording transactions associated with specific assets/events
- **Entity Concept** – accounting for an entity, not individuals
- **Going Concern Concept** – assume continuing operations
- **Matching Concept** – match costs associated with revenue in same period
- **Money Measurement Concept** – money is the common measure

Accounting Choices

The accountant does have choices how to report different transactions; the same supply chain action can appear different ways in financial reports, and these can have different impact on the financial reports depending on how they are recorded. There are three sets of choices to be aware of:

- **Product costing.** The accountant is free to determine how to allocate overhead and how to categorize expenses to create as an insightful result as possible.
- **Depreciation method.** There are many ways to set the rate and period length for depreciation. For some types of assets, however, there are established accounting practices.
- **Asset or expense.** It is not always clear whether a transaction should be classified as a capital investment or an expense. One example is a prototype. The account has leeway in deciding how to classify this.

LIFO vs FIFO

There are two very different ways to estimate the value of goods in an inventory:

- **LIFO** (last-in, first-out) uses the most recent cost of inventory to assign to sales, which results in lower stated profits (assuming material costs increase over time). As a result, the remaining inventory can be undervalued.
- **FIFO** (first-in, first-out) uses the oldest cost of inventory, which results in potentially higher profits but near term tax obligations. As a result, the remaining inventory can be valued at current replacement cost.

3.6.4 Costing Systems

Cost Accounting

While cost accounting is a type of reporting, it has a different purpose than the financial reports. The purpose of cost accounting is purely internal to the firm – it is designed to measure costs to enable performance analysis, decision-making and internal reporting. So, while in financial reporting, costs are classified based on type of transaction for external reporting; in cost accounting, costs are classified based on needs of management for internal use (decision-making support). This means that cost accounting does not have to follow generally accepted accounting practice.

Overhead and Other Types of Costs

There are many types of costs to consider in the cost accounting process:

- Fixed costs: these costs do not vary with volume.
  - Ex. Monthly payments for plant, property, equipment
- Variable and Semivariable costs: costs that vary with volume
  - Ex. Cost for materials used to produce a product; more materials are needed to make more units of production
  - Ex. Semivariable: cost for salaried employees who get commission on sales
- Direct costs: costs that can be attributed to the production of a specific product
  - Ex. Cost for raw materials or labor hours to produce the product
- Indirect costs (overhead): costs that cannot be attributed to the production of a specific product
  - Ex. Legal fees, SG&A, insurance

Much of the challenge of the cost system is to find a representative way to allocate the indirect costs – *the overhead costs*. These costs include depreciation of assets, supervision, quality control, and many other costs. To properly understand the cost of producing certain products/services, the cost to serve certain customers, or the cost to rely on certain suppliers, managers need to allocate these overhead costs in a representative way. This is not easy, since there is often no direct relationship between the cost and the product. Different cost systems apply different logic for this allocation, as you will see below.

Traditional Cost Systems

The most common way to allocate costs is using a "traditional" cost system. With such a cost system, "standard costs" are calculated and used to allocate the overhead. The standard cost is the anticipated overhead cost for a process or product. A standard rate is then calculated based on some measure, e.g. throughput or labor hours. For instance, if total standard cost for manufacturing is $100,000 and budgeted throughput is 1,000, the standard rate to be applied is $100 per unit.

While these systems often perform well in many instances, there are certain instances where they are problematic. This includes instances when depreciation and other overhead costs account for a larger share and in complex environments with many (customized) products and/or processes.

Actual Cost Systems

Actual cost systems try to use actual costs or quantities to provide a more accurate picture compared to standard traditional systems. There are different variations:

- Actual cost using actual quantities and standard prices
- Actual cost using standard quantities and actual prices
- Actual cost using actual quantities and actual prices

It is however difficult to use actual prices since this will not be available until after the period is over.

Activity-Based-Costing (ABC)

An increasingly common way to handle the shortcoming of the other costing systems is to use an activity based costing approach. With this approach, relevant activities are defined, and all overhead costs are related to these activities. Based on the nature of the activity, a cost driver is identified which is then used to calculate the overhead cost for different objects (e.g. products, customer, suppliers). The general steps to follow when using ABC are the following:

1. *Identify all relevant (repetitive) activities* (a formal approach would involve creating a process model). Note that a relevant activity is often of the form “verb+object”, e.g. “schedule production”.
2. *Identify the resources consumed in performing the activities.* Based on interviews, reports, or other information, identify the relevant resources consumed in each of the relevant activities.
3. *Determine the costs of the activities.* Based on the insights from step 2, find the total cost for each relevant activity
4. *Determine cost-drivers of the activities.* This is the “unit” that drives the cost of the activity: units produced, batches, orders, shipments, etc.
5. *Determine cost-driver rate for the activities.* Based on the total cost of the relevant activity, use its total activity level to determine the rate for the cost-driver. For instance, if “number of orders” is the cost-driver, find the total number of orders over the accounting period, this is the activity level. To find the driver rate, divide total activity cost with the activity level.
6. *Trace costs to (secondary) cost objects.* Once you have all driver rates, use the information you have about each object to multiple that objects activity level with the driver-rate.

Activity-Based Costing provides a different and potentially more accurate cost for producing products and providing services. ABC can be helpful for decision makers assessing the profitability of various products, services or segmentation of those by customer or geography.

Working Capital

The working capital is a basic measure of both a company’s efficiency and its short-term financial health. It is defined as:

Working capital = Current assets – Current liabilities

Positive working capital enables the firm to continue its operations and to satisfy both maturing short-term debt and upcoming operational expenses. Basically, they can pay their current bills when due. Negative working capital means that there are not enough current assets (cash, accounts receivable, inventory) to satisfy their current liabilities (accounts payable, maturing short-term debt and upcoming operational expenses). This may be good, if the firm collects its bills before paying suppliers. But it may also be a bad thing: the company cannot convert assets into cash quick enough to pay off liabilities. A company can have assets and profits but lack liquidity if assets can't readily be converted to cash. Keep in mind that a firm can have a negative cash-to-cash cycle and positive working capital.

Sometimes working capital is analyzed through the "current ratio". If this ratio is somewhere between 1 and 2, it is considered a healthy company.

Current ratio = Current assets/Current liabilities

Another test is called the "acid test". It looks only at the current assets that can be quickly converted to cash. If the acid test is less than the current ratio, the current assets are highly dependent on inventory.

Acid test = (Cash + Accounts Receivables + Short term investments)/Current liabilities

A third important measure is the working capital turnover. This describes how effectively a company is using its working capital to generate revenues. In general, a higher turnover is preferable.

Working capital turnover = Sales/Working capital

Cash-to-Cash Cycle (CCC)

Also known as the Cash Conversion Cycle, the operating cycle or simply the cash cycle. This is a liquidity measure that can help a company plan its timing of working capital requirements. It measures the number of days that a company's cash is tied up in the production and sales process of its operations, and the benefit it gets from payment terms from its creditors. The shorter the cycle, the more liquid the company's working capital position.

The measure consists of the following three parts:

- **Days of Inventory Outstanding (DIO)** – this is the average inventory in the system, expressed in number of days
  DIO = (Average Inventory/Cost of Sales) x 365
- **Days of Sales Outstanding (DSO)** – this is the average number of days to collect revenues from a sale, that is, the credit time given to customers
  DSO = (Average Accounts Receivable/Total Sales) x 365
- **Days of Payables Outstanding (DPO)** – this is the average number of days before paying suppliers
  DPO = (Average Accounts Payable/ Cost of Sales) x 365

The Cash-to-cash Cycle, or the Cash Conversion Cycle, is then found as:

CCC = DIO + DSO - DPO

3.6.5 Supply Chain Cash Flows

There are three flows in the supply chain: the physical flow (goods), the financial flows (money), and the information flows (data). Since we may use contractors for many of our firm's operations, physical flows may not pass through our firm at all. Financial flows, however, will. Consequently, the design of the financial flows in the supply chain will have an impact on the value we are creating and therefore how valuable our supply chain designs are.

Investments and future cash flows

The primary source for capital for a firm is from its stockholders, or equity investors. These investors purchase stock in the firm to provide the firm with capital. The firm may then contract out with debtors, banks, or bond holders to raise additional capital that can be used for investments. There is a contractual agreement with these debtors. For instance, if you borrow money from a bank you will have a contract where you pay them back over a certain period of time.

This is not the case with investors. For these, the firm hopes to make enough money to pay them back. How? By investing the capital in operational assets to generate future cash flows. The goal is to manage the assets in the supply chain well enough so that revenues are greater than expenses.

Investors compare a firm's result with the other investment options they have to create a portfolio of options that will give them future returns. So investors' investment in a firm is a part of their portfolio. Given the firm's ability to provide returns, it will be more or less attractive to investors.

Investment evaluation

Normally, those that have the money in an organization are also those that decides on which investments to make. Following general corporate finance guidelines, investments are evaluated according to the following steps:

1. Estimate the relevant cash flows – this includes also the cash outflow of making the investment
2. Calculate a figure of merit for the investment – this is to come up with the "value" of the investment
3. Compare the figure of merit to an acceptance criterion

Note that the same process may be used to evaluate projects.

EBIT and EBITDA

To separate different types of operating income, we will use the following concepts:

- **EBIT** – Earnings Before Interest and Taxes. These are the earnings, or the profit before any financial aspects are added.
- **EBITDA** – Earnings Before Interest, Taxes, Depreciation and Amortization. These are the earnings, or the profit, before financial aspects as well as depreciation and amortization. That is, it is the revenue minus COGS and SG&A.

Projected cash flows

Cash coming in is considered positive cash flow; cash going out is considered negative cash flow. To handle the cash flows in the evaluation models, they are aggregated in "bins" – normally each year is a bin. Common practice assumes cash flows occur at the end of the projected period, or bin. The appropriate time horizon to consider depends – the decision makers simply have to come to an agreement. However, one must also consider cash flows at the end of the decision horizon – assets may be divested and, when they are, incur a salvage value.

Inflation

There are two different ways to think about inflation when evaluating cash flows:

- **Nominal cash flow:** incorporate inflation in price/cost. The inflation rates may differ between different components.
- **Real cash flows:** do not include inflation in price/cost.

The choice between nominal or real cash flows influences the discount rate used when calculating the figure of merit.

Relevant Cash Flows

We need to make sure that the cash flows we are projecting are relevant to the particular investment or project we are evaluating. So how do we determine what is a relevant flow?

1. **Cash flow principle:** only cash flows where money moves are relevant. That is, a cash flow either goes out of the firm or comes into the firm. For instance, depreciation is not a movement of money, so it is not a relevant cash flow. However, depreciation also affects taxes, which is a relevant cash flow.
2. **With-without principle:** only cash flows that are different with the investment compared to without the investment are relevant to the decision. Note that this is different from "before and after", since we are looking into the future at two different worlds – we are only interested in the difference between those worlds. For instance, if revenues will not differ between our alternatives, then they are not relevant cash flows.

Note that this also means that sunk costs are not relevant cash flows. Opportunity cost of assets should, however, be included in the evaluation. It differs between the alternatives and is thus a relevant cash flow.

Free Cash Flows

The free cash flows (FCF) are the funds available to (the equity) investors after the firm invests capital and pays taxes. It provides a convenient way to calculate cash flows to compare investments, by relying on posts in the financial reports rather than separate line items. For free cash flow calculations, we need to consider revenues, COGS, operating expenses, and taxes as well as capital expenditures and working capital. Note that some of these are found in the income statement whereas others are found in the balance sheet. Starting with the income statement, relevant cash flows include taxes.

To make sure we capture the effect of taxes, we make use of the Net Operating Profit After Taxes (NOPAT).

$$NOPAT_t = (1 - \text{tax rate}) \times EBIT_t$$

Using NOPAT, which takes depreciation into account, we can find the relevant cash flows of the income statement by adding the depreciation (DA) "back again". From the balance sheet, we get the capital expenditure and the change in net working capital requirement.

Combing the data from the income statement and the balance sheet we get:

$$FCF_t = NOPAT_t + DA_t – CapEx_t – \Delta NetWorkingCapital_t$$

$$FCF_t = (1 - \text{tax rate}) \times EBIT_t + DA_t – CapEx_t – \Delta NetWorkingCapital_t$$

Working Capital Cash Flows

The working capital cash flow is the net change in working capital requirements from the previous period. For instance, a decrease in working capital requirements means that more cash is freed up (note the minus sign in the FCF formula). Note that working capital cash flows occur when the change takes place. For instance, a change in inventory level affects working capital requirements when we next replenish stock.

3.6.6 Discounted Cash Flow Analysis

We now focus on the two subsequent steps in investment evaluation: how to calculate a figure of merit based on the projected relevant cash flows and compare this figure to an acceptance criterion. The basic idea underlying these calculations is that today's money is worth more than money in the future. To correct for this when we make investment decisions, we use a discount rate to find today's value of future cash flows. Why is today's cash more valuable than tomorrow's? There are primarily three reasons:

- **Opportunity cost** – not having the money now results in foregone investment returns
- **Inflation** – this reduces purchasing power over time
- **Risk/uncertainty** – receipt of cash is not guaranteed over time

Owing to this, we need to calculate the present value of future cash flows when evaluating supply chain designs. As we saw in this lesson, there are several ways to do this. Common for all discounted cash flow analysis, is that it quantifies the value created by investments.

Figure of Merit

The figure of merit is a single number that estimates the economic value of an investment. This number can be compared with a criterion established by the firm. There is no universal criterion. There are also several different figures of merit, which you will see more of below.

Payback Period

The payback period is the time until the cumulative cash flow is equal to our initial investment. Using this figure of merit, the acceptance criterion is to invest if the payback period is shorter than the cut-off point. The firm decides the cut-off point.

This is a fairly simple measure, which is easy to communicate. The main drawback is that it does not consider the timing of cash flows. For instance, cash up front is not valued differently than cash later. Neither does it consider cash flows after the payback-period, even though these may be significant.

Return on Assets (ROA)

The return on assets (ROA) is defined by:

ROA = Net income/ Assets

The acceptance criterion using this figure of merit is to make an investment if ROA is greater than some target return. "Assets" are, for an investment, the total capital investment that will depreciate over time. This means that later incoming cash flows are given a higher weight in the calculation, even though earlier cash flows are generally more desirable.

ROA is very much linked to the financial statement. However, it is based on accounting calculations and not cash flows. Also, just like the payback period, it does not consider the timing of cash flows.

Present Value

The present value of a future cash flows is given by:

$$PV = FV/(1+r)^n$$

where PV is present value, FV is future value, r is the discount rate for the period, and n is the number of periods. Note that you must have the same time period for your rate and your number of periods. Often, we stick to annual values.

The discount rate is based on investors' expected rate of return. This is sometimes referred to as the hurdle rate – it sets the hurdle by which we have to run our operations to exceed the expectations of our investors.

Net Present Value

The net present value (NPV) is the sum of all discounted cash flows from all relevant future periods of an investment:

$$NPV = c_0 + c_1/(1+r)^1 + c_2/(1+r)^2 + \ldots + c_T/(1+r)^T$$

where c_i represents the net cash flow in period i. Note that we are not discounting the cash flow of period 0 – this is the present value already.

NPV is easily implemented in your spreadsheet software. If using the built-in function, keep in mind that period 1 is the first period in the function, period 0 cash flows need to be added separately.

The net present value is a figure of merit that can be used to evaluate an investment on its own or compare different investments with each other. The acceptance criterion for a single investment is that the NPV is greater than zero.

Internal Rate of Return

While the NPV has a lot of nice properties for a figure of merit, it is dependent on picking the right discount rate. The internal rate of return (IRR) addresses this problem by, instead of considering the NPV given a certain discount rate, considers the discount rate at which the NPV=0. Consequently, management can compare the target return with the IRR. The criterion of acceptance is thus to invest as long as the target rate of return is between zero and the IRR.

The IRR is found as the solution to:

$$0 = c_0 + c_1/(1+IRR)^1 + c_2/(1+IRR)^2 + \ldots + c_T/(1+IRR)^T$$

There is no closed form solution for this. There are however spreadsheet functions available to calculate this.

Terminal Value

We may want to consider how assets that last beyond our financial projections should be handled. We have touched upon the salvage value before. There may be cash flows to consider even if the investment is not salvaged – maintenance, certain inventory policies, etc. The terminal value handles all the cash flows that are incurred after period T, which is the last period of our projections. After period T, cash flows are assumed to be stable. There are normally two approaches:

- **Perpetuity:** estimated value of stable cash flows that continue indefinitely
- **Annuity:** estimated value of stable cash flows for definite period beyond the unique projections

You can incorporate growth into the analysis as long as growth is stable.

A perpetuity is geometric series that converges to:

$$PV = C/r$$

where C is the stable cash flow per period. An annuity is the net present value of stable cash flows over a given number of periods. The annuity is thus given by:

$$PV = C/r–(C/r)(1/(1 + r)^T))$$

Inventory Holding Cost

In inventory management we use a holding cost (or carrying cost) for calculating how much inventory to keep. This is an annual cost, and it consists of several components:

- Capital
  - Opportunity cost of capital
- Operating
  - Warehouse (power, property taxes, etc.)
  - Equipment
  - Labor (handling, stock keeping, etc.)
  - Disposal, scrap (direct or third party costs)
- Lost revenue
  - Obsolescence
  - Depreciation (real market value)
  - Deterioration
  - Shrinkage
  - Damage
  - Insurance to prevent lost revenue

Oftentimes, we refer to the last two categories as non-capital holding costs. But how is this incorporated into discounted cash flow analysis?

Well, these cash flows have to be evaluated like any other cash flows:

- Opportunity cost of capital is implicitly included in the discount rate, and is not a relevant cash flow.
- The non-capital costs (operating cost, lost revenue) are relevant, projected cash flows.

Consequently, when you reduce inventory, you get the working capital cash once to reinvest and you avoid operating expenses and/or lost revenue over time.

3.6.7 Defining Supply Chain Finance

Supply chain finance has been used as a broad term. It can describe how we look at the supply chain through a financial lens. It helps us identifies the financial impact from supply chain transactions and how supply chain decisions impact financial statements. But in recent years, the definition of supply chain finance has been formalized as a method to provide liquidity to buyers and sellers. This typically means payment terms of Net 30 or 2% 10, Net 30.

The move to a focus on liquidity stems from the global financial crisis of 2008, when the issue of getting cash to fund business operations was exasperated. In general, buyers are increasingly looking for ways to reduce their net operating working capital requirements – that is, the funds necessary to continue operation of the business. Many buyers try extend the payment cycle from 30 days anywhere to 150+ days. Therefore, many suppliers have to wait a long time to get paid and need to cash to pay bills while waiting for the invoice payment.

Supply Chain Finance (SCF) is the set of solutions that allow a company to finance its own working capital, leveraging its role within the supply chain and its relationships with other players in the chain. Supply Chain Finance has many variation among different solutions and also has different terminology across users.

SCF provides financing

- "Financing is the act of providing funds for business activities, making purchases or investing."
- Third parties often involved: Banks, other financial institutions.
- Providing funding/capital often to consumers (e.g., for mortgages), and businesses (capital improvements).
- Supply chains 'finance' or fund their payables (and sometimes inventory).

There are two general options for financing (debt and equity) when applying these options to Supply Chain Finance (SCF). Most are using debt – or loans – to provide liquidity – cash – to the firm. Some entail selling the asset (e.g. a receivable from the buyer), usually at a deep discount. There are many variations, and each have advantages and disadvantages.

Liquidity

Liquidity is the degree to which an asset or security can be quickly bought or sold in the market without affecting the asset's price or the ability to quickly convert assets into cash at value. Liquidity allows the business to operate, to pay bills, to buy materials, etc.

Liquidity and the CCC

Consider the Cash Conversion Cycle, CCC (also known as the cash-to-cash conversion cycle). There are three components in the CCC: the cycle time to pay suppliers, the cycle time to get paid by customers, and the cycle time to produce and sell a product (from order entry time to time of sale). To provide more liquidity the firm can: arrange for faster payments from customers; work to reduce the production and sales cycle times, and/or slow down payments to suppliers.

Cash Conversion Cycle Acronyms

- DIO = Days of Inventory Outstanding (how long it takes to produce and sell)
- DSO = Days of Sales Outstanding (how long it takes to collect payment)
- DPO = Days of Payables Outstanding (how long the company takes to pay suppliers)
- CCC = Cash Conversion Cycle (how to convert cash → inventory → AP → cash)

Supply Chain Finance: How does it work?

Case: Raw material purchase

- Situation: Buyer agrees to pay seller in X days
- Seller can wait X days for payment or consider other options to get cash sooner

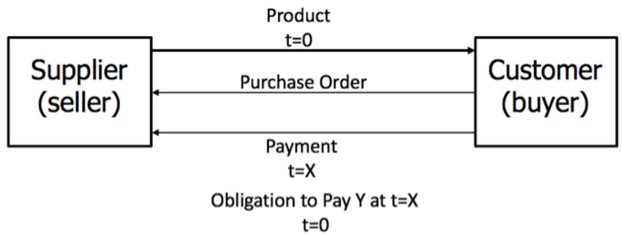

FIGURE 3.18: How does Supply Chain Finance work?

Basic Factoring

If the seller does not want to wait. One seller option is called **factoring**:

- Sell the receivable to a third party (e.g., collections agency), called the Factor
- The receivable is an obligation of the buyer to pay the seller

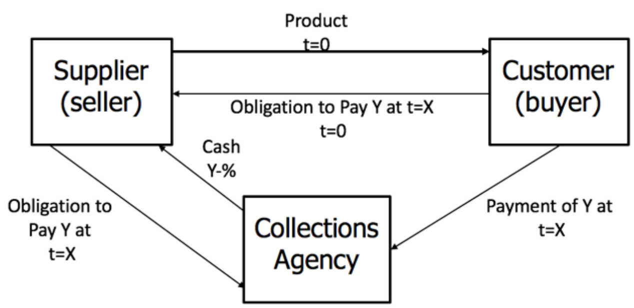

FIGURE 3.19: Basic factoring
t=time, X=payment date, Y=amount

Factoring with Recourse: The Factor can go back to seller to collect if the buyer fails to pay

- If the Buyer fails to pay the Factor, then the Factor can collect from the seller.
- The percentage off invoice will be lower because the seller still bears the risk if the customer does not pay.

Factoring without Recourse: The Factor has no option to go back to the seller to collect, can only collect from the buyer.

- If the Buyer fails to pay the Factor, then the Factor is stuck.
- No option to go back to the seller to collect.
- The percentage off invoice will be higher because the Factor bears the risk if the customer does not pay.

There are many options to finance receivables such as factoring. This is probably the most common but can come at a high cost, percentage of the invoice. Other options vary based on factors including: loan or sale of the receivable, timing of the payments, which party bears the risk of repayment, use of different types of collateral, who initiates the arrangement. Each option has different impact on net operating working capital.

Options for Liquidity in the Supply Chain

Options for liquidity	Description
Invoice Discount	-Initiated by the seller -No third party involved -Seller offers lower price for early payment -Buyer chooses to exercise the option to pay early at a discount -The payment amount Z will be lower than the invoice amount -The timing of the payment in exchange for the discount is determined by the seller, but it will be shorter than the original payment term
Financing a loan (Invoice Discount)	-Use the receivable as collateral for the basis of a loan (e.g. from a bank) -Initiated by the seller -Third party involved -Seller uses invoice as collateral for loan with 3rd Party (e.g. Bank) -Third party issues loan to seller, uses seller credit risk to set the rate
Forfaiting	-Used for exports/imports -Initiated by the seller -Third party involved, usually with expertise in export/import -Seller sells the receivable to the third party at a discount -Third party buys receivable at a discount, collects from buyer
Reverse Factoring	-Initiated by the buyer who also initiates the financing options (loans) -Third party involved through the buyer -Seller has the option to use the financing options offered -Third party offers to finance receivable based on the buyer's credit risk, meaning lower cost loan
Dynamic (Invoice) Discount	-Initiated by the seller -No third party involved -Seller offers lower price for early payment, timing of payment and discount rate varies -Buyer negotiates timing and rates and chooses to exercise the option to pay early at a discount -The payment amount Z will be lower than the invoice amount
Inventory Financing	-Initiated by the seller -Third party involved -Seller arranges line-of-credit or loan in order to purchase materials -Third party offers loan with seller's inventory as collateral -Useful when seller must pay its suppliers before being able to sell FGI; also used to build inventory prior to peak demand season
Invoice Auction	-Initiated by the seller -Third parties invited -Seller arranges an auction process to solicit bids for the invoice (buyer obligation to pay); seller can use existing auction platform -Highest bidder wins -Like Factoring without Recourse, but using an auction to find third party

FIGURE 3.20: Options for liquidity in the Supply Chain

Advanced Options

Utilize automated processes

- Less uncertainty in timing
- Lower cost loan operations

Solicit and use more detailed data about seller and buyer

- More informed risk assessments

- Can result in lower risk and therefore lower cost loans
- Favorable rates for increased visibility and faster process due to automated info sharing, better risk assessment

Proactive use of information technology for faster and more informed decision making.

Other Options to Finance the Supply Chain

Use operational measures to reduce the need for working capital:

- Consignment inventory (customers push the capital requirement onto the supplier)
- Direct shipments (reduce the need to tie up capital in warehouses)
- Various forms of collaboration – vendor managed inventory (VMI), sales and operations planning (S&OP), collaborative planning, forecasting and replenishment (CPFR), continuous replenishment (CRP), etc. Each of may reduce the amount of inventory necessary to maintain service levels

Benefits of Supply Chain Finance

There are many benefits of supply chain finance. They include:

- Potential to reduce the Net Operating Working Capital Requirements (NOWC): lower for both seller and buyer via quicker cash to seller, lower inventories
- Potential lower cost materials: via discounts; lower operating costs with use of technology for information
- Potentially faster cycle times: via faster cash, faster operations; reducing CCC
- Potentially more robust supply chain: via collaborative financial connections and ensuring supplier health
- Benefits to third parties: More customers for loans, potentially lower risk

3.6.8 Analyzing Financial Performance

We might ask ourselves, who cares about a firm's financial performance? The obvious answer is investors, executives in the firm, and employees of the firm (if it affects their bonus). Beyond who this matters to, many supply chain professionals ask further questions including: How do you analyze financial performance? How do you measure it? How do financial analysis and measurement approaches apply to supply chain management? We will discuss these questions in the next section.

The main goal of a CEO is to provide shareholder value. Drivers of shareholder value include: revenue growth, operating margin, and asset productivity. To monitor this, CEOs and other executives generally need a scorecard to assess these drivers.

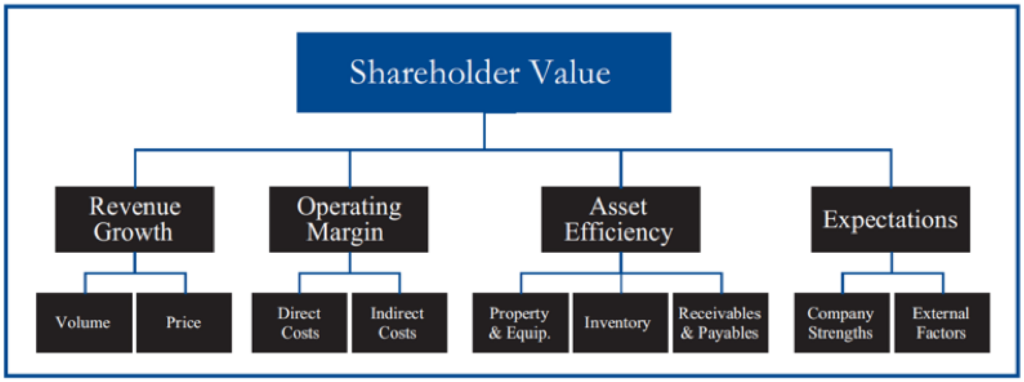

FIGURE 3.21: High-level shareholder value map (Deloitte methods)

Balanced Scorecard
Ultimately, causal paths from all the measures on a Scorecard should be linked to financial objectives. We have found that companies use three financial themes to achieve their business strategies:

- Revenue Growth and Mix
- Cost Reduction / Productivity Improvement
- Asset Utilization / Investment Strategy

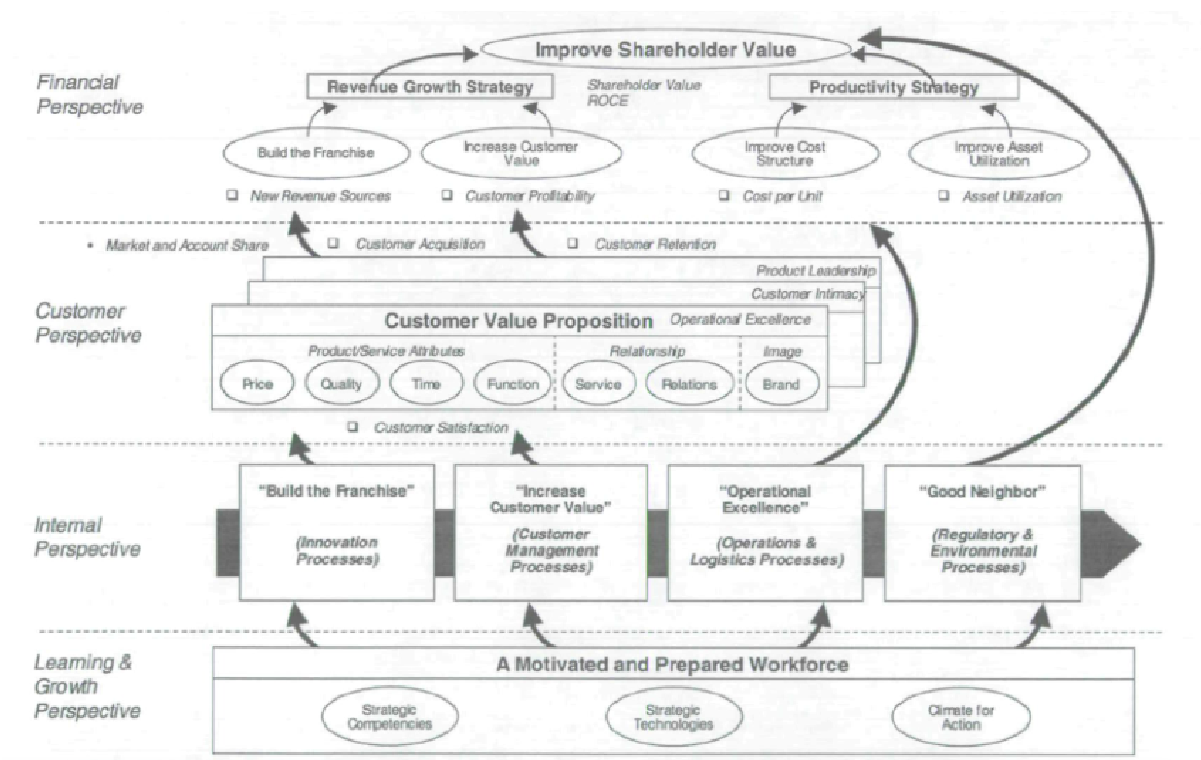

FIGURE 3.22: The Balanced Scorecard strategy map

Drivers of Shareholder Value

The drivers of shareholder value are: Revenue Growth, Operating Margin and Asset Productivity (see figure 3.23).

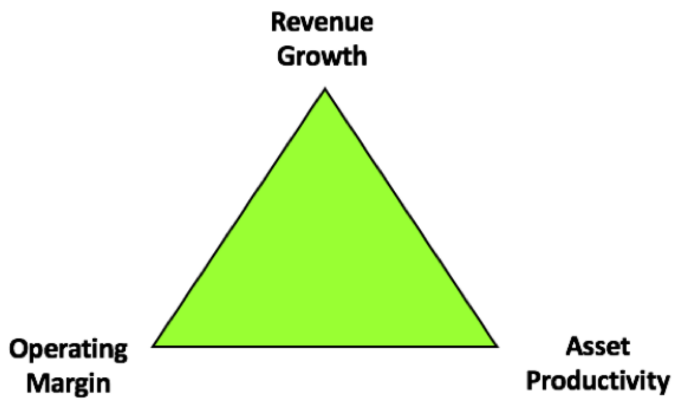

FIGURE 3.23: Drivers of shareholder value

Ratio Analysis

Ratio Analysis is the mathematical comparison of a financial statement accounts or categories. These relationships between the financial statement accounts help investors, creditors, and internal company management understand how well a business is performing and of areas needing improvement. See table 3.5 for potential comparison areas on each statement or sheet.

Income Statement	**Balance Sheet**
Sales	Cash and Equivalents
Cost of Goods	Accounts Receivable
Gross Profit	Inventories
SG&A Expenses	Total Current Assets
Depreciation & Amortization	Net Property/Plant/Equipment
EBIT	Total Long-Term Assets
Interest Expense	Total Assets
Income Taxes	Accounts Payable
Net Income	Total Current Liabilities
	Long Term Debt
	Total Liabilities
	Common Stock
	Retained Earnings
	Total Stockholder's Equity
	Total Liabilities & Equity

TABLE 3.5: Potential comparison areas on each statement

Overall measure of financial performance

Is there a single metric that can reflect the firm's financial performance for investors? Or executives?

Return on Equity

$$ROE = NetIncome/Equity$$

Return on Assets

$$ROA = NetIncome/TotalAssets$$

Specific measures of financial performance

Revenue growth

$$\text{Sales growth} = (Sales_t/Sales_{t-1}) - 1$$

$$\text{Compound Annual Growth Rate (CAGR)} = (Sales_t/Sales_{t-n})^{1/n} - 1$$

Operating margin

- Gross margin = Gross Income / Sales
- Operating margin = Operating Income / Sales
- Net margin = Net Income / Sales

Asset productivity

- Asset turnover = Sales / Total Assets
- Inventory turnover = COGS / Average Inventory
- Accounts receivable turnover = Credit Sales / Average Accounts Receivable
- Accounts payable turnover = COGS / Average Accounts Payable
- Days of inventory outstanding = Average Inventory / (COGS/365)

Ratio Analysis Advice

There are not any "correct" values for ratios. Ratio values need to be understood in context such as: compare with industry averages, compare with specific competitors; and/or observe trends over time. You will need to develop a framework of several ratios to monitor. In combination, these clues may tell an interesting story.

DuPont Analysis

DuPont analysis is based on a return-on-investment formula developed in 1914 by a DuPont explosives salesman named Donaldson Brown and used by the company. Mr. Brown later used it as CFO at General Motors, but it was already known as the DuPont formula.

$$R = T \times P$$

Where:
R = rate of return on capital invested
T = rate of turnover of invested capital
P = percentage of profit on sales

In essence, it is a simple combination of two ratios:

$$\text{ROA} = \text{Net Margin} \times \text{Asset Turnover}$$

$$\text{Net Margin} = \text{Net Income/Sales}$$

$$\text{Asset Turnover} = \text{Sales/Total Assets}$$

The formula commonly used today focuses on ROE for Dupont analysis incorporates financial leverage:

$$\text{ROE} = \text{Net Margin} \times \text{Asset Turnover} \times \text{Financial Leverage}$$

$$\text{Net Margin} = \text{Net Income/Sales}$$

$$\text{Asset Turnover} = \text{Sales/Total Assets}$$

$$\text{Financial Leverage} = \text{Total Assets/Equity}$$

GMROI: Gross Margin Return on Investment

GMROI has a narrower focus than DuPont and is useful in evaluating inventory decisions, and their impact on profitability.

$$\text{GMROI} = \text{Gross Margin} \times \text{Inventory Turnover}$$

$$\text{Gross Margin} = \text{Gross Profit/Sales}$$

$$\text{Inventory Turnover} = \text{Sales/Inventory}$$

ROIC

ROIC (Return on Invested Capital) is an overall measure of financial performance. This includes returns on all capital for investors seeking a return, not only on equity. ROIC is a good measure for supply chain performance. It is not confounded by financing strategies. It shows the fundamental earning power of the firm, i.e. created by operations.

$$\text{ROA} = \text{Net Income / Total Assets}$$

$$\text{ROE} = \text{Net Income / Equity}$$

Both measures can be distorted by financial leverage, i.e. more debt.

$$\text{ROIC} = \text{NOPAT / Invested Capital}$$

Net Operating Profit After Tax (NOPAT)

- NOPAT = EBIT (1 – Tax rate)
- Earnings after tax as if it were all equity financed (i.e., not considering interest expense or tax books)

Invested Capital (IC)

- IC = Interest-Bearing Debt + Equity
- Sum of all sources of cash on which a return must be earned (i.e., not including accounts payable)
- You may want to subtract excess cash
- Prefer the book value of Invested Capital (i.e. the value invested) rather than the market value

RONA

RONA (Return on Net Assets) is similar to ROIC and similar to ROCE (Return on Capital Employed). It is a return on assets that are expected to produce profit.

$$\text{RONA} = \text{NOPAT/ Net Assets}$$

Net Assets

- NA = Fixed Assets + Non-cash Working Capital
- NA = Total Assets – Current Liabilities – Cash

Learning Objectives

- Understand the basics of financial reporting and accounting
- Understand the impact of depreciation on the financial reports
- Introduce cost accounting/cost systems
- Become familiar with the principles of the different cost systems
- Learn how to perform simple activity based costing (ABC)
- Become familiar with working capital requirements and the Cash Conversion Cycle
- Understand relevant cash flows and how to estimate them
- Understand and learn how to find free cash flows from the financial reports
- Determine the free cash flows over time for an investment/project
- Understand the underlying reasons for discounted cash flow (DCF) analysis
- Review how to calculate the Net Present Value (NPV) and the Internal Rate of Return (IRR) of an investment, and how to use these figure of merits for evaluating investments
- How to handle inventory reductions in DCF analysis
- Understand change in payment terms to get cash to fund operations
- Recognize supply chain finance definition and that it has variations
- Become familiar with the Cash Conversion Cycle
- Review different types of finance receivables
- Recognize variations across factoring in financing receivables
- Understand different options to finance supply chain
- Introduction to effective analysis to drivers of shareholder value
- Recognize financial metrics that can assess supply chain performance and guide strategy
- Review which metrics are better at assessing total supply chain performance

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- Anthony, R.N. and Breitner, L.K. Core Concepts of Accounting. 10th ed. Prentice Hill, 2010, see pages 1-43 [this book provides a summary of the Essentials book, but the reader should go through the Essentials book first for complete coverage of the material]
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3.7 Organizational, Process, and Performance Metric Design

3.7.1 Summary

We also review three areas of "soft" design: organizations, processes, and metrics.

Organizational design for supply chains has evolved as the profession has changed. Supply chain organizations started as separate silo-ed functions or activities spread out across different larger divisions, such as finance, manufacturing, and marketing. The areas started combining into materials management (covering the flow of inbound materials) and physical distribution (covering the movement of final products to customers). Logistics groups emerged later to bring these two functions together in order to work out trade-offs between the sometimes conflicting goals of inbound and outbound management. Currently, there are various forms of organizations used across different companies. The main perspectives differ in terms of viewing supply chains or logistics as a set of functions, as a program, or within a matrix. The matrix form is most common in larger global firms. In these cases, logistics is a horizontal function that interacts directly with each vertical business unit providing shared services.

The key trade-off involved with supply chain organizational design is whether to centralize or decentralize different activities. Centralization implies that all decisions are made at a headquarters while decentralization moves this to the regions or individual business units. Each has a role with the general rule of thumb shifting most procurement, long-range planning, and new product activities to central and keeping daily operations decentralized.

Processes within supply chains are essentially transformations of inputs into outputs. Procurement transforms an order into product. Transportation converts material at a distribution center to product at the customer location. Each of these processes can be managed individually or looked at as within a greater system. The different processes within the supply chain should be designed to complement each other.

Metrics follow the process structure. All metrics can be boiled down to utilization, productivity, or effectiveness metrics. Utilization measures inputs compared to some norm (capacity, standard, etc.). Productivity or efficiency compares inputs to outputs. Effectiveness compares output to some norm value. The different metrics can be evaluated for robustness, integration, usefulness, and validity. There is no single best metric, however, they will all have trade-offs between these criteria. This is why systems of metrics should be used in a balanced scorecard framework.

3.7.2 Key Concepts

3.7.3 Supply Chain Organizational Design

Centralization vs. Decentralization

Organizational design centers on whether to centralize or decentralize an activity. Each has a place and there are basic trade-offs involved.

Centralized

- Leverages economies of scale
- Harmonizes policies and practices
- Allows for optimal (global) solutions

Decentralized

- Allows decision making to be closer to customer
- Allows decisions to reflect local cultures and customs
- Allows for optimal solutions within a region
- Allows business units to act autonomously

Supply Chain Processes

Supply chain processes can be thought of transformations of one flow unit to another.

Process	**Flow Unit**	**Transformation**
Order Fulfillment	Orders	Receipt of order => Delivery of product
Production	Products	Receipt of materials => Completion of product
Outbound Logistics	Products	End of manufacturing => Delivery to customer
Procurement	Supplies	From issuing a purchase order => Receipt of supplies
Customer Service	Customers	Arrival of customer ⇒ Departure
New Product Development	Projects	Product development start ⇒ Launch
Cash Cycle	Cash	Expenditure of funds ⇒ Collection of revenue

FIGURE 3.24: Supply chain processes

3.7.4 Supply Chain Metric Design

Performance Metrics

Metrics can be divided into Utilization, Productivity, and Effectiveness measures.

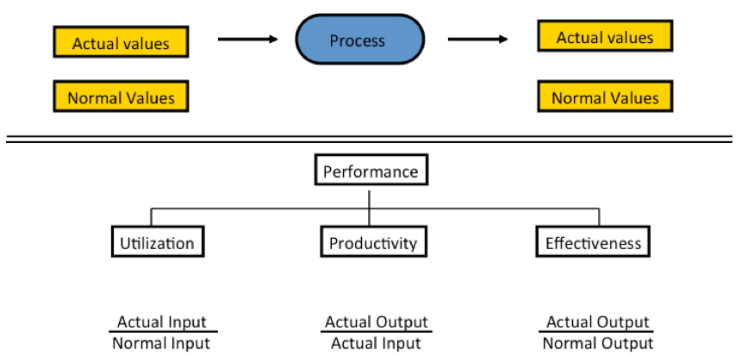

FIGURE 3.25: Performance dimensions

Performance Metric Criteria and Trade-offs

Metrics can be evaluated according to several metrics – but they will always have trade-offs.

- **Robust:** The metric is interpreted similarly by all users, is comparable across time, location and organizations, and is repeatable.
- **Valid:** The metric accurately captures events and activities measured and controls for exogenous factors.
- **Integrative:** The metric includes all relevant aspects of the process and promotes coordination across functions and divisions (and even enterprises).

- **Useful:** The metric is readily understandable by decision makers and provides a guide to action.

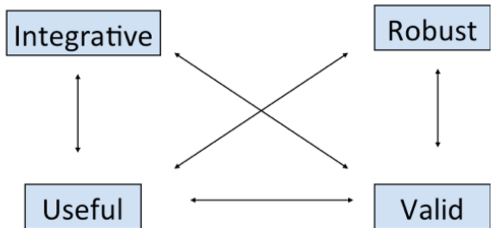

FIGURE 3.26: Performance criteria trade-offs

Balanced Metrics

Metric systems should be designed to balance across three key areas: Asset utilization (utilization metrics), Efficiency (productivity metrics), and Customer Response (effectiveness metrics). Different industries will emphasize different aspects of process performance.

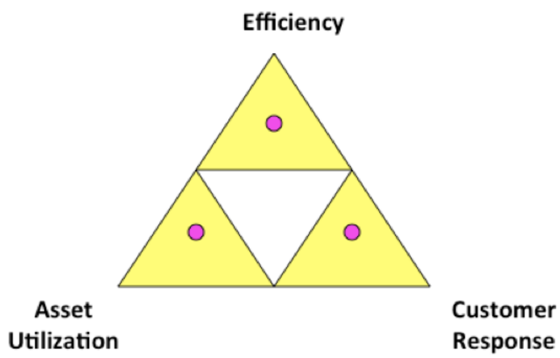

FIGURE 3.27: Balanced metric system

Learning Objectives

- Understand how supply chain organizations are typically and can be organized and why
- Learn strengths and weaknesses of centralized versus decentralized organizations
- Gain insights into supply chain processes
- Able to develop and design performance metric systems for supply chains

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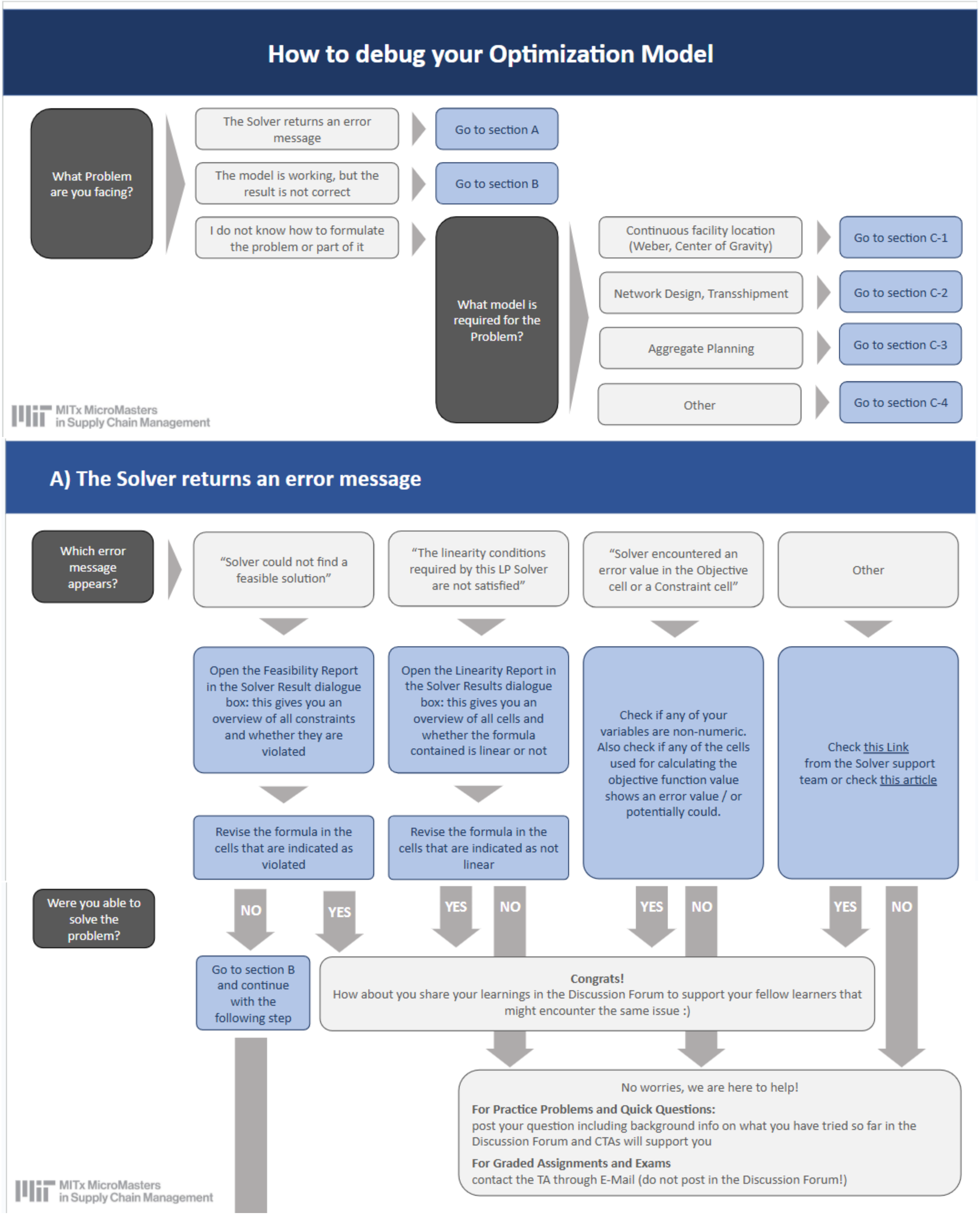
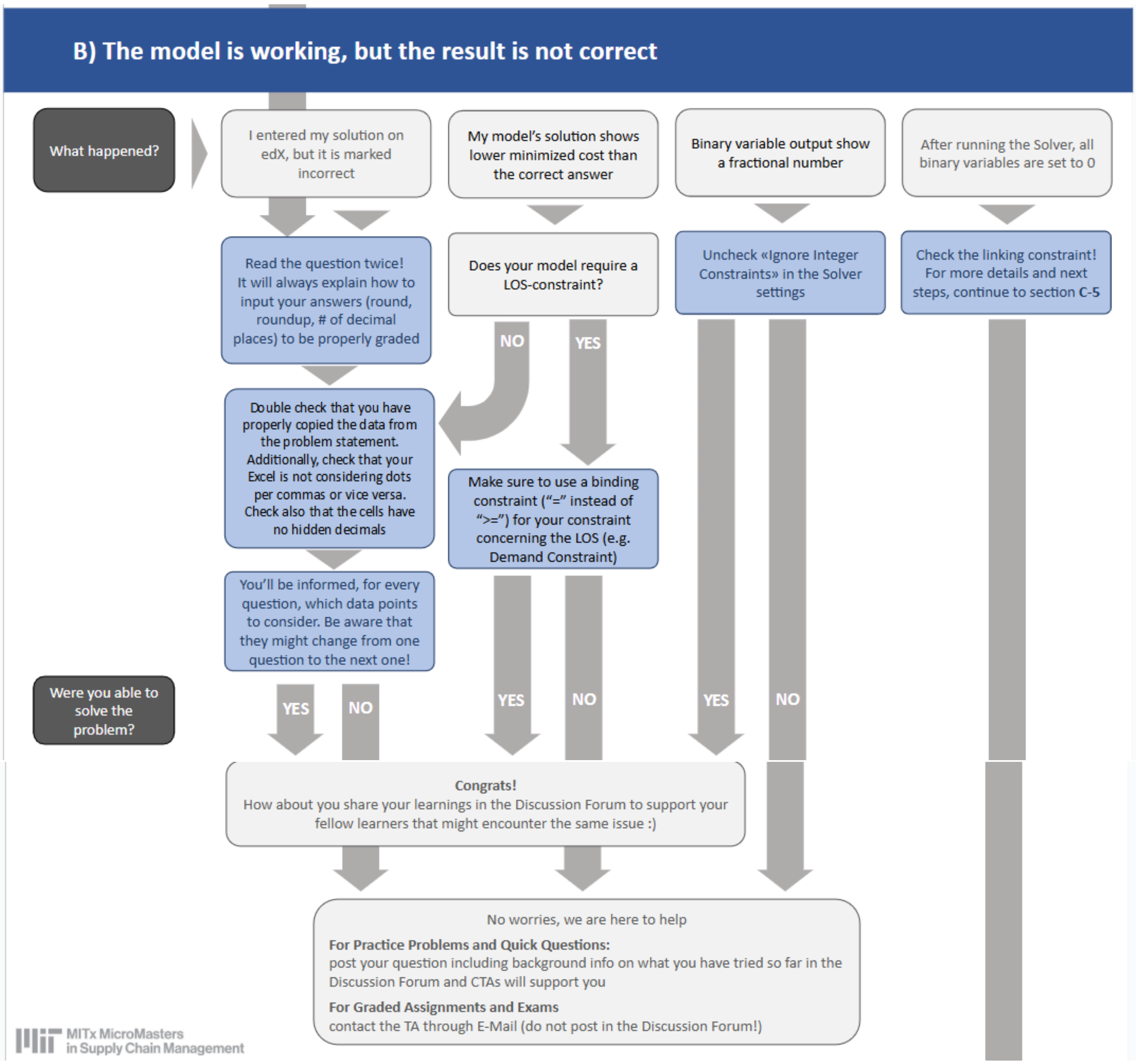
3.8 Appendix A How To Debug Your Model - Flowchart


How to debug your Optimization Model
What Problem are you facing?
The Solver returns an error message
Go to section A
The model is working, but the result is not correct
Go to section B
I do not know how to formulate the problem or part of it
What model is required for the Problem?
Continuous facility location (Weber, Center of Gravity)
Go to section C-1
Network Design, Transshipment
Go to section C-2
Aggregate Planning
Go to section C-3
Other
Go to section C-4
MITx MicroMasters in Supply Chain Management
A) The Solver returns an error message
Which error message appears?
"Solver could not find a feasible solution"
"The linearity conditions required by this LP Solver are not satisfied"
"Solver encountered an error value in the Objective cell or a Constraint cell"
Other
Open the Feasibility Report in the Solver Result dialogue box: this gives you an overview of all constraints and whether they are violated
Open the Linearity Report in the Solver Results dialogue box: this gives you an overview of all cells and whether the formula contained is linear or not
Check if any of your variables are non-numeric. Also check if any of the cells used for calculating the objective function value shows an error value / or potentially could.
Check this Link from the Solver support team or check this article
Revise the formula in the cells that are indicated as violated
Revise the formula in the cells that are indicated as not linear
Were you able to solve the problem?
NO
YES
YES
NO
YES
NO
YES
NO
Go to section B and continue with the following step
Congrats!
How about you share your learnings in the Discussion Forum to support your fellow learners that might encounter the same issue :)
No worries, we are here to help!
For Practice Problems and Quick Questions:
post your question including background info on what you have tried so far in the Discussion Forum and CTAs will support you
For Graded Assignments and Exams
contact the TA through E-Mail (do not post in the Discussion Forum!)
MITx MicroMasters in Supply Chain Management


B) The model is working, but the result is not correct
What happened?
I entered my solution on edX, but it is marked incorrect
My model's solution shows lower minimized cost than the correct answer
Binary variable output show a fractional number
After running the Solver, all binary variables are set to 0
Read the question twice! It will always explain how to input your answers (round, roundup, # of decimal places) to be properly graded
Does your model require a LOS-constraint?
Uncheck «Ignore Integer Constraints» in the Solver settings
Check the linking constraint! For more details and next steps, continue to section C-5
NO
YES
Double check that you have properly copied the data from the problem statement. Additionally, check that your Excel is not considering dots per commas or vice versa. Check also that the cells have no hidden decimals
Make sure to use a binding constraint ("=" instead of ">=") for your constraint concerning the LOS (e.g. Demand Constraint)
You'll be informed, for every question, which data points to consider. Be aware that they might change from one question to the next one!
Were you able to solve the problem?
YES
NO
YES
NO
YES
NO
Congrats!
How about you share your learnings in the Discussion Forum to support your fellow learners that might encounter the same issue :)
No worries, we are here to help
For Practice Problems and Quick Questions:
post your question including background info on what you have tried so far in the Discussion Forum and CTAs will support you
For Graded Assignments and Exams
contact the TA through E-Mail (do not post in the Discussion Forum!)
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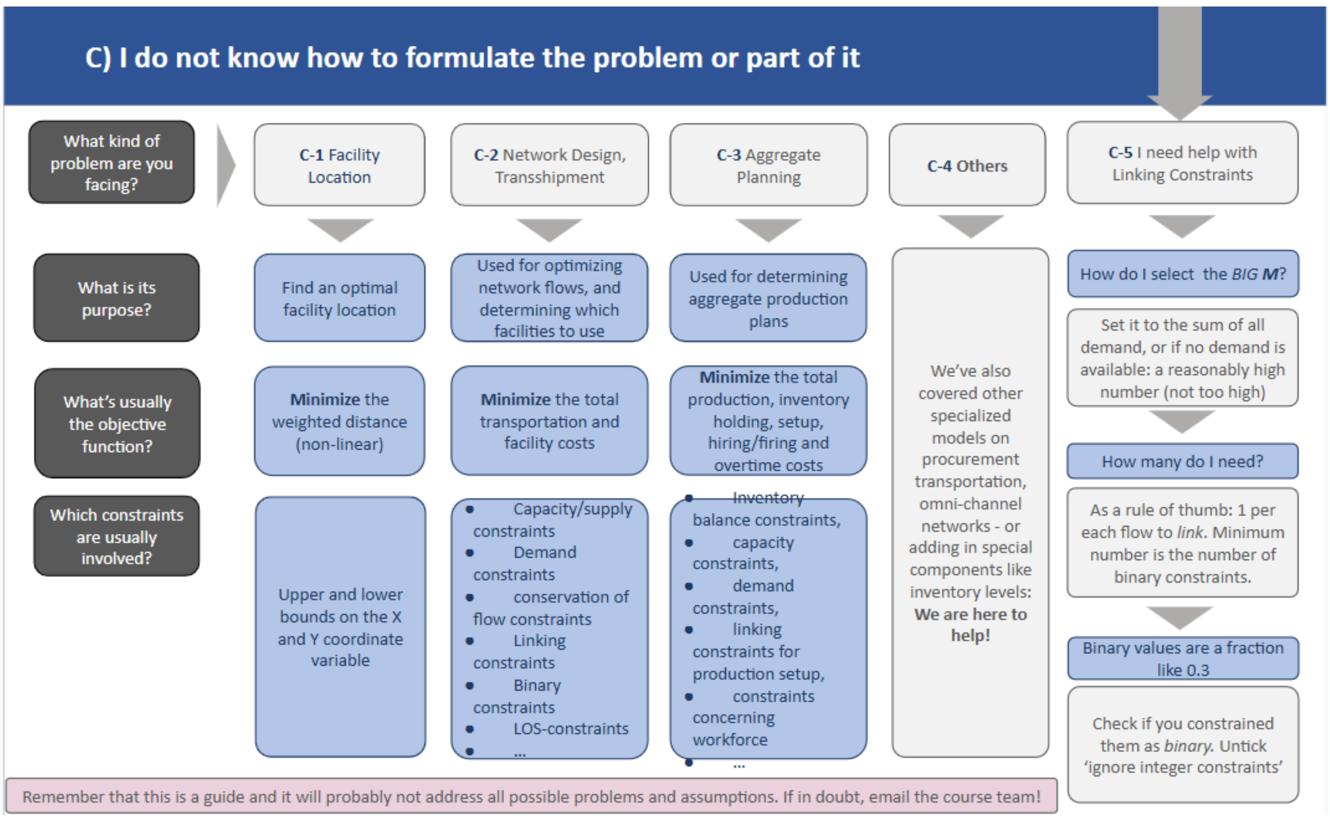

FIGURE 3.28: How To Debug Your Model Flowchart

Part IV

SC3x - Supply Chain Dynamics

4

Supply Chain Dynamics

CONTENTS

4.1 Complex Systems

4.1.1 Summary

Supply chain dynamics illustrates how reality can impact supply chains. We introduce the idea of complexity and we identify five main drivers of supply chain complexity. Supply chains are complex systems. A system is more than just the sum of its components, it involves the product of their interactions and the need for their alignment.

4.1.2 Complexity

Supply chains are complex, are only getting more complex, and complexity adds costs to the supply chain! Therefore, we need to mitigate or minimize complexity. First we need to understand what complexity is!

Two types of complexity (Senge, 1990):

- Detail complexity: Distinct number of processes or parts within the system
- Dynamic complexity: Unpredictability of response of the system due to interactions

Product complexity	**Customer complexity**
Small batch sizes	Customized products
Long set-up times	Short lead times
Unique components	Unpredictable orders
Special tests/inspections	Extensive technical support
Extensive material handling	Extensive post-sales support
Special vendors	Special tests or requirements

TABLE 4.1: Source of Complexity

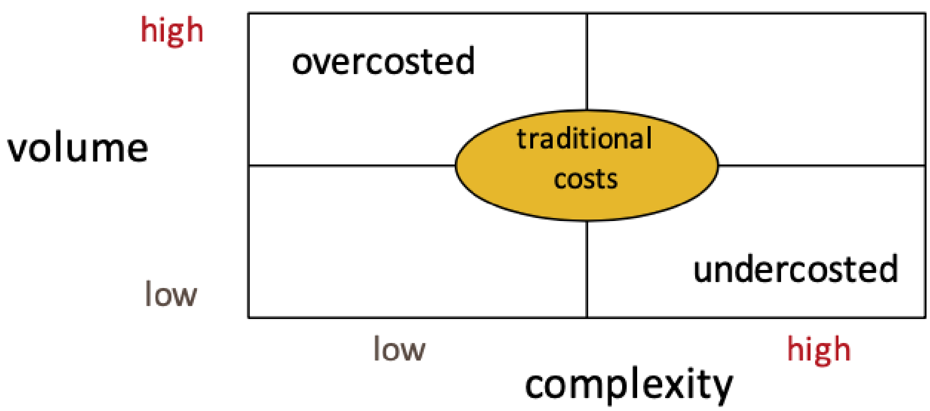

FIGURE 4.1: Traditional costs estimation underestimated additional complexity

According to Herbet Simon (1962): "A system is complex if it is made up of a large number of parts that interact in a non-simple way".

Drivers of Complexity

Drivers of SC complexity (complexity increases according to these drivers):

- Numerousness - number of suppliers, products, customers. . .
- Variety/diversity of the different entities or components
- Interconnections/interactions between those entities
- Opacity of interactions
- Dynamic effects

Why do we care what the divers of complexity are? Drivers of complexity = the drivers of profitability.

Drivers of profitability:

- Increase revenue per unit
- Increase the number of customers
- Increase number of units sold
- Decrease cost per unit

Source of Complexity

Most complexity enters from the ends!

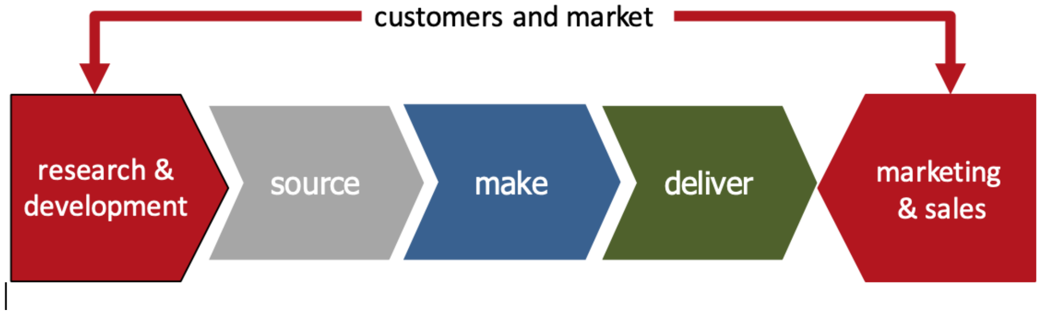

FIGURE 4.2: Complexity enters the SC through new products or through marketing and sales

Start by mapping the complexity fingerprint: identifying where complexity lies by identifying the potential complexity drivers and count total number used.

Any initiative to reduce complexity in the SC needs to involve people from marketing and sales as well as from new product development under the R&D process.

Complexity-adjusted margin

$$\text{Complexity ROI} = \frac{(\text{Incremental Margin - Variable Complexity Cost})}{(\text{Fixed Complexity Cost})}$$

Variable Complexity Cost

Low volume of a SKU drives costs

- Volume discounts for procurement
- Excess costs (obsolescence, storage, etc.)
- Shortage costs (expedite, lost sales, etc.)

Fixed Complexity Cost

High SKU variety drives costs

- Resource costs (R&D, testing, etc.)
- External cash outlays (tooling, etc.)
- Indirect impacts (manufacturing switching, returns, etc.)

Learning Objectives

- Learn what complexity is and how to recognize it
- Understand how to manage complexity in supply chains
- Identify how to assess complexity and mitigation options
- Help understand the dynamics within a system

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4.1.3 Bullwhip Effect

The Bullwhip Effect is a phenomenon where upstream variability of demand is greater than downstream variability. This can occur for many reasons including order batching, demand forecasting updates, rationing and shortage gaming, and price fluctuations. The concept was pioneered initially by Jay Forrester and was observed in practice in the supply chain by P&G in its disposable diaper line. The effect in supply chains was first described and quantified by Lee, Padmanabhan, and Whang in 1997.

The Bullwhip Effect is essentially a signal that a supply chain is not coordinated well. The costs of this lack of coordination includes:

- Increased manufacturing costs
- Higher inventory levels & costs
- Longer replenishment lead times
- Higher transportation costs
- Lower product availability
- Deteriorates trading partner relationships
- Lowers supply chain profitability

This lesson discusses approaches to counteracting the Bullwhip Effect including improving forecasting methodology, designing single-stage replenishment control, shortening lead and review period times, reducing batching of orders, reducing the incentive of forward buying, and sharing information more effectively.

The Bullwhip Effect

The Bullwhip Effect is the situation where the variability of demand in the supply chain increases as one moves upstream from the consumer to suppliers. It is primarily caused by four factors and is exacerbated by the observation that supply chains consist of multiple independent firms and that individual firms will tend to operate in order to maximize their own profits. These are the four factors that cause the bullwhip effect:

- Demand Forecasting – where forecasting relies on the demand each firm sees from its immediate downstream partner or customer and not the end downstream demand.
- Rationing and Shortage Gaming – where suppliers ration supply and customers, knowing this, inflate orders or submit phantom orders however, orders evaporate when supply is made available. The net effect is false demand signals that ripple and are amplified upstream.
- Order Batching – where customers bunch or batch orders for many different reasons including:
  - Ordering set up costs
  - Optimal lot-sizing
  - Periodic review policies
- Price Fluctuations – where the retailer incentivizes behavior from its consumers by changing prices that in turn causes batching of orders. These include:
  - Volume discounts
  - Minimum order quantities
  - Limited transportation mode options
  - Forward buying

Quantifying the Bullwhip Effect

We can measure the rough impact of the Bullwhip Effect under certain conditions:

- One retailer selling one item replenished by one wholesaler DC
  - Daily demand at store is $\sim$ N(100, 10)
  - Daily review period with (R,S) inventory policy (order up to)
  - $S_t = \mu_t L + k\sigma t\sqrt{L}$
  - μ_t = expected daily demand estimated at time t
  - σ_t = standard deviation of daily demand estimated at time t
  - L = lead-time in days
  - k = Safety factor
- Forecasting uses simple moving average of the last M time periods which implies that μ and σ will change each period based on new forecast and impact the order up to level and safety stock.

  Then, we can see that:

$$\frac{Var[\text{Retail Orders}]}{Var[\text{Customer Demand}]}] \geq +1 + \frac{2L}{M} + \frac{2L^2}{M^2}$$

Counteracting the Bullwhip Effect

There are several methods or approaches to counteracting the Bullwhip Effect. These include:

- Improve forecasting methodology
  - Eliminate multiple forecasts that only use immediate partner order data
  - Employ point-of-sale or end consumer data, if possible
  - Avoid "nervous" forecasting techniques
- Design single-stage replenishment control
  - Have upstream partner manage its downstream partner's inventory
  - Referred to as Vendor Managed Inventory (VMI) or Continuous Replenishment Programs (CRP)
  - Bypass the downstream stages – consumer direct policies
- Shorten lead and review period times
  - More frequent review and faster delivery reduces impact
  - Less time for uncertainty to build
  - Incent orders to be better distributed over time
- Reduce batching of orders
  - Reduce the fixed cost of order set up and delivery (lower friction)
  - Shift from minimum order quantity (MOQ) of individual SKUs (or families) to minimum volume quantity of a wider assortment of products
  - Reduce transportation costs by using: milk-runs, multi-zone trucks (ambient, refrigerated, and frozen), 3PL solutions...
- Reduce the incentive of forward buying
  - Be selective on the use of price promotions
  - Analyze the true costs of a promotion using ABC accounting
  - Shift sales incentives from "Sell-To" to "Sell-Through"
  - Use supply chain risk and other contracts to coordinate sales
- Better sharing of information
  - Allowing visibility into POS or end customer demand
  - Sharing of plans and intentions – sometimes called Collaborative Planning, Forecasting, and Replenishment (CPFR)

4.1.4 System Dynamics

In this lesson we review the basic components of system dynamics, what it is, how to apply it, and how to use models to understand it. We go over feedback loops, causal loop diagrams, time lags & delays, and stock & flow diagrams. All approaches to visualize system complexity. We then move into a discussion on modeling the systems. Throughout the course of the lesson, we build a "toolbox" for system dynamics which includes causal loop diagrams, behavior over time charts, stock & flow diagrams, and models.

"A system is not the sum of its parts, it is the product of their interactions." Russell Ackoff

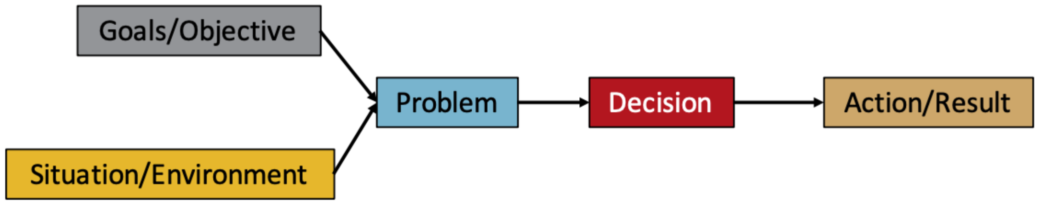


FIGURE 4.3: Event-oriented approach to solve problems

Event Oriented Thinking

Event-oriented thinking is problematic because:

- Assumes problem is an isolated event to be solved in isolation
- Linear thinking - does not consider feedback from others
- "pragmatic, action oriented, alluringly simple, and often myopic" - sometimes the solution is worse than the original problem!!!

Moving from linear to circular thinking

Move away from linear thinking to be able to incorporate our effects on others as well as the actions of others.

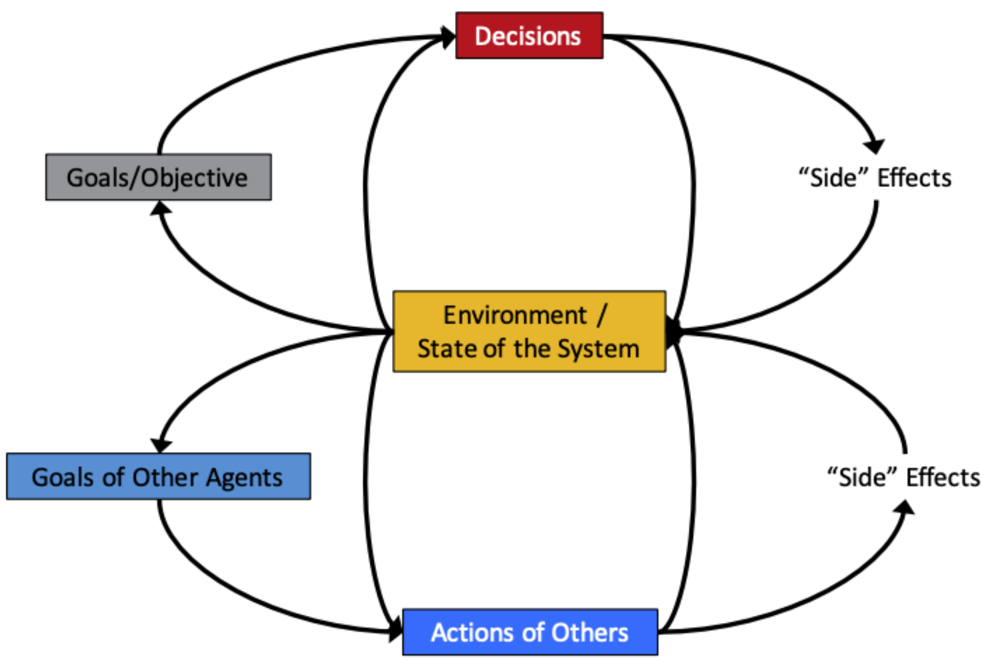


FIGURE 4.4: Circular thinking and "side" effects

Modeling Process

1. Articulate the Problem

2. Formulate the Dynamic Hypothesis
3. Formulate the Simulation Model
4. Test the Simulation Model
5. Design and Evaluate the Policy

Causal Links

- Causal link (arrows): Causal Links capture the relationship between two variables and must have either Positive (+) or Negative (-) polarity.
- Positive Polarity (+ or S): variables move or change in the **same direction**
  - An increase in one variable causes an increase in the other variable
  - A decrease in one variable causes a decrease in the other variable
- Negative link (-): variables move or change in the **opposite direction**
  - An increase in one variable causes a decrease in the other variable
  - A decrease in one variable causes an increase in the other variable

Feedback or Causal Loop Diagrams

A diagram consisting of variables connected by causal links representing relationships in a complex system. The type of loops include:

- Reinforcing loop: a collection of links that form a loop that provides positive feedback. Results in exponential growth (or decline) over time.
- Balancing loop: a collection of links that form a loop that provides negative feedback. Generally results in some sort of equilibrium or state of balance over time.

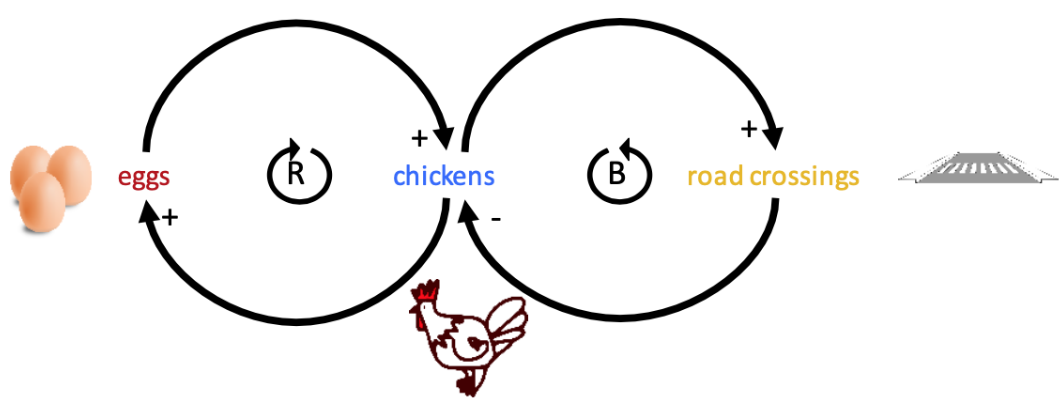


FIGURE 4.5: Causal Loop Diagram notation

Time Lags & Delays

The longer the delay the more "aggressive" the response and the longer to reach steady state. Delays between actions and consequences are everywhere. . . such as the bullwhip effect! An example of a time lag in the shower:

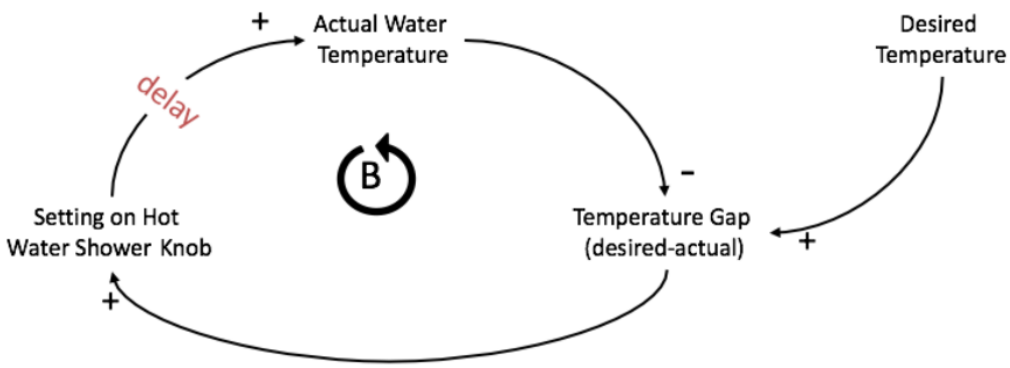


FIGURE 4.6: Example of a time lag and delay implemented using CLD

"Information transferred in the form of orders tends to be distorted and can misguide upstream members in their inventory and production decisions… the variance of orders may be larger than that of sales, and the distortion tends to increase as one moves upstream" Lee, Padmanabhan and Whang (1997)

Commonly Recognized Patterns in Supply Chains: Oscillation – fluctuation in orders increases as we move upstream Amplification – the size of the fluctuations increases as we move upstream Phase Lag – the impact is delayed longer as we move upstream

Stock and Flow Diagrams

Stocks	Flows
Define the "state" of the system	Define the rate of change of system states
Examples: • Balance sheet • Wealth • Water in a bath tub • Inventory in a DC • Integrals	Examples: • Cash flow statement • Income –Expenses • Flows in through faucet and out drain • Throughput (replenishment – shipments) • Derivatives

FIGURE 4.7: Stocks and Flows

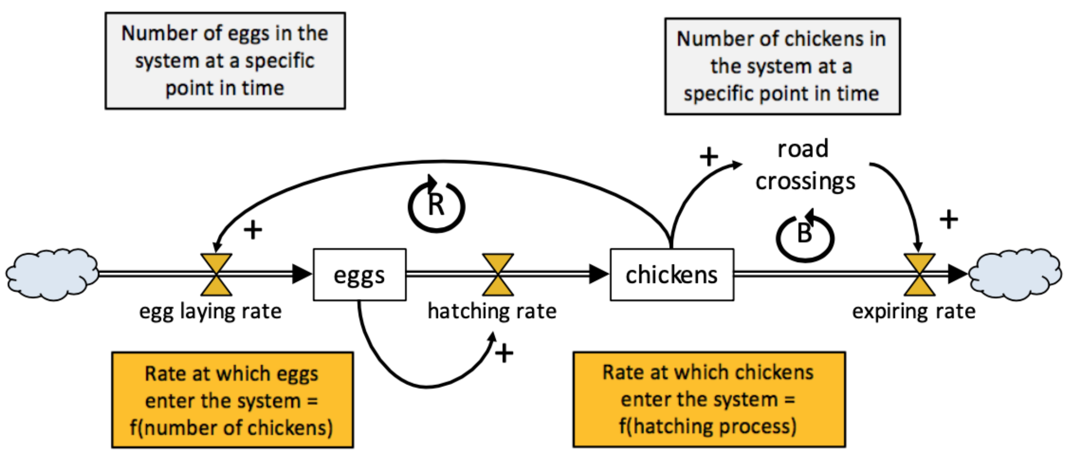
Stocks:

- Define the "state" of the system
- Stocks have memory
- Stocks change the time path of flows
- Stocks decouple flows
- Stocks create delays
- Examples:
  - Balance Sheet

  - Wealth
  - Water in a bath tub
  - Inventory in a DC
  - Integrals

  Flows:

- Define the rate of change system states
- Examples:
  - Cash Flow Statement
  - Income – Expenses
  - Flows in through faucet and out drain
  - Throughput (replenishment - shipments)
  - Derivatives



4.1.5 Modeling System Dynamics In Action

So far in this lesson we have reviewed Causal Loop Diagrams and Stock & Flow Diagrams. We will now add two more approaches to your system dynamics tool box including Behavior Over Time Charts and Models.

Stock and Flow Diagrams

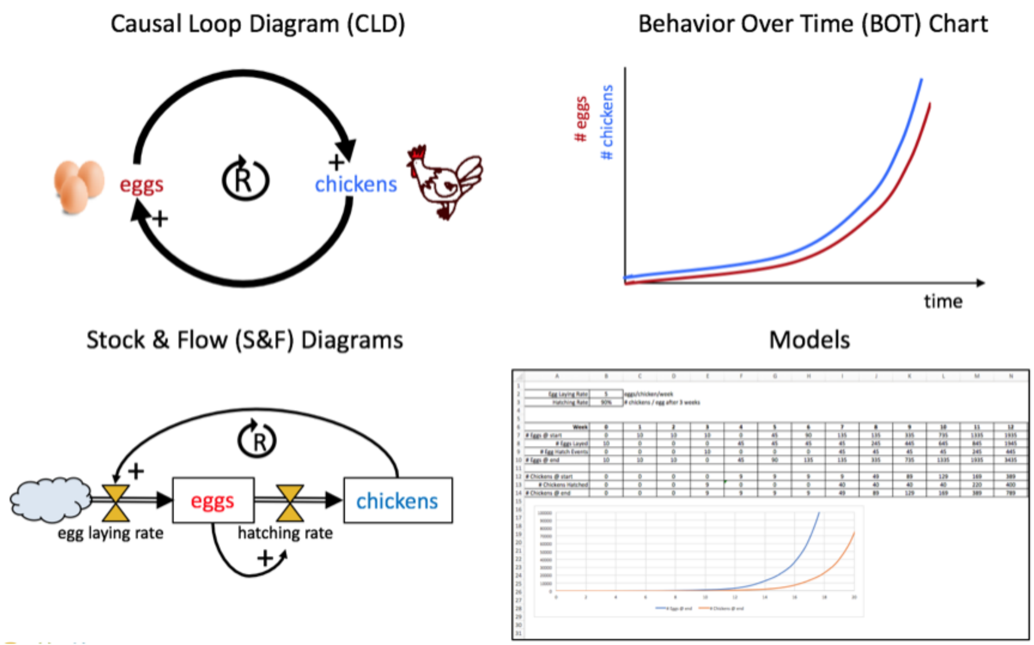


FIGURE 4.8: CLD, BOT and SF of a reinforcing loop

Behavior over time charts

- BOT charts arise from the relationships described in the CLD
- BOTs track a primary attribute of the system (# customers, profit, sales, # eggs, etc.)
- A BOT chart is NOT a point forecast, instead the critical aspect is its shape

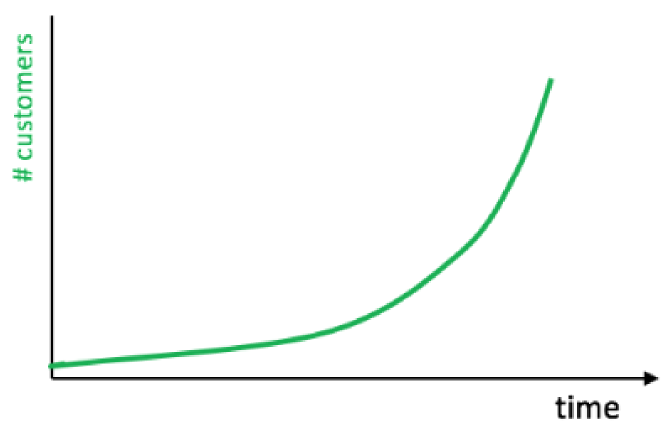


FIGURE 4.9: Behavior over time chart

Examining the behavior over time can prevent unintended consequences for the future. For instance, focusing myopically can lead to a poor outcome. BOT charts inform this progression over time to assess this outcome. See example below regarding high raw materials costs. The BOT chart was able to visualize the unintended consequence of a shortsighted fix.

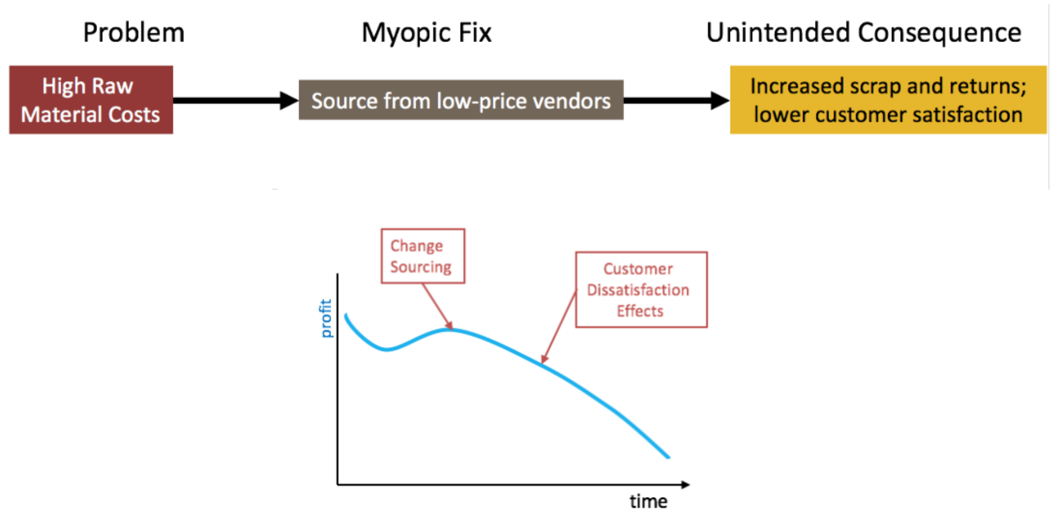


FIGURE 4.10: Preventing unintended consequences using BOT chart

Converting Stock & Flow Diagrams into a Model

In the example discussed in the lecture, of egg laying and hatching, we want to simulate the behavior of the system:

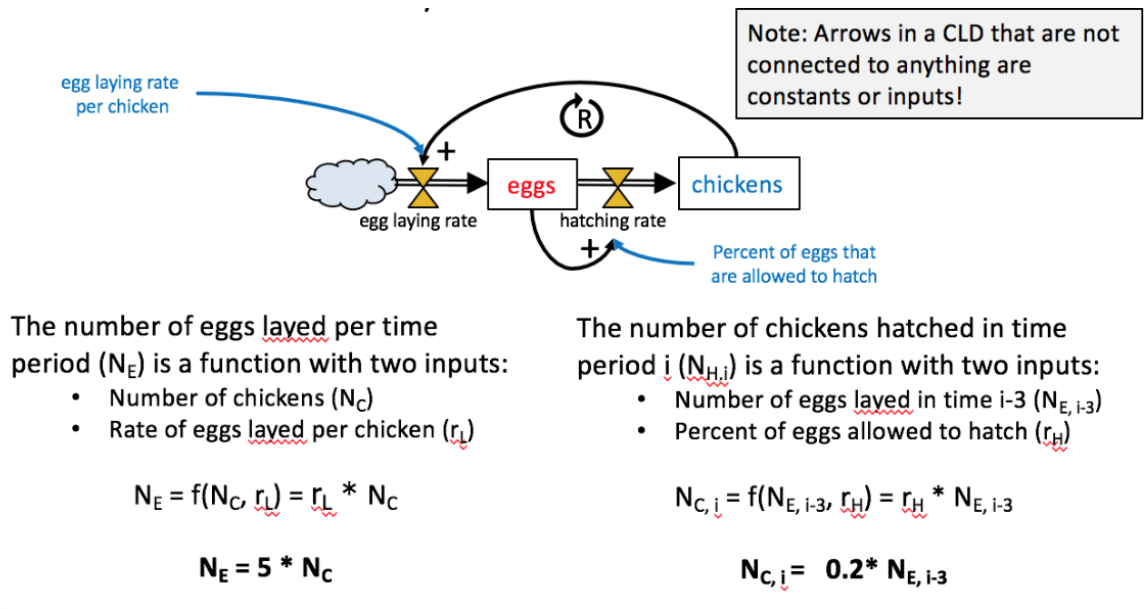
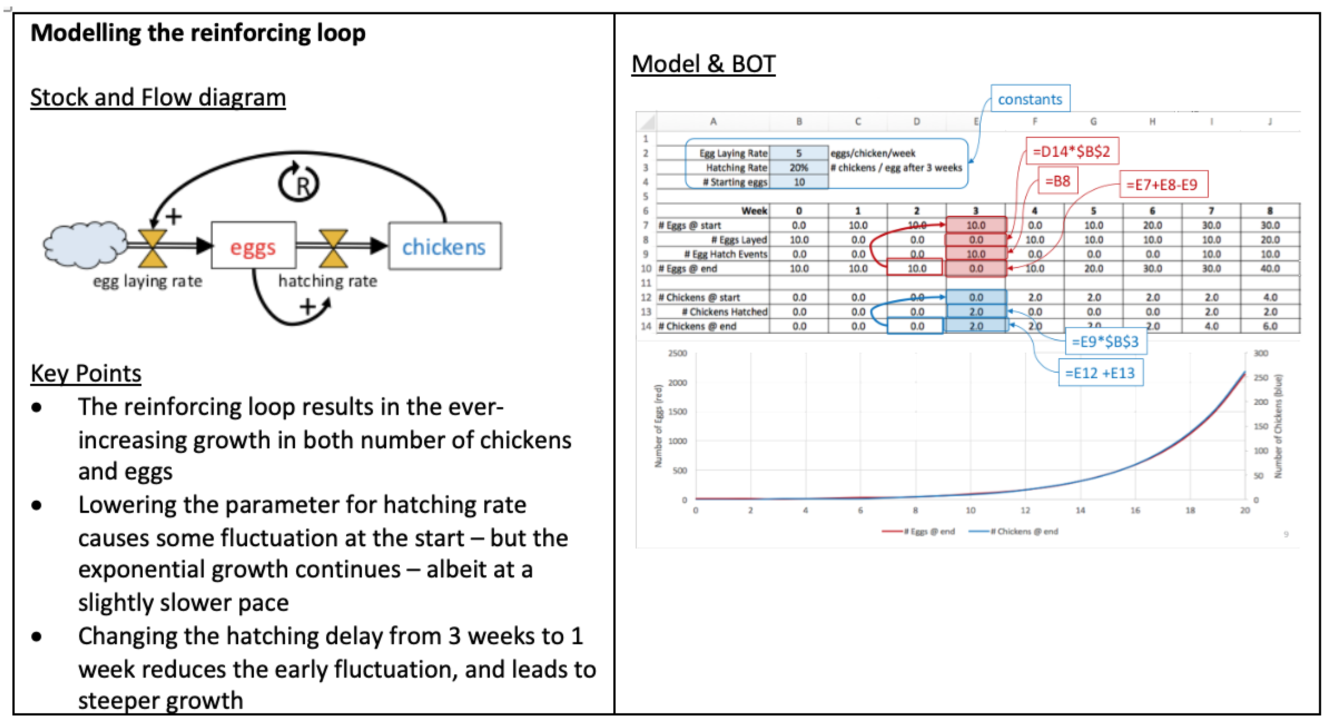


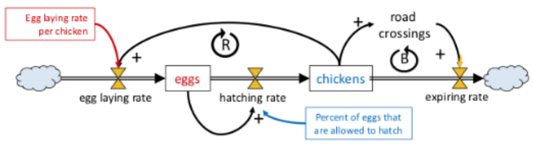
FIGURE 4.11: Preventing unintended consequences using BOT chart

Simulation Model Example



Modelling the reinforcing & the balancing loop

Stock and Flow diagram



Key Points

- There is oscillation between chickens and eggs over time
- The amplitude of peaks and valleys decrease over time
- The overall pattern can be virtuous, vicious, or stable – depending on interplay of parameters!
- The delay exacerbates this oscillation

Model & BOT

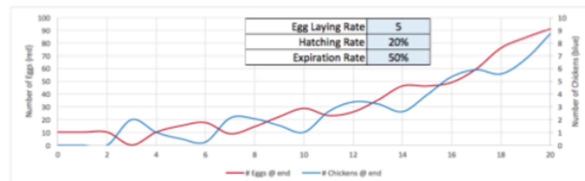
	A	B	C	D	E	F	G	H	I	J
1										
2	Egg Laying Rate	5	eggs/chicken/week							
3	Hatching Rate	20%	# chickens / egg after 3 weeks							
4	Expiration Rate	50%	Percent chickens run over in road							
5	# Starting eggs	10								
6										
7	Week	0	1	2	3	4	5	6	7	8
8	# Eggs @ start	0.0	10.0	10.0	10.0	0.0	10.0	[illegible]	[illegible]	8.8
9	# Eggs Layed	10.0	0.0	0.0	0.0	10.0	5.0	[illegible]	[illegible]	10.6
10	# Egg Hatch Events	0.0	0.0	0.0	10.0	0.0	0.0	0.0	10.0	5.0
11	# Eggs @ end	10.0	10.0	10.0	0.0	10.0	15.0	17.5	8.8	14.4
12										
13	# Chickens @ start	0.0	0.0	0.0	0.0	2.0	1.0	0.5	0.3	2.1
14	# Chickens Hatched	0.0	0.0	0.0	2.0	0.0	0.0	0.0	2.0	1.0
15	# Chickens Run Over	0.0	0.0	0.0	0.0	1.0	0.5	0.3	0.1	1.1
16	# Chickens @ end	0.0	0.0	0.0	2.0	1.0	0.5	0.3	[illegible]	2.1

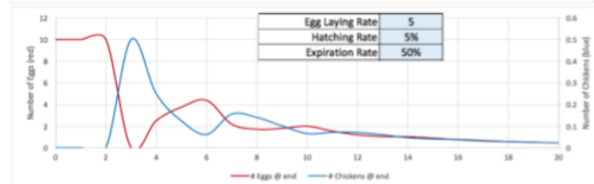
constants

=F10*$B$3

=F13*$B$4

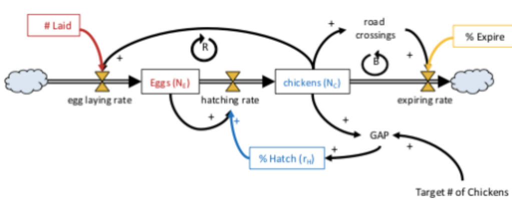
=F13+F14-F15





Behavior with a Management Goal

Stock and Flow diagram



Key Points

- Having a goal or target enables a system to be better controlled
- The magnitude of the response and "tightness" of the control bounds dictate the
- volatility of the behavior
- Making decisions on myopic conditions can lead to volatility
- Tracking what is potentially in the pipeline can lead to a more stable outcome
- Parallels to supply chain and inventory management – track the inventory position, not just what is on hand!

Model & BOT

Current: $r_{H,i} = f(N_{C,i})$

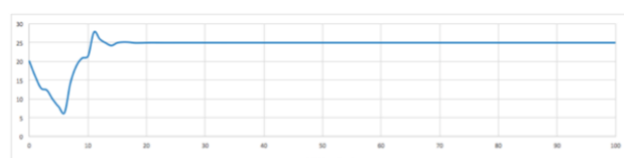
If N_C	Then r_H
≤ 22	20%
$22 < N_C \le 28$	5%
>28	0%

Proposed: look at what is "in the pipeline" in terms of the expected number of eggs ready to hatch and the expected number of surviving chickens.

$r_{H,i}$ = f(Expected # Eggs to Hatch, Expected # 'live' Chickens) = Average($N_{HE,i-2}$, $N_{HE,i-1}$, $N_{HE,i}$)+N_C*(1 – r_E)

	A	B	C	D	E	F	G	H	I	J
1							Criteria (Nc)	r(H)		
2	Egg Laying Rate	5	eggs/chicken/week				0	20%		
3	Hatching Rate	XX	# chickens / egg after 3 weeks				20	15%		
4	Expiration Rate	20%	Percent chickens run over in road				75	4%		
5	Chicken Goal	25.00	Targ							
6	# Starting eggs	10								
7										
8	Week	0	1	2	3	4	5	6	7	8
9	# Eggs @ start	0.0	10.0	10.0	10.0	0.0	61.2	110.2	149.3	119.5
10	# Eggs Layed	10.0	0.0	0.0	0.0	61.2	49.0	39.2	31.3	71.0
11	# Egg Hatch Events	0.0	0.0	0.0	10.0	0.0	0.0	0.0	61.2	49.0
12	# Eggs @ end	10.0	10.0	10.0	0.0	61.2	110.2	149.3	119.5	141.5
13										
14	# Chickens @ start	25.0	20.0	16.0	12.8	12.2	9.8	7.8	6.3	14.2
15	Hatch Rate f(GAP)		15%	20%	20%	20%	20%	20%	15%	15%
16	# Chickens Hatched	0.0	0.0	0.0	2.0	0.0	0.0	0.0	9.2	7.3
17	# Chickens Run Over	5.0	4.0	3.2	2.6	2.4	2.0	1.6	1.3	2.8
18	# Chickens @ end	20.0	16.0	12.8	12.2	9.8	7.8	6.3	14.2	18.7

=VLOOKUP(AVERAGE(C11:E11)+E14*(1-$B$4),$G$2:$H$4,2)



Modelling a Shared Resource

System Dynamics can model systems with a shared resource. These models can be used to determine the best allocation strategy for the shared resource. Such Systems are common in supply chain systems. An example will be to allocate a shared workforce (also known as Flex Team) among Fulfillment center and a Transport operation for a retailer.

While modelling a shared resource, we need to take in to consideration the following three aspects:

- **Performance of a function:** i.e. How does performance improve as the management assign the Flex Team to the Fulfillment Center or Transportation?
  - The performance degrades over time if no additional workforce is allocated. Adding a workforce proportionally increases the performance until we reach a saturation point. Beyond this point, increase in workforce doesn't lead to additional increase in performance.
- **Management response:** i.e. How does management respond to poor performance in the Fulfillment Center or Transportation operations?
  - A management response must be modeled by taking into consideration the following three parameters:
    * Timing i.e. if the KPI drops to a threshold, when should the management pay attention?
    * Magnitude i.e. how much should the management respond?
    * Symmetry i.e. should the management respond the same way if the performances are eroding, trending down, or if the performance is trending up?
- **Sharing rule:** i.e. What is the sharing rule (proportional or dominance)?
  - In a proportional rule, the departments get allocation proportionally to what they have requested while in dominance, a certain department get most of what they have requested for and the remaining is allocated to the other department(s).

The picture below shows the casual loop diagram, behavior over time and the stock and flow diagram of a system with shared resource:

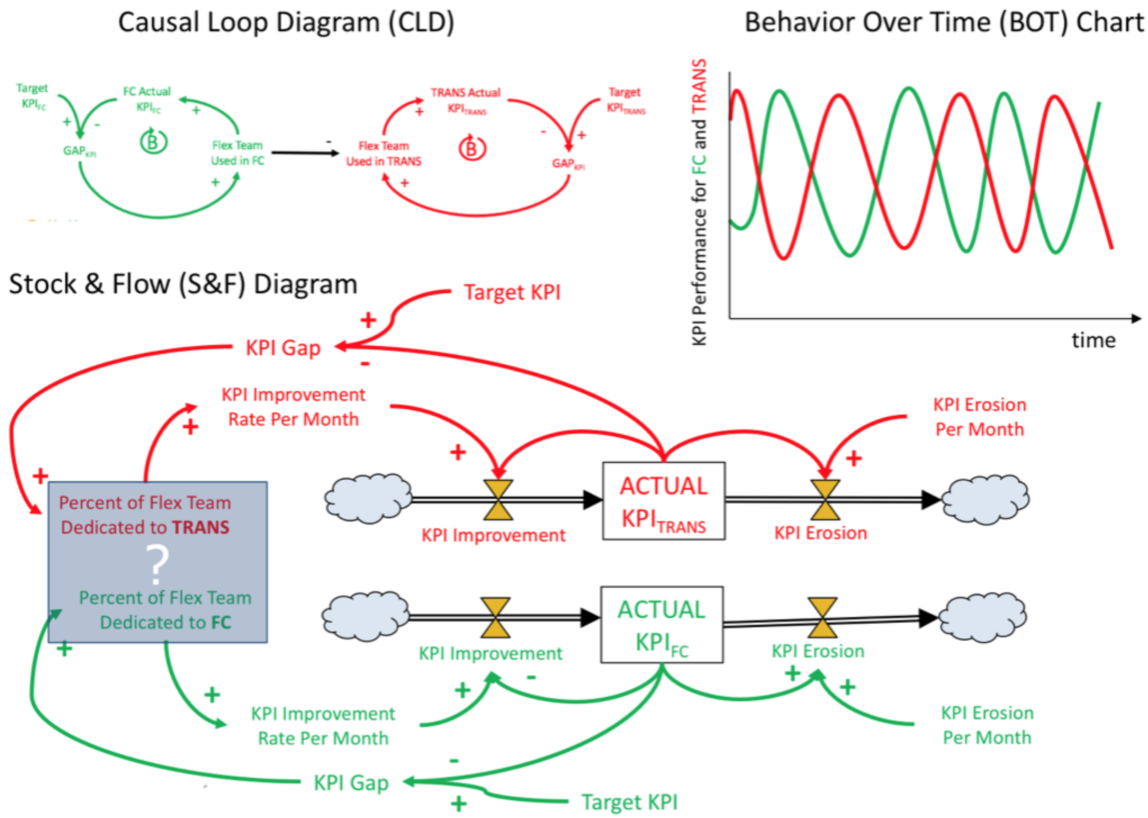
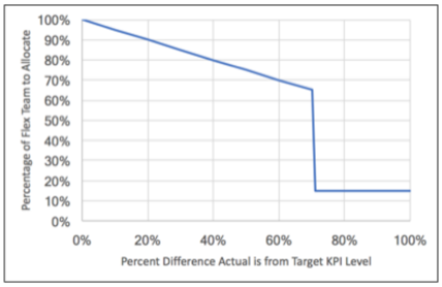
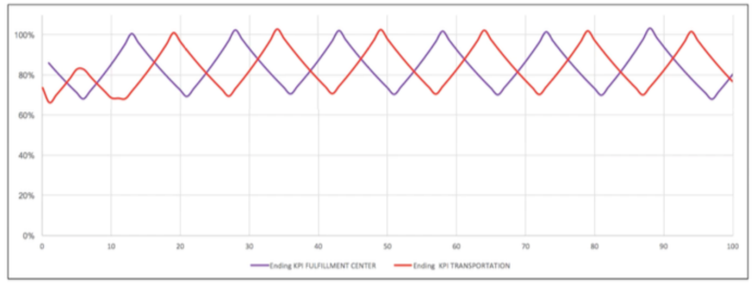


FIGURE 4.12: CLD, BPT and S&F for a system with a shared resource

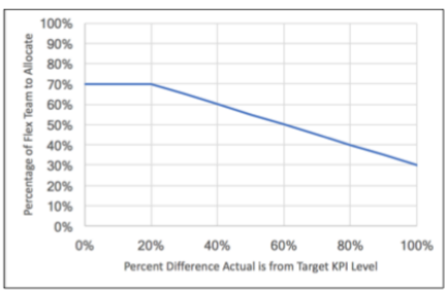
The below table shows two management response options and the corresponding outcome:

<table>
<tr><th>Management Response</th><th>Outcome of the model</th></tr>
<tr><td>


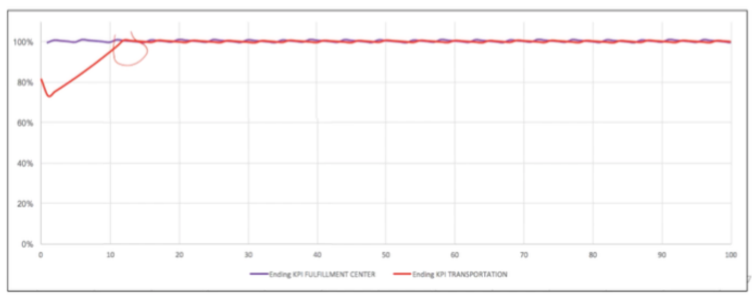
Minimal allocation when KPI is ”OK”
Swift and drastic action when KPI falls below threshold
Effort increases significantly in proportion to the KPI level

</td><td>


Severe oscillation between KPI target and dropping below threshold
Alternating resource between two competing needs: Transportation & Fulfillment Center
Management response is sudden and dramatic – goes from ignoring to full attention

</td></tr>
<tr><td>


Always keep above a “sustaining” allocation even at high KPI levels
Gradual escalation immediately when KPI starts degrading
Peak effort is capped to prevent “robbing” from other functions

</td><td>


Firefighting rarely fixes the underlying problem.
Comparing measured versus dramatic management responses

Immediate shifting from “darkness to spot light” introduces volatility
It is better to allocate a proportion of shared resource for flexibility – not all of it.
Avoid cycles of over and under investing/supporting.
Measure consistently and act earlier – do not wait for crisis

</td></tr>
</table>

Learning Objectives

- Introduction to the concepts and tools of system dynamics
- Review Feedback Loops and their role system dynamics
- Recognize Time Lags and Delays
- Learn Tools to Capture and Model Systems, Causal Loop Diagrams, and Stock and Flow Diagrams

- Review the tools of system dynamics including behavior over time charts and modeling
- Learn how to model system dynamics stocks and flows including management practices

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- Morecroft, John, (2015), Strategic Modelling and Business Dynamics: A Feedback Systems Approach, Wiley.
- Senge, Peter, (2006), The Fifth Discipline, Doubleday. http://www.systemdynamics.org/
- Don Woodlock's videos on System Dynamics https://www.youtube.com/playlist?list=PL7490F4FA4B45DA26

4.2 Supply Chain Strategy

4.2.1 Summary

Strategy is key to an effective supply chain. To understand this, we review the most common business strategy frameworks and methodologies. We provide some examples that help to understand how supply chains need to align to the selected company's strategy in order to enable it. There is not a single best supply chain for every firm. Typically, each company should develop and utilize a portfolio of different supply chains.

We also introduce a novel way of thinking about supply chain strategy. It demonstrates a way of tackling the problem of supply chain strategizing. Our primary resource for this lesson is the material developed by the MIT CTL Supply Chain Strategy Lab led by Dr. Roberto Perez Franco. We discuss the nature of the problem that supply chain strategizing presents to supply chain practitioners and present ten supply chain strategy evaluation criteria.

4.2.2 Key Concepts

Strategy is an "art", not exactly a science, which means focusing on the big picture and overall results of an effort. Here are some definitions:

According to the Oxford English Dictionary: "The art of a commander-in-chief; the art of projecting and directing the larger military movements and operations of a campaign." Perspectives and challenges of the general and soldier are very different.

According to Merriam-Webster: "The science and art of military command exercised to meet the enemy in combat under advantageous conditions."

4.2.3 Supply chain strategy

The importance of strategy for supply chain management has been recognized for a long time. Shapiro and Heskett (1985) highlighted that "logistics' most important role is strategic". They recommended that logistics managers apply a variety of perspectives while running their logistics systems:

- **Internal perspective:** understand structure, economics and requirements of the logistics systems as well as constraints and components
- **Inter functional perspective:** interact constantly with other functional managers in areas such as marketing, production, and finance

- **Channel perspective:** think in terms of maximizing the total channel benefit, considering that the firm's decisions affect and is affected by channel partners
- **Strategic perspective:** well-designed logistics management can influence company's strategy and has the potential to "advance a company's strategic goals"

A dichotomy: The two faces of logistics (supply chain management)

Successful supply chain management requires attention to detail, to day-to-day control and coordination, and to the tactical and analytic. However, managers should also be able to see "the big picture," and be cognizant of the broad, qualitative, long-term aspects of supply chain management. This awareness will ensure that the firm's supply chain function, combined with other functional areas, can further the overall objectives of the organization.

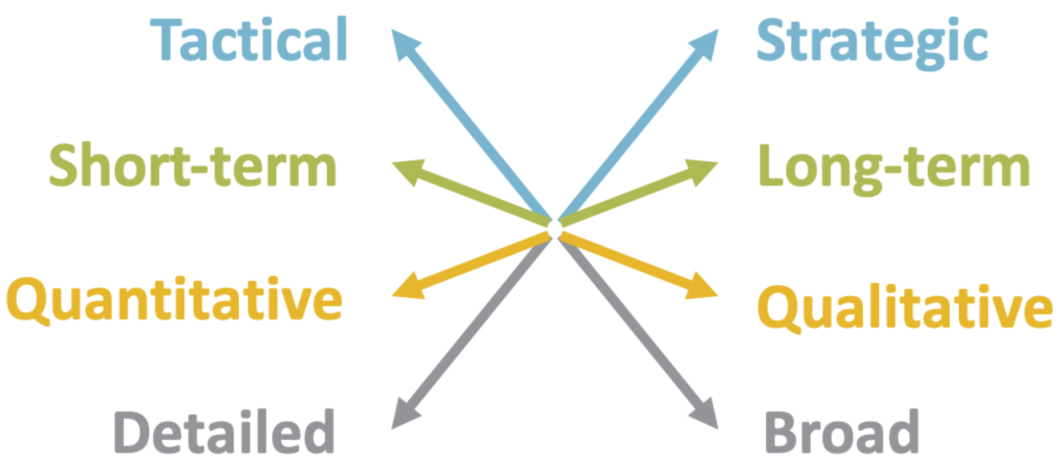

FIGURE 4.13: "A difficult, yet fundamental dichotomy"
Based on Shapiro and Heskett (1985)

Shapiro and Heskett (1985) point out that this "ever-present tension between the strategic and the tactical, the broad and the detailed", "all conspire to make the job of a supply chain manager a challenging one."

Two strategic roles for the supply chain

- **As input to formulate new overall strategy:**
Supply chain must be an input when formulating a new overall strategy. Organizations usually have – and should have – an *overall strategy* that gives them direction and helps them succeed. This overall strategy is called by many names, depending on the type of organization we are referring to. In the case of organizations that compete against others, this is often called their *competitive strategy*. In the case of business units, it is often called their *business strategy*. But in all cases, the overall strategy seeks to provide the organization with a path or direction to success.
- **As enabler of an existing overall strategy:**
Implementing an overall strategy means translating objectives into decisions, goals into action. An overall strategy that provides a set of high-level strategic objectives has to be elaborated into more specific objectives, policies and choices across a wide range of areas of activity. This may include many diverse functions that are relevant to the supply chain, such as purchasing, logistics, operations and sales. The means that the strategy executed has to be in line with the principles and values the

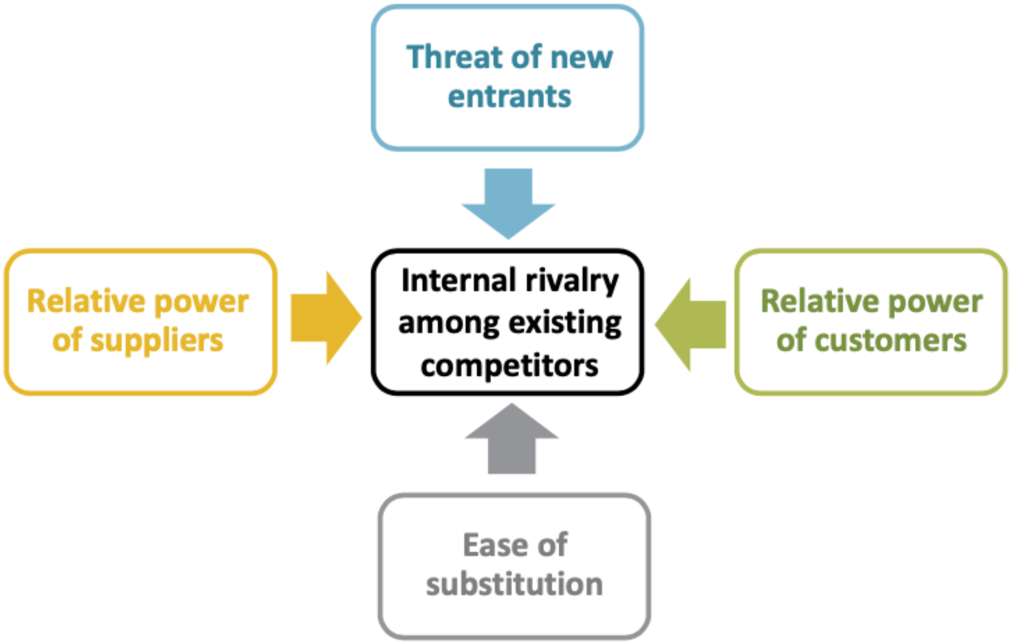

FIGURE 4.14: Porter's Five Forces

organization embraces, across a series of areas of interest such as quality, sustainability, service level, safety, etc. There is a gap between the overall strategy of an organization and the execution of this strategy. Supply chain strategy helps to bridge this gap.

Strategic tools

Porter's Five Forces: Porter argues that a company's ability to increase profit is affected not only by rivalry of immediate competitors but also by four other forces, which determine the intensity of competition and the attractiveness of an industry. These five forces are:

1. Internal rivalry among existing competitors, which is influenced by different factors such as number of existing competitors, rate of market growth, balance between supply and demand, among others.
2. Threat of new entrants, which refers to how difficult it is to become a competitor in a given market and is influenced by access to specific technology, brand credibility and recognition, laws and regulations, etc.
3. Ease of substitution, which depends on alternative products or services available that customers could choose to buy instead.
4. Relative power of suppliers, which is influenced by the number of suppliers available, availability of substitutes for the material or component you are procuring, number of customers the supplier has, cost of switching to another supplier, among others.
5. Relative power of customers, which is determined by the quantity bought by each customer, availability of similar products from competitors, cost of switching from one seller to another, amount of information that is available to customers, customers' profit margin, etc.

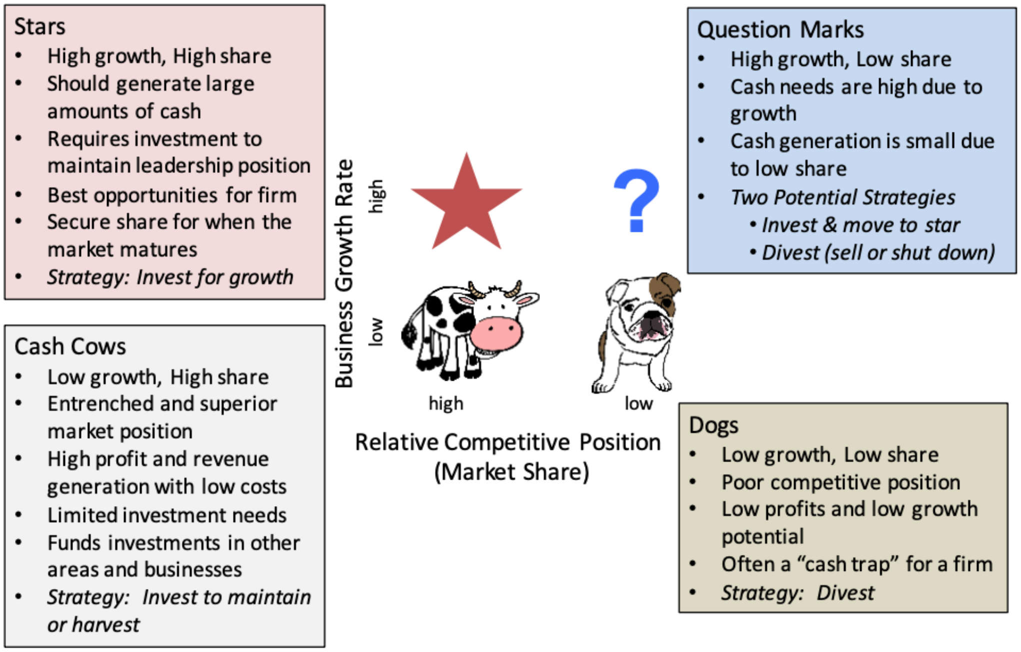

FIGURE 4.15: Growth share matrix
Source: adapted from Hedley (1977)

Growth-Share Matrix

The Growth-Share Matrix is a tool that was developed by the Boston Consulting Group in the 70's, used to manage a portfolio of businesses within a firm. It is important to understand the categorizations for each business line because the supply chain required for each category will be very different. According to the market share and the business potential growth rate, we can classify the lines of businesses in a 2x2 matrix. The four types of businesses are:

- **Stars:** are lines of business with high growth rate and high market share, which generate a lot of cash and require investments to maintain the market share.
- **Cash Cows:** are lines of business with high market share but low growth rate. Like the stars, cash cows generate a lot of cash but, because of the low growth rate, investments do not make sense.
- **Dogs:** are lines of business with low market share and low growth rate, which do not generate any profit; the strategy for these is to divest.
- **Question Marks:** are lines of business with high growth rate and low market share, there are two potential strategies for these lines of businesses: invest to increase market share or divest and reallocate your resources.

SWOT Analysis

The SWOT analysis, developed by Kenneth Andrews, is used to categorize internal and external factors that influence strategic decisions. SWOT is an easy way to have a structured brainstorming session with a team to better understand the lay of the land. SWOT stands for Strengths, Weaknesses, Opportunities and Threats.

FIGURE 4.16: SWOT Analysis

Strategic Fit

Porter introduces the concept of "fit", which means that activities are consistent with each other and reinforce each other to create a competitive advantage and superior profitability.

Porter identifies three types of "fit", which "are not mutually exclusive":

- First order fit ensures that "competitive advantage of activities cumulate and do not erode"
- Second Order fit "occurs when activities are reinforcing"
- Third order fit is "optimization effort"

What is Strategy?

According to Porter's famous paper:

- Strategy is creating fit among a company's activities
- Successful strategy is a result of doing many things well, and integrating them
- If there's no fit among activities, there's no distinctive strategy and the strategy will be unsustainable

Supply Chain Strategy

According to Narasimhan et al. (2008): "supply chain strategy can be viewed as the pattern of decisions related to sourcing products, capacity planning, conversion of raw materials, demand management, communication across the supply chain, and delivery of products and services."

According to Cigloni et al. (2004): "what companies actually did, rather than what they claimed their strategic intent to be, is the best clue to reveal their very supply chain management strategies."

Match and Mismatch in Supply Chain Strategies

Marshall Fisher introduced the concept of match and mismatch of supply chain strategies in 1997. Figure 3-5 includes his 2x2 matrix.

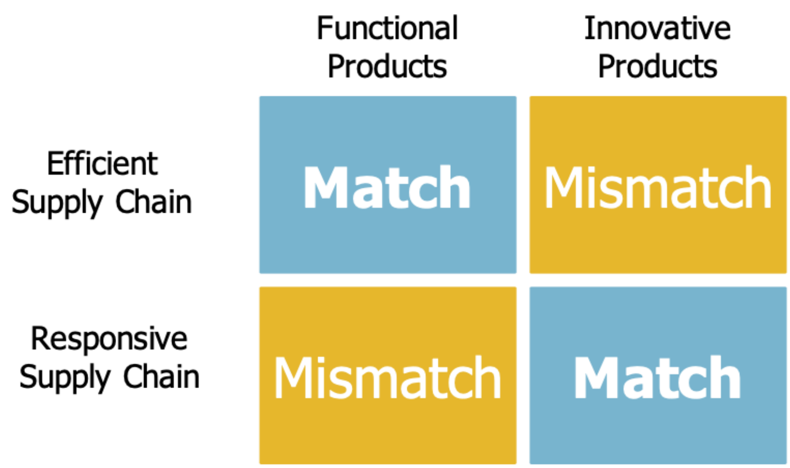

FIGURE 4.17: Fisher's 2*2 matrix
Source: Fisher, 1997

Lee's Matched Strategies

Lee (2002) uses the uncertainty framework as a way to classify risks. Figure 4-18 presents this 2x2 matrix.

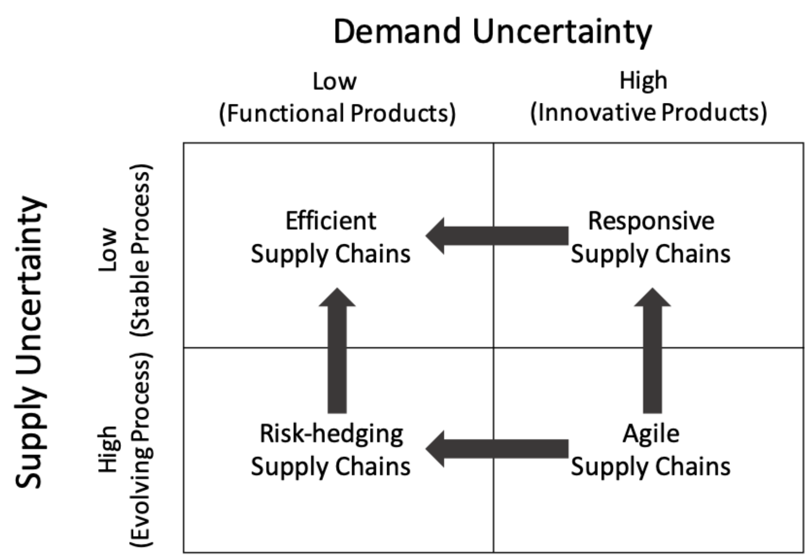

FIGURE 4.18: Lee's matched strategies
Source: Lee, 2002

1	Coverage
2	Clarity
3	Feasibility
4	Internal Consistency
5	Support
6	Sufficiency
7	External Consistency
8	Advantageousness
9	Parsimony
10	Riskiness

TABLE 4.2: Supply chain strategy evaluation criteria

4.2.4 Rethinking your Supply Chain Strategy

The Basic Challenges

Supply chain strategy presents a set of interrelated challenges:

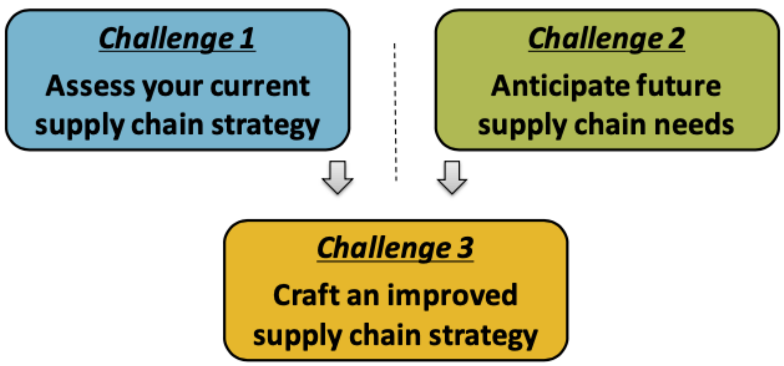

FIGURE 4.19: The basic challenges

Ten supply chain strategy evaluation criteria

The first challenge is to assess the current supply chain strategy, which means understanding what you have in place today. To understand the strategy, you must map the current supply chain strategy and evaluate it. To evaluate whether the current supply chain strategy is good enough there are 10 evaluation criteria.

First criterion is **coverage**. A good supply chain strategy must be comprehensive and cover every area that matters for a company's supply chain. It must have **clarity** – policies and choices in a supply chain strategy must be unambiguous and understood by all of the decision makers. It must be **feasible**, realistic and able to be achieved. **Internal consistency** is key to supply chain strategy with different objectives, policies and choices and can be thought in three different levels: *compatibility, coherence*, and *synergy*.

Each component of a good supply chain strategy must **support**, enable, advance or help realize some element of the overall organization's strategy. Supply chain strategy must also have: **sufficiency, external consistency, advantageousness, parsimony**, and **riskiness**.

Evaluation criteria can be summarized as: "a good supply chain strategy should be comprehensive, clear, feasible, consistent both internally and externally, sufficiently supportive, competitively advantageous, not wasteful, and not riskier than is acceptable to the organization."

Scenario Planning

The second challenge is anticipating future supply chain needs, which in a short-term timeline are identified using forecasting techniques. However, for a longer timeline we use the scenario planning technique. Scenario planning is a method that shifts from predicting events to preparing for multiple potential futures. It is defined as: "a structured brainstorming methodology that immerses decision makers of an organization in different potential future scenarios in order to better understand potential risks, blind spots, opportunities, and future needs."

Creating scenarios

A scenario is essentially a story about a future based on a set of driving forces. They have high impact and high uncertainty for your selected time frame. Some criteria for creating a good set of scenarios are:

- Avoid the preferred and/or probable future
- Capture the right decision
- Plausible, within realistic limits
- Include real alternatives
- Consistent
- Different
- Memorable names
- Challenge the status quo

In addition to creating scenarios, we need to translate events into effects and apply the scenarios.

Five categories of effects for supply chains:

- Impact on sourcing patterns
- Impact on flow destination
- Impact on routing
- Impact on flow volume
- Impact on value density

Applying scenarios

1. Immerse a large group of people in each scenario
2. Bring everyone together and evaluate, compare and contrast the solutions/strategies proposed across the different scenarios
3. Eliminate the no gainer strategies
4. Monitor with sensors in the ground whether to apply strategies that are only applicable to some scenarios

Progressive formulation

The third challenge is to craft an improved supply chain strategy. These are the steps to perform a progressive formulation:

- Identify a starting point: a revised overall strategy
- Identify areas of decision or activity and interest, add or drop areas accordingly
- Identify sequence of events that you want to follow
- Use evaluation criteria on each area of decision
- Repeat this process in each level of abstraction

Learning Objectives

- Learn basic concepts of business and supply chain strategy
- Identify the most common business strategy frameworks
- Understand that there is not a single best supply chain for every firm
- Outline a process to develop and improve a supply chain strategy
- Learn how to tackle the problem of supply chain "strategizing"
- Identify the main criteria for evaluating a supply chain strategy

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FIGURE 4.20: Supply chain process variability

4.3 Process Analysis & Applications in Practice

4.3.1 Process Analysis in Practice

Process analysis is an important perspective in supply chain management. According to Wallace Hopp (2011), "a supply chain is a **goal-oriented network** of **processes** and **stock points** used to deliver goods and services to customers."

Impact of variability

Variability within a process occurs due to either arrival and/or process variability.

Sources of input or arrival variability include:

- Scheduling
- Transportation delays
- Quality issues
- Upstream processing
- Random demand

Sources of process variability include:

- Variety of items
- Operator speed
- Failures
- Set ups
- Quality problems

Supply chain process variability

One of the key problems in supply chains is dealing with variability. There are two main sources of variability: interarrival variability and process variability.

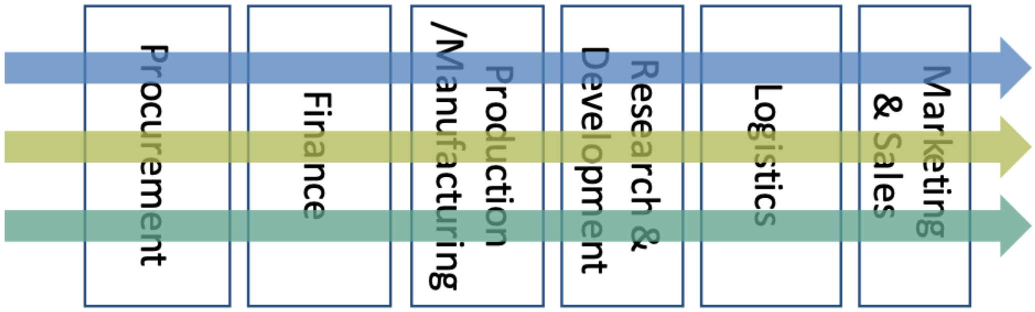

FIGURE 4.22: Coordination across the functions in a SCM

Reducing variability

There are several approaches for reducing variability:

- Identify & measure sources of variability and defects
- Develop plans and courses of action
- Segment customers, SKUs, suppliers, etc.

In addition, the approaches for buffering against variability are:

- Inventory – traditional approach, implies high inventory levels OR flexible inventory – module parts, common platforms, generic stock
- Capacity – maintain excess capacity at facilities to handle peaks OR flexible capacity – cross trainer personnel, multi-use equipment/facilities
- Time – plan time for system to recover OR flexible time dynamically allocate waiting time across customer segments

Core Supply Chain Processes

Main actors and processes in a supply chain:

FIGURE 4.21: Main actors and processes in a supply chain

Processes follow the work flow so coordination across functions is the key to SCM.

According to Lambert (2014) there are seven core supply chain processes. We can categorize them in internal and external processes.

Internal facing processes:

- Demand management

- Order fulfillment
- Manufacturing flow management
- New product development
- Returns management

External facing processes:

- Customer management
- Supplier relationship management

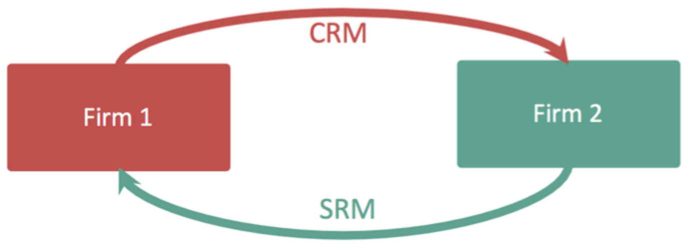

FIGURE 4.23: External facing processes

These are critical links in the supply chain, they mirror process – to a large degree. A rough process to assess:

1. Review/align firm strategy
2. Identify segmentation criteria
3. Establish product and service agreement (PSA) guidelines for different segments
4. Develop metrics – primarily profitability
5. Create guidelines for sharing improvement benefits

To work on CRM and SRM – you can also segment based on different criteria. For customers it could be profitability, growth, stability, volume, competitive positioning, buying behavior, sophistication, and location. For suppliers, they can be segmented on profitability, growth, volume purchased, criticality, innovation, quality, sophistication, and potential to co-create value.

Internal Facing Processes

Internal Facing Processes including: demand management, order fulfillment, manufacturing flow management, new product development, returns management.
Demand Management – potential variability drivers

Order Fulfillment

Strategic Tasks

1. Review/align firm strategy – coordinate with CRM teams
2. Define requirements – specific lead team and customer service requirements
3. Evaluate network – how/where customers will be service
4. Define Plan – establish rules for allocating scarce product, informational flow
5. Develop metrics to monitor – cash-to-cash cycle time, order fill rates, perfect order

Source	Potential problems	Possible solutions
Promotions	Creates lumpy demand; Cannibalizes future demand; Misdirects scarce resources	Plan and coordinate timing, duration, and level with operations and customers in advance
Sales metrics	Creates hockey stick effect at end of periods; Creates surges and lumpy demand	Design sales metrics to lessen end of quarter effect
Minimum order quantities	Creates lumpy demand; Increases potential for obsolescence and spoilage	Incorporate all costs when determining MOQ; Work to minimize the MOQ in order to speed up inventory velocity

FIGURE 4.24: Main sources of variability for demand management

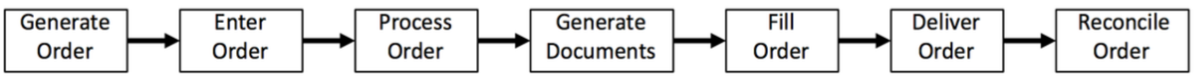

FIGURE 4.25: Order Fulfillment tasks

Manufacturing Flow Management

Strategic Tasks

- Review/align firm strategy
- Determine level of flexibility
- Determine Push/Pull boundaries
- Identify manufacturing constraints & capabilities
- Develop metrics to monitor

Returns Management

Strategic Tasks

1. Review/align firm strategy
2. Define avoidance, gatekeeping, and disposition
3. Create network for return flows
4. Define credit/refund rules
5. Develop metrics to monitor

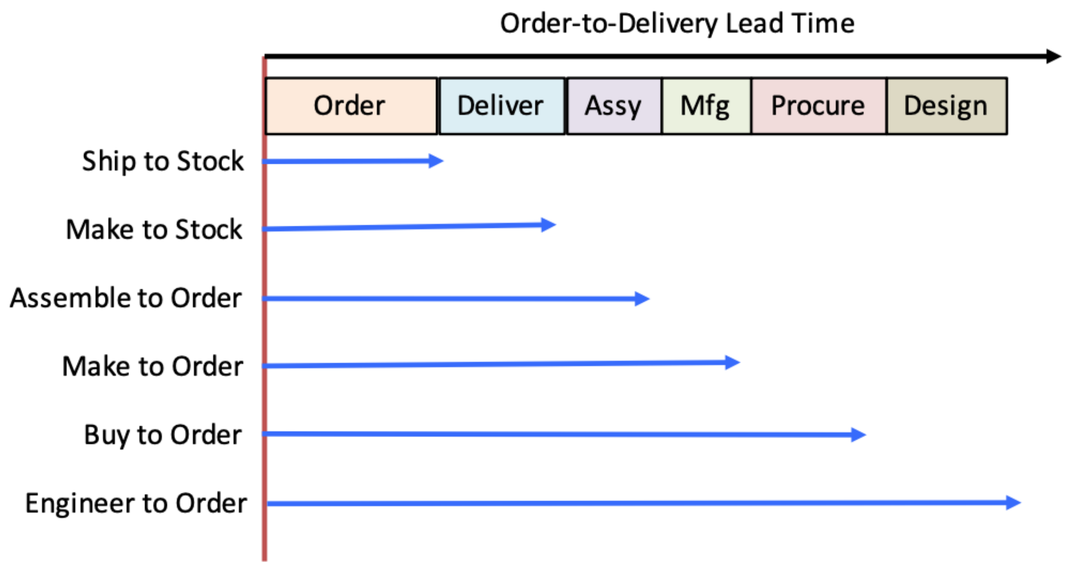

FIGURE 4.26: Type of manufacturing processes

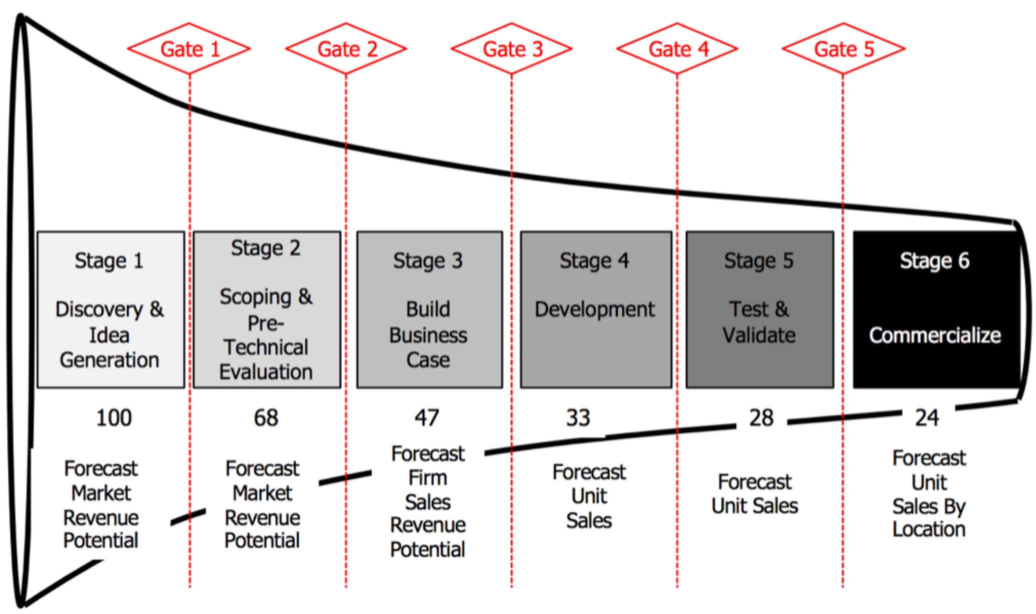

FIGURE 4.27: Product Development cycle

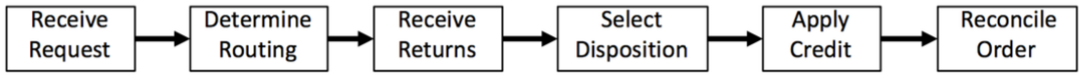

FIGURE 4.28: Returns Management tasks

Process map

A process map is a model that is a symbolic representation of the workflow used to better understand, communicate, level-set, codify, and converge on how a process works. There are different types: flowcharts, relationship maps, cross-functional (swimlane) maps, value stream maps, etc.

Main steps to create a process map:

- Determine the scope and level of detail
- Based on scope, identify and list the people or functions involved
- Using sticky pads, brainstorm the steps involved
- Work through the process chronologically, placing the sticky pads in the appropriate swim lanes
- Discuss and debate the draft process map and adjust accordingly
- Do functions touch the same items multiple times?
- Are there repeated and redundant handoffs?
- Are steps missing or extraneous?
- Transfer the diagram to paper; add a date and version

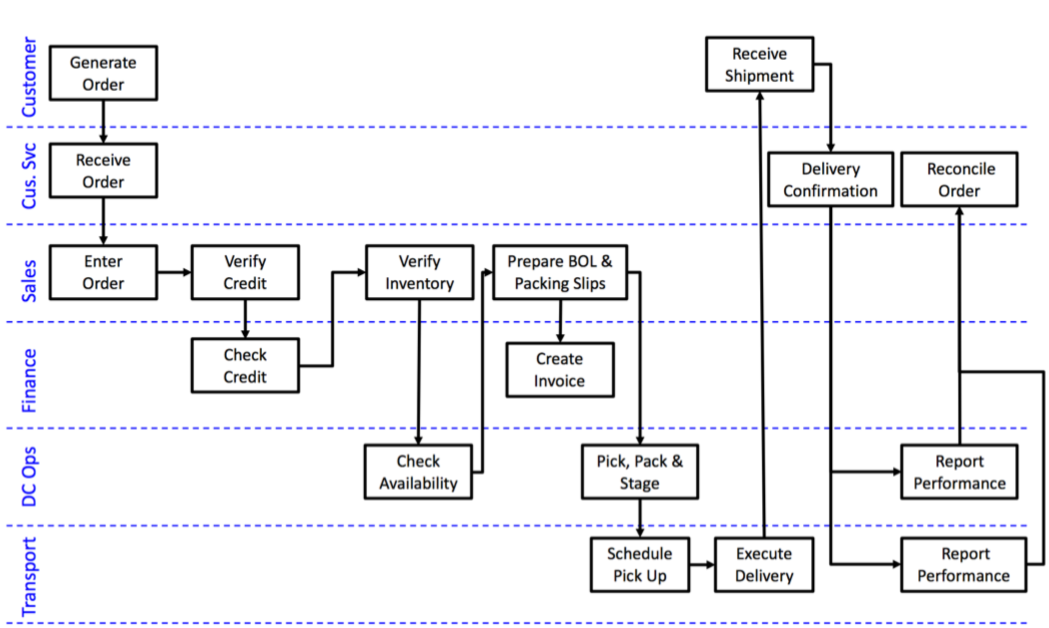

FIGURE 4.29: Swim Lane Diagram

Process improvement tools

Tools for checking variability

- Histograms: help understand outliers
- Time series charts: help understand where to focus

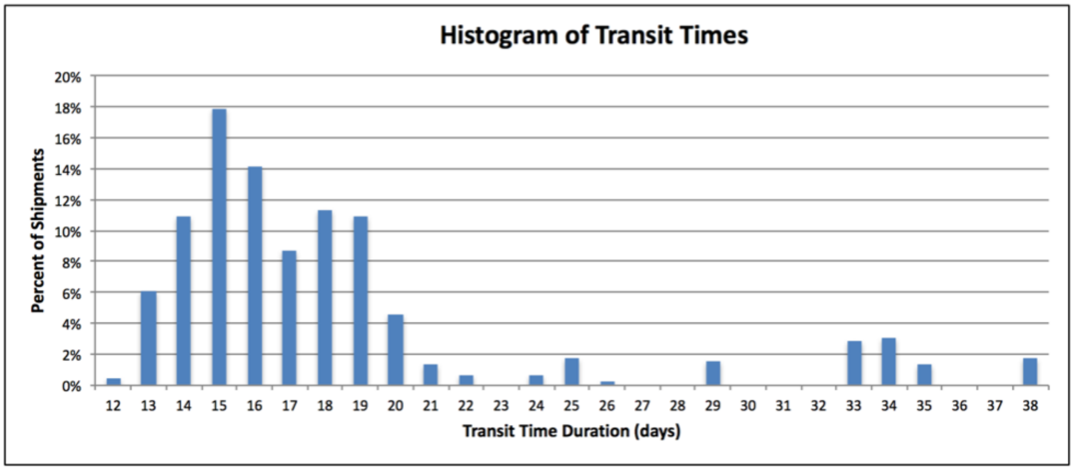

FIGURE 4.30: Histogram

Tools for identifying causes of variability

- 5 Whys
  - tool for encouraging brainstorming
  - forces team to look beyond superficial solutions
- Cause and effect diagrams (fishbone / Ishikawa):
  - Tool that provides structure for understanding root causes
  - Ensures that a balance list of ideas have been considered

Learning Objectives

- Understand the Supply Chain as a process
- Learn how to measure the performance of a process
- Identify sources of process variability
- Quantify how process variability impacts the performance of a queue
- Understand supply chain processes and variability
- Identify strategies to reduce variability
- Describe the seven core processes in a supply chain
- Learn tools and techniques to help with process analysis

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4.4 Global Supply Chain Management

4.4.1 Summary

This review covers global supply chain management concepts and explain the challenges and opportunities of trading between countries. Since most supply chains are global, it is important to consider the many factors that will influence your supply chain. It is also important to note that while guidance is provided here on how to work in global supply chains, some of the rules and data changes all the time.

4.4.2 Global Supply Chain Management

From 1950 to now, global population has increased by 2.8x and World trade has increased 20x.

World Trade Organization (WTO)

The World Trade Organization (WTO) was founded in 1995 to promote fair trade. Its principals include trade without discrimination, freer trade, predictable, more competitive and more beneficial for less developed countries.

Problems with the WTO include Trade barriers have shifted and LDC abuse of rules.

Duty determination

Duty is the amount of tax paid on an imported good. The amount of duty that an importer has to pay is determined by three factors:

- The type of goods (their **classification**)
  - Harmonized Tariff Schedule (HTS) is a 10-digit code used to describe all goods in trade for duty, quota, etc. The first 6 digits are standardized internationally; the last 4 digits are country specific.
  - Which code to use based on codes, duty rates differ, everyone tries to twist the definitions to their own advantage
- The value of the goods (their **valuation**)
- The country from which the goods originated (the rules of **origin**)

Duty Drawbacks

There are three types of duty drawbacks:

- Same condition drawback: when you import an item and then re-export this item in the same condition
- Different condition drawback: or sometimes also called manufacturing drawback, when you use an imported item in the assembly of another product
- Domestic goods returned in different condition: when an item that was previously exported comes back as part of an assembly of another product
- Below shows how Duty Drawbacks work:

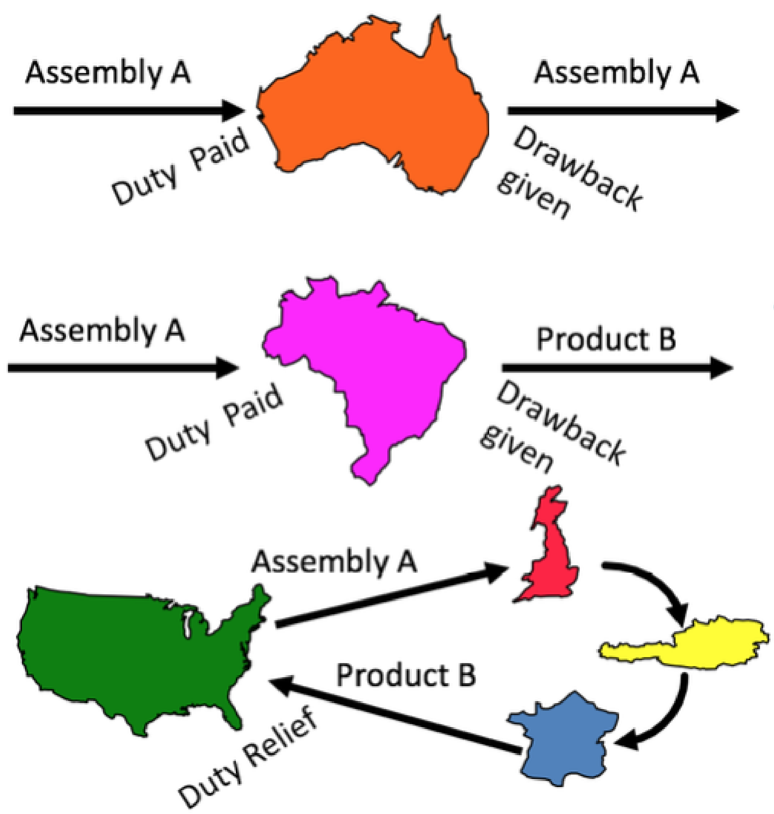

FIGURE 4.31: Duty Drawbacks

Sales Taxes on Imports: VAT, GST, HST

- Besides charging Duty on imports, many countries charge a Sales Tax also:
- VAT – value added tax - used in Europe, tax based on a % of the (dutiable value + duty)
- GST – goods and services tax – used in Canada, Australia, N Zealand and many others . . . defined the same as VAT but may apply to cost of inland transport also
- HST – harmonized sales tax – used in Canada to represent a combined federal GST plus a local provincial sales tax

Valuation

Valuation is important because most of the duties are collected as a percentage of the value of goods, this is called ***ad valorem***. For Customs' purposes, valuation is generally the amount billed by the exporter and shown on the invoice. However, there are situations where the amount shown in the invoice may not represent an "arms-length transaction." This occurs when the exporter and importer are "related parties" such as a parent company and its subsidiary. There are four alternative methods used by Customs authorities worldwide to determine the value of goods:

- Comparative method: use the value of similar goods
- Deductive method: work backwards from price
- Computed or reconstructive method: build up from manufacturing cost
- Method of last resort: make an educated guess

Dutiable Value: FOB vs CIF

- Countries calculate "Dutiable Value" based on either the FOB value or the CIF value.
- FOB value – "Free on Board"
  - Price paid for the goods plus the cost of assists, extra packing, commissions, royalties, transportation, loading, unloading, handling, and insurance to deliver the goods from the Seller onto the ship at the Port of Export
- CIF value – "Cost, Insurance, & Freight"
  - Equals FOB plus marine insurance and international (ocean) freight

Rules of Origin

Rules of origin are important because duties may differ by origin countries. There are two methods used to figure out the rules of origin:

- Substantial transformation: last country where the product underwent a substantial transformation
  - Lots of opportunities for multinationals to rearrange where they make things and route shipments to take advantage of differing duty rates.

Free Trade Zone	Bonded Warehouses
Main purpose: duty-free access to low cost labor	Main purpose is to delay the paying of duties
No duty paid if re-exported	No duty paid if re-exported
Duty delayed if imported	Duty delayed if imported
Work can be performed on the goods	Little / no work can be performed on the goods
Great for processing goods in low cost labor countries	Great for delaying duty on imported $$$ goods
Great for showing off products to international buyers	

FIGURE 4.33: Free Trade Zones and Bonded Warehouses

- Change in Harmonized System Code: last country where the harmonized system code of the product changed
  - This method is the one currently followed by the United States for textile products.

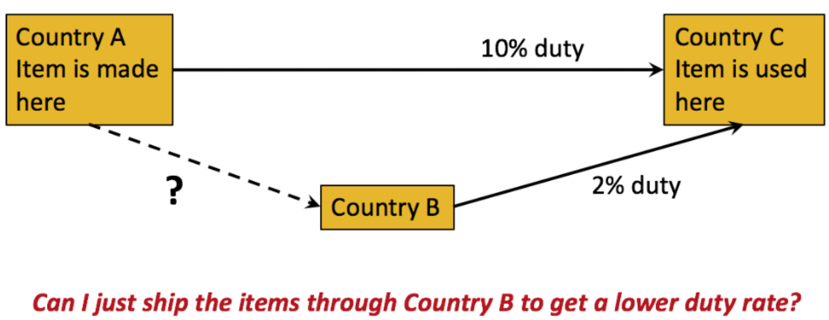

FIGURE 4.32: Rules of Origin

Generalized System of Preferences

Developed countries usually provide opportunities to developing countries to grow their economies by designating them as Generalized System of Preference (GSP). Countries designated as GSP benefit from reduced duty rates and quotas for their goods.

- US: Generalized System of Preferences
  - Promotes economic development by eliminating duties on up to 5,000 types of products when imported from one of 122 designated beneficiary countries and territories.
- Europe: Generalized Scheme of Preferences:
  - Standard Level – about 30 countries, reduced duties
  - GSP+ Level – about 14 countries, no duties
  - EBA Level – about 49 countries, no duties or quotas (Everything but Arms)
- Most Favored Nation Trading Status
  - Designation for normal trading partner

As of 2020

Name	Countries
European Union (umbrella)	27 European countries (UK exited)
Caribbean Community	15 Caribbean countries
African Union	55 African countries
MERCOSUR	Arg, Brz, Par, Uru (Ven suspended in 2016)
ASEAN	10 SE Asian Nations (+ China)
USMCA	US, Mex, Can

Term	Definition
Trade Bloc	Umbrella term
Preferential Trade Area	Reduced tariffs between members
Free Trade Area	Zero tariffs between members
Customs Union	Zero tariffs between members AND the same external tariffs
Monetary (Currency) Union	Same currency

This data changes! Learn from this table but do your own checking.

FIGURE 4.34: Major Trading Blocs

Dumping

Dumping is a form of predatory pricing where producers charge a high price in the home market but a very low price in another market to gain market share. In international trade there are two types of penalties:

- **Anti-dumping Duty:** import duty to bring the price up to the price charged at the home market
- **Countervailing Duty:** or anti-subsidy duty, import duty to products that have subsidies at home

Landed Cost Analysis

Landed cost usually means all the costs leading up to the point where you are ready to take the goods out of the port of import.

The calculations must be done in THIS ORDER:

1. Calculate the FOB value
2. Calculate the CIF value (= FOB + ocean freight + marine ins.)
3. What is dutiable value for the importing country (FOB or CIF)?
4. Calculate the Duty (= duty rate x dutiable value)
5. Calculate the VAT taxable value (= dutiable value + duty)
6. Calculate the VAT tax (= VAT rate x VAT taxable value)
7. Calculate the Landed Cost (= CIF + duty + VAT tax)

4.4.3 International Transportation

International transportation differs significantly from domestic transportation. The main ways are reviewed including INCO terms, transportation options, and ports.

INCO terms

INCO stands for International Chamber of Commerce. These terms define which responsibilities belong to the buyer and which responsibilities belong to the seller during an international shipment. They also define who pays for freight cost, insurance, duties, cost of goods, and other various fees. There are four main groups of INCO terms: E, F, C, and D which are groups based on responsibility transfers between parties.

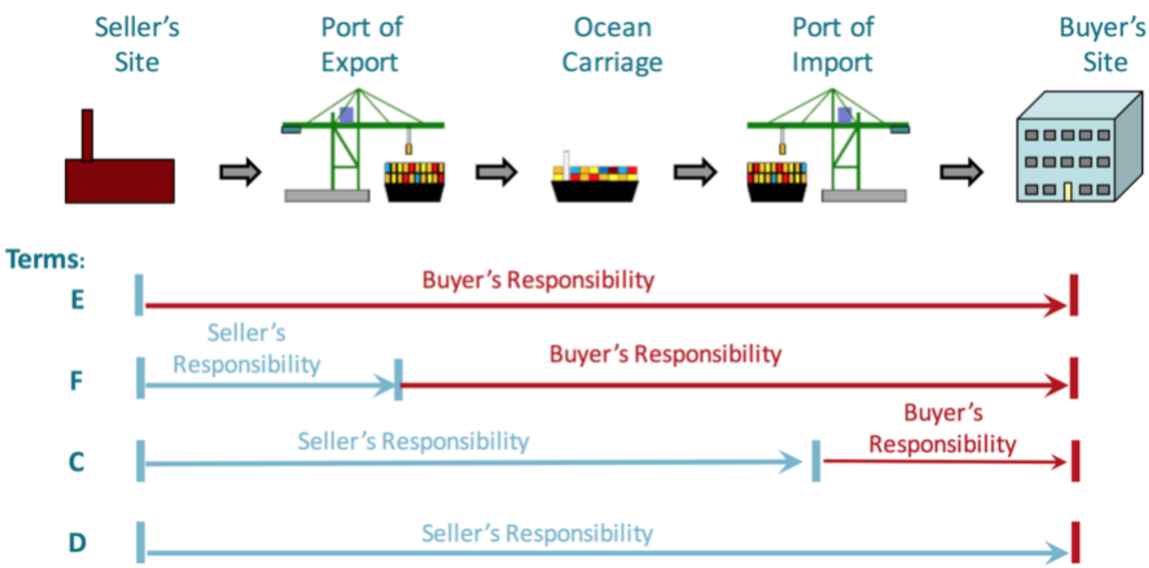

FIGURE 4.35: Four Main Groups of INCO Terms

2020 INCO Terms Chart

Incoterms® 2020 Rules Responsibility Quick Reference Guide

IncoDocs

	Freight Collect Terms					Freight Prepaid Terms					
Groups	Any Mode or Modes of Transport		Sea and Inland Waterway Transport				Any Mode or Modes of Transport				
Incoterm®	EXW Ex Works (Place)	FCA Free Carrier (Place)	FAS Free Alongside Ship (Port)	FOB Free On Board (Port)	CFR Cost and Freight (Port)	CIF Cost Insurance & Freight (Port)	CPT Carriage Paid To (Place)	CIP Carriage & Insurance Paid to (Place)	DAP Delivered at Place (Place)	DPU Delivered at Place Unloaded (Place)	DDP Delivered Duty Paid (Place)
Transfer of Risk	At Buyer's Disposal	On Buyer's Transport	Alongside Ship	On Board Vessel	On Board Vessel	On Board Vessel	At Carrier	At Carrier	At Named Place	At Named Place Unloaded	At Named Place
Obligations & Charges:											
Export Packaging	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller
Loading Charges	Buyer	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller
Delivery to Port/Place	Buyer	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller
Export Duty, Taxes & Customs Clearance	Buyer	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller
Origin Terminal Charges	Buyer	Buyer	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller
Loading on Carriage	Buyer	Buyer	Buyer	Seller	Seller	Seller	Seller	Seller	Seller	Seller	Seller
Carriage Charges	Buyer	Buyer	Buyer	Buyer	Seller	Seller	Seller	Seller	Seller	Seller	Seller
Insurance	Negotiable	Negotiable	Negotiable	Negotiable	Negotiable	*Seller	Negotiable	**Seller	Negotiable	Negotiable	Negotiable
Destination Terminal Charges	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Seller	Seller	Seller	Seller	Seller
Delivery to Destination	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Seller	Seller	Seller
Unloading at Destination	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Seller	Buyer
Import Duty, Taxes & Customs Clearance	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Seller

IncoDocs Create your Sales & Shipping Documents at www.incodocs.com

*CIF requires at least an insurance with the minimum cover of the Institute Cargo Clause (C) (Number of listed risks, subject to itemized exclusions)
**CIP now requires at least an insurance with the minimum cover of the Institute Cargo Clause (A) (All risk, subject to itemized exclusions)

This is general information for guidance purposes only. IncoSolutions Pty Ltd is not responsible for these contents nor do the contents listed above contain all details. For a full and complete description, refer to the full version of Incoterms 2020 by the International Chamber of Commerce at the ICC website.

FIGURE 4.36: Incoterms 2020 Quick reference Guide

Attribute	Air Freight	Ocean Freight
On-time reliability	Very reliable, 1-2 days delivery window	Not reliable, 1-2 weeks delivery window
Freight Cost (ignoring inventory)	Expensive 5 to 10+ times Ocean	Cheaper
Basis of Freight cost	Weight is most important	Volume is most important factor
Transit time	1-3 days to (almost) anywhere	5 to 40 days
Pollution	Very high	Much lower

FIGURE 4.37: Air Freight Vs: Ocean Freight

Type of ship	Type of Cargo
Roll-On/Roll-Off or Ro-Ro ship	Wheeled cargo
Break-Bulk Ship	General cargo
Oil tanker	Oil in bulk
Chemical Carrier	Chemicals in bulk
Dry-Bulk Carrier	Unpackaged bulk cargo
LNG ship or Gas Carrier	Liquefied natural gas (LNG)
Container Ship	Non-bulk cargo

FIGURE 4.38: Types of Ships

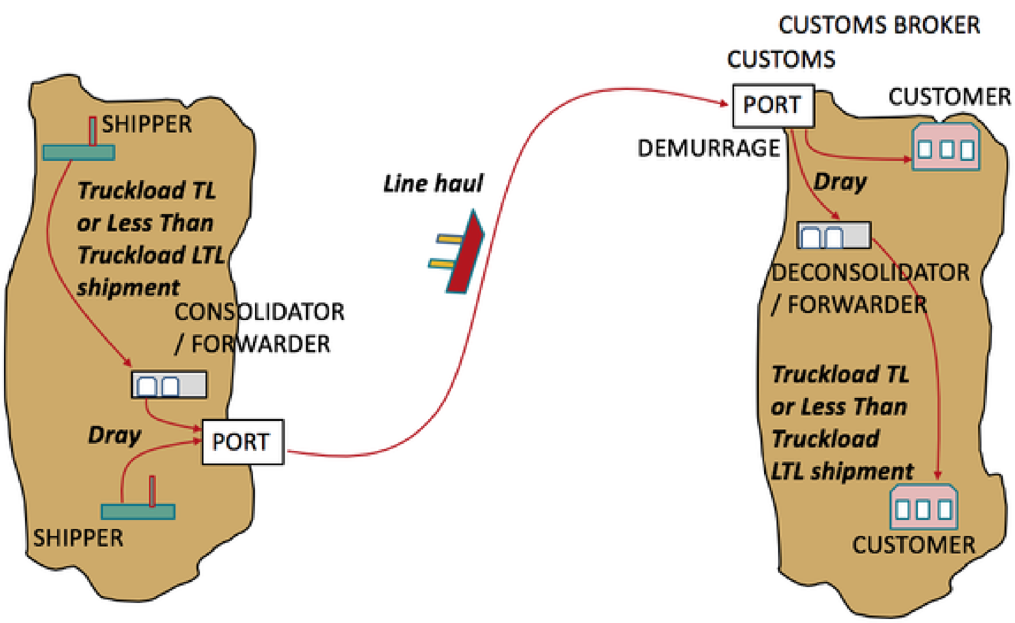

FIGURE 4.39: Legs of an ocean shipment

Ocean Shipment Overview

- Dray: local haul to the port
- Demurrage: penalty for extra days that containers stay at a dock
- Cabotage: when a foreign-flagged vessel picks up and drops off in the same country
- TEU: twenty-foot equivalent unit
- Weekly service: string of ships following each other

The Steamship Industry

The steamship industry is a volatile industry in which few carriers handle most of the freight. Carriers

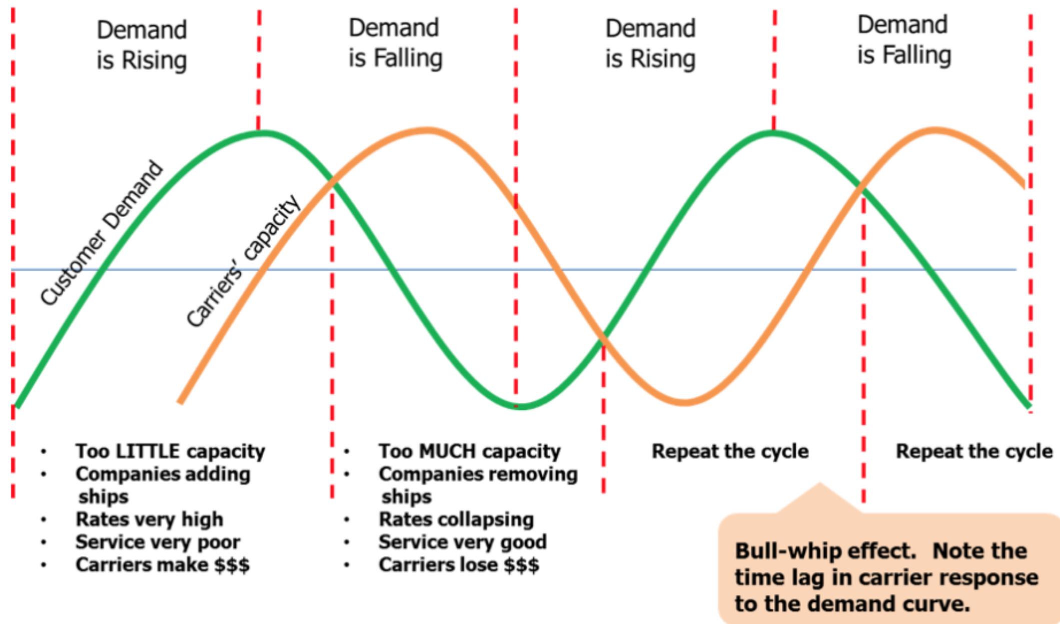

FIGURE 4.40: Bull-Whip effect in the steamship industry

collaborate and cooperate among each other, there are known as alliances. This industry is characterized by having very different freight rates depending on the direction of the trip. Because of volume discount pricing, this industry attracts consolidators and Non-Vessel Operating Common Carrier (NVOCC). Historically the steamship industry has faced the Bull-Whip effect, which is the time lag in carrier response to the demand curve.

Top Ocean Carriers

- There are 50-75 ocean carriers of significance.
- Top few carriers handle the vast majority of shipments.
- They form "Conferences" to collude on rates and manage capacity to set rates. (outlawed in Europe)
- They form "Alliances" to cooperate on capacity by lane – same string will have ships from different carriers.

Volume Discount Pricing – the more you buy, the cheaper per unit.

- This pricing scheme is ripe for Consolidators
- In transportation:
  - Truckload Broker (CH Robinson, Coyote Logistics)
  - Air Freight Forwarder (Damco)
  - NVOCC (non-vessel operating common carrier) (Expeditors Intl, UTi, APEX, Damco)

International Surface Transport

International Surface transports are railroads and trucks that cross international borders.

MAJOR RAILROAD GAUGES IN THE WORLD					
Gauge	Name	Installation (km)	Installation (miles)	Percent of track in world	Where used:
1,000 mm (3 ft 3 3⁄8 in)	Metre gauge	95,000	59,000	7%	SE Asia, India, Argentina, Brazil, Bolivia, northern Chile, Switzerland, East Africa
1,067 mm (3 ft 6 in)	Three foot six inch gauge	112,000	70,000	9%	Southern and Central Africa, Indonesia, Japan, Taiwan, Philippines, New Zealand, Queensland Australia, Western Australia
1,435 mm (4 ft 8 1⁄2 in)	Standard gauge	720,000	450,000	55%	Europe, Argentina, United States, Canada, China, Korea (South), Korea (North), Australia, India, Indonesia, Middle East, North Africa, Mexico, Cuba, Panama, Thailand, Venezuela, Peru, Uruguay, Philippines, Japan, Taiwan and Spain.
1,520 mm (4 ft 11 27⁄32 in)	Russian gauge	220,000	140,000	17%	Russia, Finland, Estonia, Georgia, Latvia, Lithuania, Mongolia, Ukraine
1,524 mm (5 ft)	Finnish gauge	5,865	3,644	small	Finland
1,600 mm (5 ft 3 in)	Irish gauge	9,800	6,100	small	Ireland, Australia, Brazil
1,668 mm (5 ft 5 21⁄32 in)	Iberian gauge	15,394	9,565	small	Portugal, Spain
1,676 mm (5 ft 6 in)	Indian gauge	134,008	83,269	11%	India, Pakistan, Argentina, Chile, Sri Lanka, Bangladesh, United States

FIGURE 4.41: Major railroad gauges in the world

- Biggest challenges
  - Truck transportation: safety standards, protecting local jobs
  - Railroad transportation: different safety requirements, work rules and interoperability (different widths between tracks)

Ports and Terminals

A port is a general term referring to a big community of players, which include:

- Port Authority
- Terminal Operators
- Warehouse Operators

- Consolidators / De-consolidators
- Customs Officials
- Railroads
- Dray Operators
- Freight Forwarders
- Customs Brokers

A **terminal** is the location within a port where ships load & unload, it has:

- Quay, ship to shore cranes, RTG cranes
- Container storage yard
- Administration buildings

4.4.4 Currency Issues and Financing

In global supply chains it is essential to understand currency issues and financing. To understand this, we review several important concepts including: exchange rate risk, transfers pricing, and profit repatriation and its effects on corporate income taxes. We also briefly review description of trade finance and international trade payment methods.

Exchange Rate Risks

There are two factors that will trigger exchange rate risk when together:

- Two different currencies (i.e., buy in one currency but get paid in another)
- Time delay between beginning and end of transaction

Currency Futures Contract: A contract to exchange one currency for another at some future point in time at a pre-specified exchange rate. Normally there is a fee to do this.

The most common ways to protect against exchange rate risk are:

- Use only one currency
- Use a bank to hedge the transaction - pay a fee to the bank to guarantee the price you will get paid
- Hedge yourself:
  - Currency Futures Contract: a contract to exchange one currency for another at some future point in time at a pre-specified exchange rate (this usually requires a fee)
  - Foreign currency hedging: is a risk reducing strategy where two offsetting, opposite positions, are taken in two different parallel markets. The positions are such, that their end results offset each other.

Internal ways to hedge foreign currencies within your own company:

- Leading Expenditure: to pay in advance if currency is expected to rise
- Lagging Expenditure: to pay late if currency is expected to fall
- Netting Receipts and Payments: to hedge the net exposure

External ways to hedge currencies:

- Forward Contracts: a contract where a buyer and seller agree on an exchange rate in advance
- Currency Swaps: two parties with two different currencies take an equivalent loan in their home currency and trade the cash
- Foreign Currency Options: pay a fee to have the optional right to buy or sell a specific amount of a specific currency at a specific price at a specific future time
- Spot Contracts: contracts that are completed in 1-3 days to avoid high fluctuations

Corporate Income Taxes

Companies can be charged taxes by any government for corporate profit but also inventory value, value of property plant and equipment.

1. Taxes in each country are charged based on how much profit was made in each country.
2. Tax rates vary significantly from country to country.
3. Profitability can vary significantly from product to product.
4. Margins can vary significantly from region to region (e.g. same product priced differently in different markets).
5. Transfer pricing is used to establish intracompany selling prices (and thus establishes the profit margin in each country).

International Income Taxes

Taxes in each country are charged based on how much profit was made in that country. The tax rate varies significantly from country to country. Profitability varies significantly by type of product and profit margins can vary significantly from region to region.

Tax Haven

A Tax Haven is a country with a low corporate income tax rate such as Ireland, Switzerland, Singapore, or Puerto Rico.

Permanent Establishment

A Permanent Establishment (PE) is a fixed place of business that generates income or value added tax liability in a particular jurisdiction. The tax systems in some civil law countries impose income and value added taxes only where an enterprise maintains a PE in the country. These are the things that may get a business labeled as having a PE in a country:

- A branch
- A warehouse
- A factory
- A mine or oil or gas well
- A management office
- Owning inventory in a country
- Buying or selling in the country

Simplified example scenarios only – will differ by country

Country A	Country B	Tax Relief in Country A?	Taxes Paid in B	Profit (Div.) Repatriated	Taxes Paid in A	Comments
Parent	Branch / Office	none	25% x 1000 = $250	N/A	35% x 1000 = $350	Foreign profits by a local branch are taxed by both countries.
Parent	Branch / Office	Foreign Tax Credit (FTC)	25% x 1000 = $250	N/A	(35%x1000) - $250 = $100	FTC reduces the tax due in A by the amount of tax paid in B. Limited so the tax due in A cannot go below zero.
Parent	Sub	N/A since $0 repatriation	25% x 1000 = $250	$0	$0	Assuming profits earned in B are not connected to business in A, there is $0 tax A
Parent	Sub	none	25% x 1000 = $250	All Profit of $1000	35% x 1000 = $350	With no tax relief in A, repatriated income is taxed at the full rate
Parent	Sub	Foreign Tax Credit (FTC)	25% x 1000 = $250	All Profit of $1000	(35%x1000) - $250 = $100	FTC reduces the tax due in A by the amount of tax paid in B. Limited so the tax due in A cannot go below zero.
Parent	Sub	Participation Exemption (PEX)	25% x 1000 = $250	All Profit of $1000	$0 if FULL exemption, $0 to $350 with PARTIAL	PEX provides partial to full exemption of tax in A if paid tax in B and parent has at least X% ownership of entity in B
Parent	Sub	Double Taxation Agreemt (DTA)	25% x 1000 = $250	All Profit of $1000	Range from $0 to $350 dep. on terms of DTA	DTA treaties vary by country pair and provide partial to full exemption from double taxation

FIGURE 4.42: Taxes on Foreign Earnings

- Being the importer of record, being the exporter of record
- Owning property in the country
- Having employees in the country

Types of Tax Relief

Foreign Tax Credit (FTC): when countries allow companies to deduct the amount of tax paid in another country.
Participation Exemption (PEX): a partial to full exemption of taxes for companies that are subsidiaries of companies with a certain level of ownership in a specific country.
Double taxation agreement (DTA): treaties between a pair of countries to provide partial or full tax exemption.

Transfer Price

A transfer price is a "market price" for intracompany transfer of goods. It is needed because duties are charged based on the value of the goods being imported. This means that the invoice amount between related parties is not always a good indication of the value of the good. Transfer price is also a way to shift profit from one country to another, given that income tax rates might be very different.

- What are the implications of a HIGH transfer price?
  - More profit is realized in the exporting country, less profit is realized in the importing country
  - More import duty is paid if the goods move into the importing country.
- What are the implications of a LOW transfer price?
  - Less profit is realized in the exporting country, more profit is realized in the importing country
  - Less import duty is paid if the goods move into the importing country.

- What does the exporting country want you to do with regard to Transfer Price?
  - Set it very high – so they get more TAXES from you.
- What does the importing country want you to do with regard to Transfer Price?
  - Mixed effect – if set high might get more duty, but less income tax. If set low get less duty but more income tax.

Profit Repatriation

Profit repatriation is bringing profits earned in a foreign country back to the home country. Usually paid as dividends to the parent company Barriers to Repatriation:

- Government limits
- Reporting requirements
- Withholding taxes on dividends, royalties, interest payments
- Double Taxation

How do companies Legally Repatriate Income while avoiding double taxation and withholding taxes? Key: don't convert profits in county B into dividends and then pay the dividends to parent in country A Key: have lower profits in country B and have higher expenses paid to parent company in country A

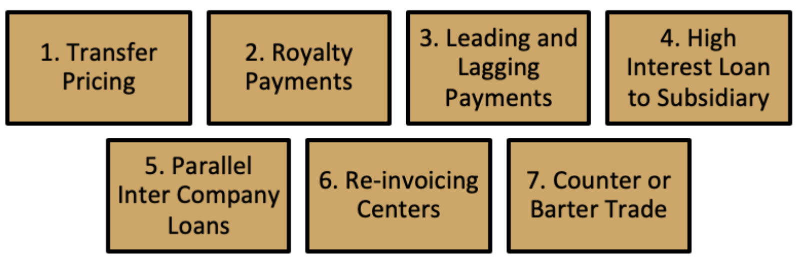

FIGURE 4.43: Legal ways of avoiding double taxation and profit withholding

Mechanics of Profit Repatriation:

- Transfer Pricing (avoid the need to repatriate): Have transfer prices "rigged" so a lot of profit occurs in the home country. Limitations: governments review for "an arms-length transaction."
- Royalty Payments: charge a fee to foreign-subsidiary for the use of the parent company's name and brand
- Leading and Lagging Payments: use currency fluctuations to favor the home country
- High Interest Loan to Subsidiary: loan money to the foreign-subsidiary with high interest
- Parallel Inter-Company Loans: partner with another company to interchange loans with foreign subsidiaries of the other company in the parent company's home country
- Re-Invoicing Centers (RIC): use a Tax Haven to route buy-sell transactions
- Counter or Barter Trade: exchange goods instead of money between parent and foreign-subsidiary

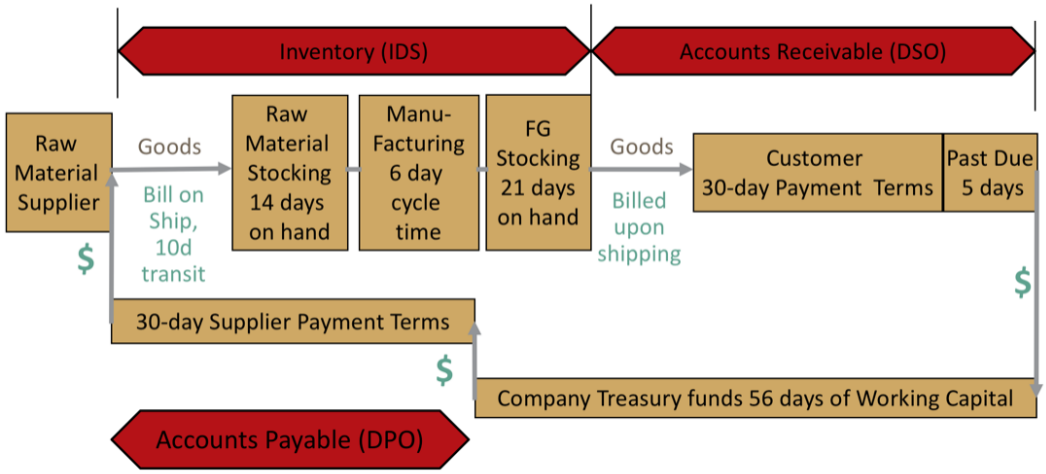

FIGURE 4.44: Example of a Cash Flow Model

Managing Working Capital

Cash-to-Cash Cycle Time

$$CTC = IDS + DSO - DPO$$

Where,

- CTC is Cash to Cash Cycle Time
- IDS is Inventory Days of Supply
- DSO is Days of Sales Outstanding (Accounts Receivable)
- DPO is Days of Payables Outstanding (Accounts Payable)

Ways to Free Up Working Capital

These are ways to free up working capital and reducing the CTC. Although their effect is to decrease working capital, a thorough analysis of the side effects of each one of these should be performed. Most common ways used:

- Lower transit time
- Return excess to suppliers
- Lower inventory targets, increase deliveries and runs
- Improve manufacturing cycle time
- Use smaller lots
- Sell off excess and obsolete
- Use shorter payment terms with customers
- Have vigorous collection of past due payments
- Ask for longer payment terms to suppliers (not usually recommended)
- Delay payments to suppliers (not usually recommended)

Innovative Ways

- Bill on receipt
- Use vendor-owned adjusted time inventory
- Implement Lean Manufacturing techniques
- Build to order
- Offer early payment discount
- Offer a pay by credit card option
- Ask about payments prior to due date
- Electronic Funds Transfer (EFT)

Trade Financing Actions

- **Early Payment Program:**
  - Offered by OEM: OEM pays Supplier early but charges the Supplier at a discounted rate
  - Using a Funder: Funder pays supplier immediately, and charges a percentage to supplier, the OEM also gets a discount for using the Funder
- **A/R Financing -** Selling Receivables: Supplier sells its receivables to a Funder who pays the Supplier immediately but gets a discount, this method is very similar to the Early Payment Program Using a Funder but the OEM is not involved in this scheme
- **Revolving Line of Credit:**
  - Supplier goes alone: Supplier borrows cash from a Funder, which charges an interest rate (based on the supplier's financial health) and takes supplier's invoices as collateral for the loan
  - OEM helps supplier: OEM publishes a list of "approved invoices" to show that it is committed to pay. Therefore, when the supplier borrows cash from the Funder the interest rate is lower as the interest rate is based on the OEM's financial health
- **Early Payment Program to Extend DPO:** Funder is used to pay to the Supplier immediately and to extend the payment terms to the OEM, the Funder charges to both Supplier and OEM

Methods of Payment in International Trade

- **Cash-in-Advance:** means that the buyer pays before receiving the goods; this method is used in high-risk trade relationships or export markets.
- **Letters of Credit:** is a document issued by the buyer's bank committing to pay to the seller even if the buyer fails to pay to the bank; this method is used in new or less-established trade relationships.
- **Documentary Collections:** occurs when the Seller's bank collects all the export documents (BOL, Commercial Invoice, etc.) of the goods shipped by the buyer, the Seller's bank sends these documents to the Buyer's bank demanding payment. The Buyer's bank will hold these documents until the payment is made, this method is used in established trade relationships.
- **Open Account:**is when goods are shipped before payment along with all the export documents. This arises in secure trading relationships or markets or in competitive markets to win customers.

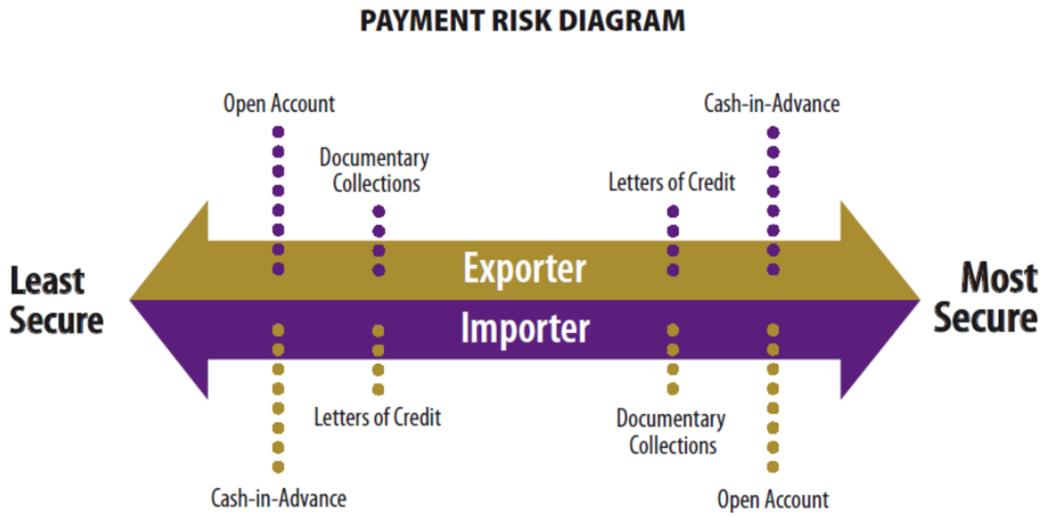

FIGURE 4.45: Payment Risk Diagram

4.4.5 Sourcing and Shoring

In a global world, the topics of sourcing and shoring are hotly debated. To break apart these topics, we identify the motivations and the challenges to move manufacturing abroad. Beyond the initial decision to move abroad, there are additional key elements on social responsibility that companies lose grasp of when work moves farther away such as slavery or conflict minerals. Finally, we provide examples of different global supply chain strategies.

Approaches for Manufacturing Internationally

- **Contract Manufacturing:** a company hires another company to manufacture goods, usually in a foreign country
- **Licensing:** a company allows another firm to use its intellectual property to make products in exchange for a royalty
- **Franchising:** a company gives another firm a "bundle" of intellectual property items and allow the firm to reproduce its entire business model in exchange for royalties
- **Joint Venture:** two or more companies join to set up a new company to enter a new marketplace
- **Subsidiary:** a wholly owned foreign company that is independent from the parent company and gives total control to the foreign company

Motivations and challenges of offshoring

- Here are some of the reasons that explains why manufacturing moved to Asia & China:
- Very low labor cost, huge labor pool
- Very few restrictions on pollution, working conditions, quality, and exports
- New factories, new equipment
- New infrastructure

- Herd mentality
- Supply based moved - self-fulfilling trend.
- Corporate profit motive & competitive pressure outweighed loyalty to community
- Consumer savings outweighed loyalty to community

Some of the challenges related to the previous decision are:

- Very long lead times
- Long supply lines - long distance ocean transportation
- Time zone challenges
- Communication challenges - language differences
- Need for higher inventory levels
- Intellectual property theft, counterfeiting of products
- Extra cargo security
- Emergency air shipments are now a very long way

Changes in the last 20 years: China is more prosperous; labor costs have skyrocketed; labor shortage in coastal cities; Chinese Currency appreciation; restrictions on quality, working conditions, environment and safety; huge coastal factories have gotten bad reputation; green movement favors local sourcing; contamination scares; political backlash; U.S. Energy prices have fallen. Please note that COVID-19 is shifting these industry trends and there are a broad and generalized representation that may change in the future.

Industry	Response
Automotive Parts	Growth in Mexico
Aerospace Parts	Growth in Mexico
Appliances	Growth in USA, Mexico
Apparel	China still dominant but Growth in Bangladesh, Vietnam, other SE Asia
Footwear	Growth in Vietnam
Consumer Electronics	Staying in China

FIGURE 4.46: Industry Response to Changes

Global Supply Chain Design Strategies

Vertical integration

The following metrics are helpful to analyzing the effect of vertical integration:

Strategy	Example	Sourcing
Pursue absolute Lowest Cost	Apparel, Most Footwear	Most still in China but Vietnam, Bangladesh are up & coming
Low Cost but also closer to demand	Aerospace parts, Auto parts Some footwear	Moving more to Mexico Moving to Dom Rep
Fast, responsive supply chain for Low Volume, High Margin	High end consumer goods	Slowly returning to USA
Bound by huge installed supplier base	Consumer electronics	Stay in China

FIGURE 4.47: Where is Manufacturing Moving to

$$InventoryTurn = \frac{AnnualSales}{AverageInventory}$$

$$CombinedInventoryTurn = \frac{1}{\sum_{i=1}^{n} \frac{1}{InventoryTurn_n}}$$

where n is the number of plants vertically integrated

Outsourcing

- Typical Situation
  - Companies import with Transfer Price from subsidiary to parent or to another subsidiary
  - They have a PERMANENT ESTABLISHMENT (PE) in each country
  - They pay taxes in each country
- Tax avoiding strategy:
  - Design a network that complies with export restrictions, minimizes taxes, and provides the flexibility to ship from and to anywhere
  - Avoid PE in all but one country (headquarters location)
  - Use subcontractors for manufacturing and exporting, use distributors for importing and distributing
  - OEM buys from manufacturers and sells to distributors only in international air space or international waters

How to avoid permanent establishment

- Do not Manufacture in a country: *Use Contract Manufacturers (CM) instead*
- Do not be the Exporter of Record: *CM to be the exporter of record*
- Do not be the Importer of Record: *Customer or Distributor is the importer of record.*
- Do not own inventory in a country: *Inventory is owned by CM, Distributors, Suppliers, Customers.*
- Do not sell goods inside a country: *All sales of goods occur in int'l space*

Learning Objectives

- Understand most common practices used in global supply chain management
- Introduce international trade concepts and understand challenges and opportunities
- Understand the basics of international transportation
- Identify the main challenges faced in Air Freight, Ocean Freight and Surface transport
- Understand currency issues and how to prepare against exchange rate risks
- Overview concepts related to corporate income taxes, transfer pricing and profit repatriation
- Understand trade financing and how companies can help each other
- Identify four methods of international trade payment and letters of credit
- Understand different ways for manufacturing internationally
- Identify social issues, slavery and conflict minerals

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4.5 Exogenous Factors

Key Concept

4.5.1 Enterprise Resilience

Companies are reliant on their supply chains, as a result of this, they are vulnerable to disruptions which pose a risk to their business. While all businesses are at risk, there are some more resilient than others, meaning that they recover quickly. To understand these two concepts, we focus on how to estimate risks, how to deal with them in terms of prioritizing and how to prepare companies for disruptions. Finally, we analyze how to respond and mitigate the problems once a disruption occurs.

There are two different ways to look at disruptions, i) thinking about causes and ii) thinking about effects or modes of failures. Cause thinking helps estimate likelihoods, while effects thinking helps estimate impacts and consequences.

Causes of Disruptions

These are different types of risks or causes of disruptions:

- **Random Phenomena:** weather phenomena or natural disasters
- **Accidents:** any undesirable event, such as explosions
- **Governments & Politics:** trade barriers, trade embargo, military coups, political instability, changes in regulations
- **Non-Compliance:** non-compliance of standards such as air pollution, water contamination, safety, etc.
- **Competition:** technological innovation, process innovation
- **Economy:**macroeconomic shocks that lead to economic contractions such as financial economic recessions and depressions
- **Social disconnect:** companies not connecting with customers' will and desire
- **Intentional disruptions:** when there is a smart attacker on the other side, who will attack in the worst time and worst place, including terrorist attacks, strikes, cyber attacks

Modes of failure

These are ways how supply chains can fail and/or effects of disruptions:

- Inability to acquire supplies
- Inability to ship/loss of shipment
- Inability to communicate
- Inability to convert
- Loss of personnel
- Unavailable credit
- Brand/Trust diminution

Bow-Tie Risk Analysis Framework

One of the classical frameworks for thinking about risk and disruption is the Bow-Tie Risk Analysis Framework (Figure 4.48).

Classification of Risks

The traditional classification used to prioritize risk, includes two dimensions: probability of disruption and consequences. Usually supply chain managers focus on the severe consequences-high probability quadrant (upper right). However, the most dangerous events are those in the severe consequences-low probability quadrant because companies are typically not prepared and do not know how to prepare for these events.

In addition to disruption probability and consequences, a three-dimensional classification includes detectability. Dependability is defined as how long it takes from the time we know that an event is going to happen until it happens.

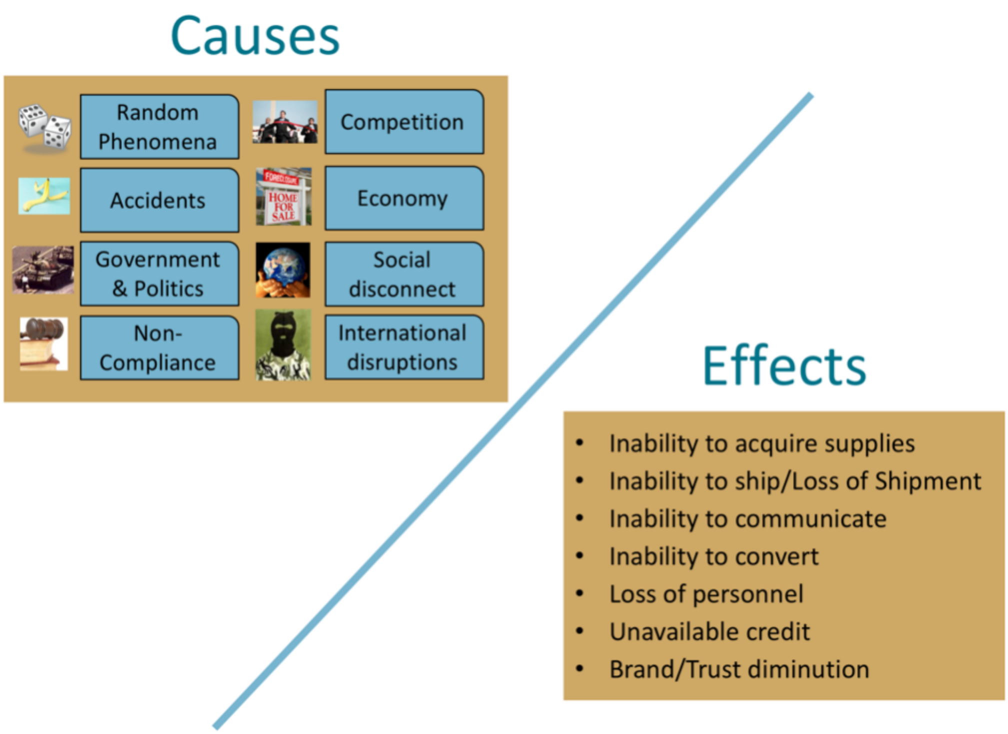

FIGURE 4.48: Causes and Effects of Disruptions

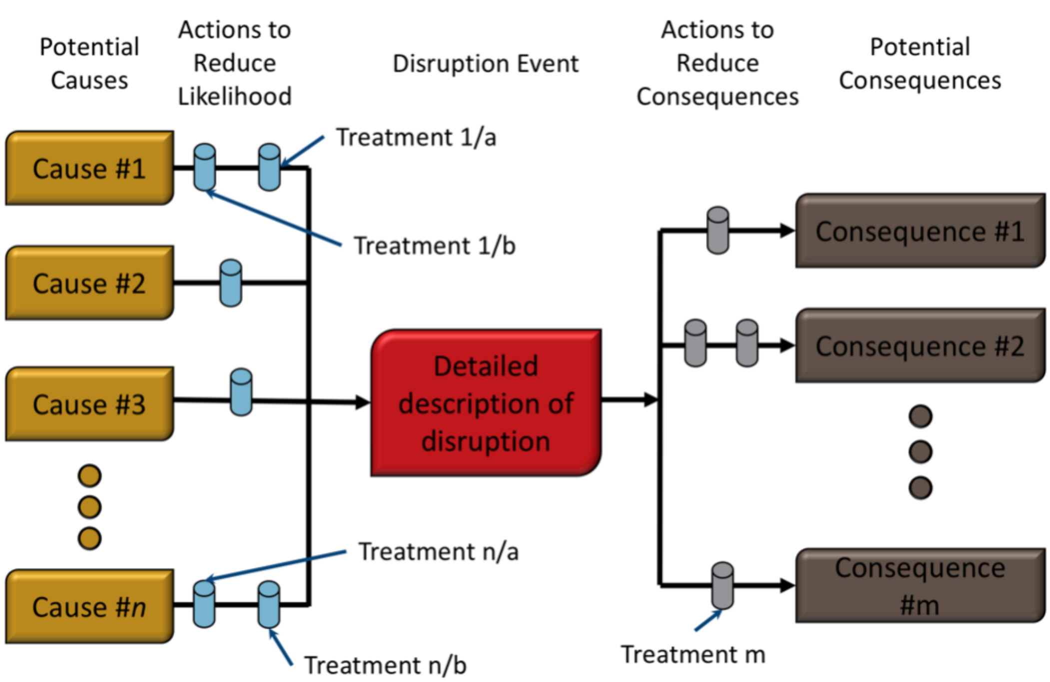

FIGURE 4.49: Bow-Tie Risk Analysis Framework

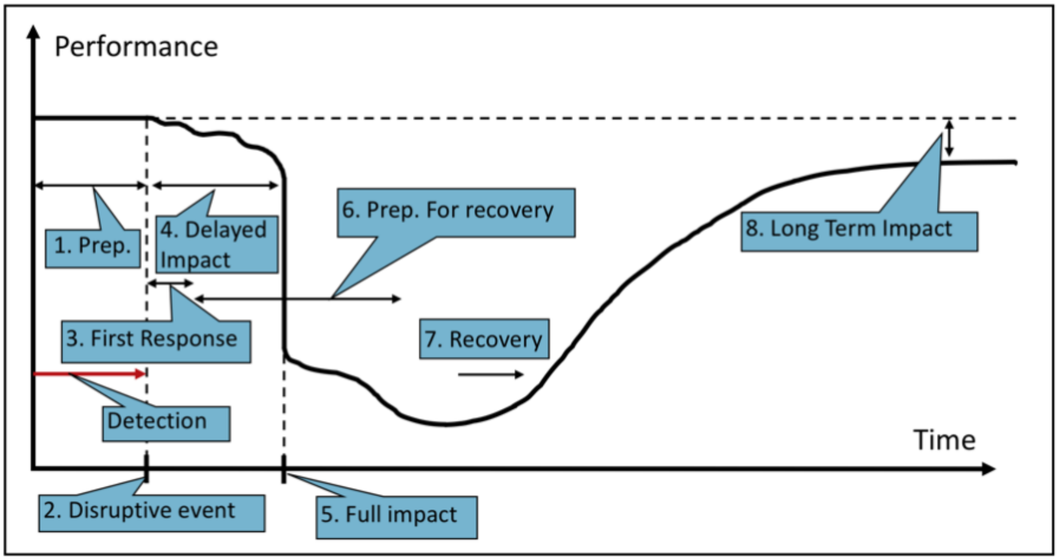

FIGURE 4.50: Disruption Profile

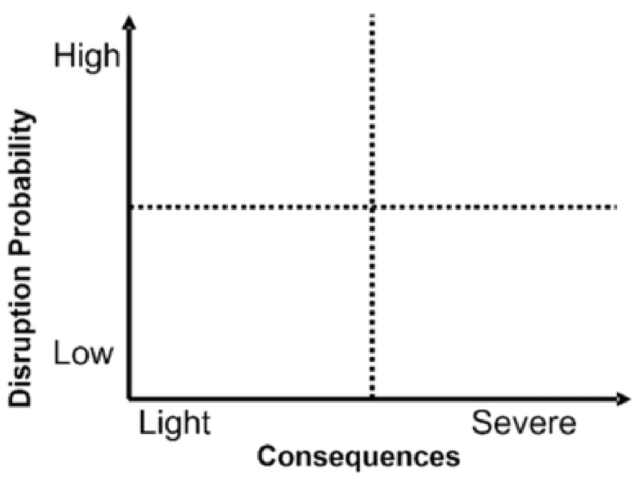

FIGURE 4.51: Two-dimensional classification: Disruption Probability vs. Consequences

- **Probability:** investigate the causes and estimate their likelihoods
- **Consequences:** identify impact, prepare options, drills, mitigate options and ways to "bounce back"
- **Detection:** develop detection ability and define decision rules

Power Law: Likelihood and Impact

Likelihood and impact of many disruptions obey the power law, which can be used to estimate the relationship between the frequency and damage of events.

$$f(x) = x^{-k}$$

Detectability and Preparation

There is a third dimension to characterize disruptions: detectability. The warning signs to identify suppliers are presented. the three types of elements to prepare for disruption are discussed: redundancy, flexibility and preparation.

Warning Lag

The warning lag is the time between alarm and event. It can occur:

- **After the attack:** in this case the time of detection is negative, you discover that something happened only after the event occurred
- **Immediate:** the time between the alarm and the event is zero, you realize that the event is happening when its occurring
- **Short term:** there is some time between the alarm and the event but not much to prepare
- **Medium term:** there is time between the alarm and the event, allowing the company to prepare
- **Long term:** the time between the alarm and the event is long so there is opportunity, enough time to prepare and take actions

Warning Signs of Supplier Failure

These might be signals that will allow you to sense that something is happening with the supplier:

- Financial: failure to prepare timely financial reports, multiple adjustments to annual reports, frequent negotiations of banking covenants, deteriorating working capital ratios, lengthening accounts payable
- Operational: high employee turnover in key positions, failed projects/failed acquisitions, operating loss, lack of capital investment, late/missed deliveries, quality issues, billing and invoicing errors, carrier selection errors

4.5.2 Resilience Fundamentals

Companies should weigh the benefits of uncertain future cost against certain current costs. Current costs are derived from the three ways to prepare for disruption:

- Redundancy: is creating/having some extra capacity, inventory, suppliers

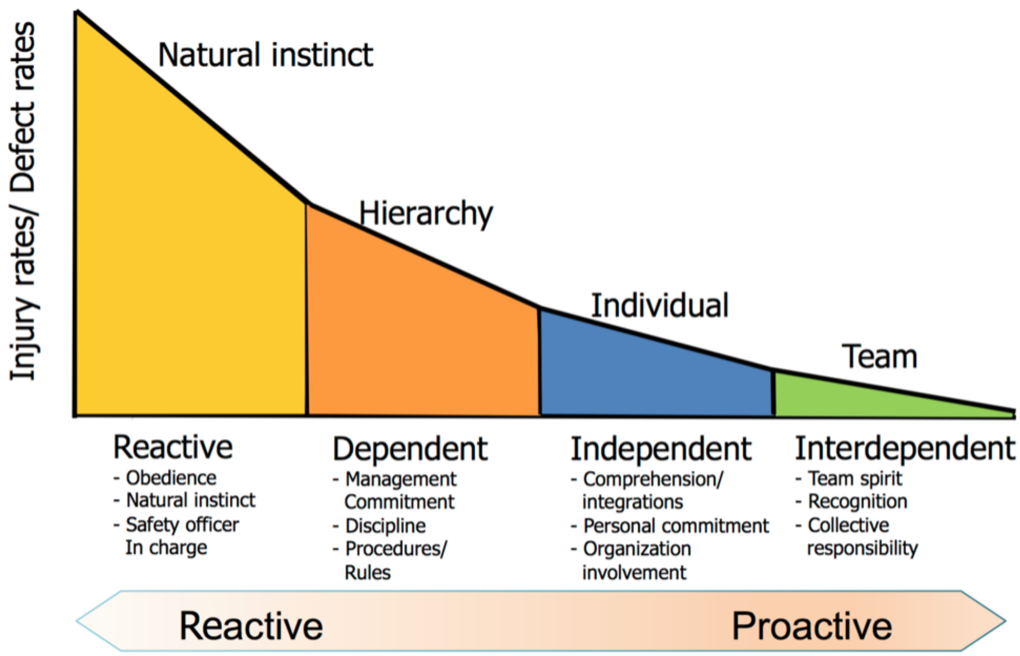

FIGURE 4.53: DuPont Bradley Curve

- Flexibility: means interchangeability, is being able to change processes fast, cross-training people to be able to perform more than one task, using postponement and standardization
- Readiness/preparation: is having real options or the tools to respond

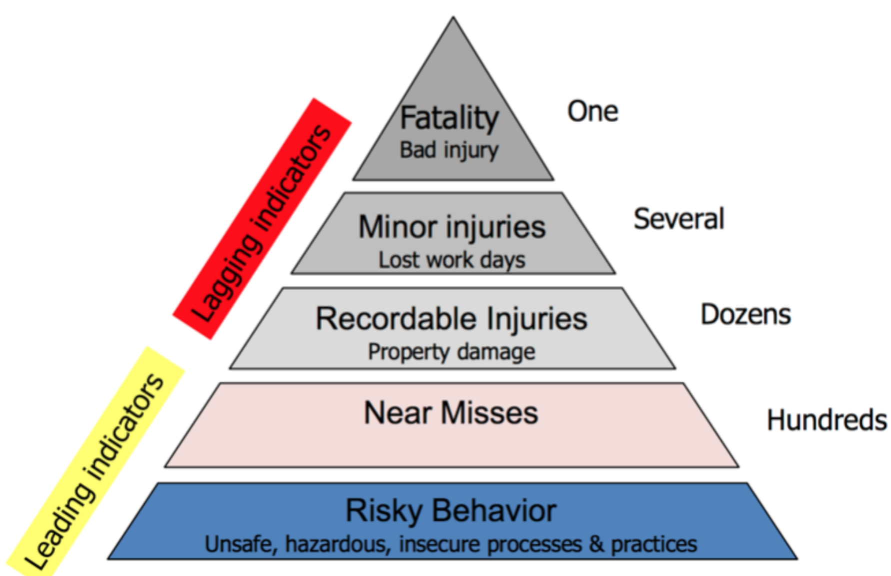

FIGURE 4.52: Safety Pyramid

Ways to deal with limited supply

- Allocation: use some criteria to allocate what is available
- Auctions: give the product to the customer who wants it most
- Dilution
- Substitution
- Demand shaping

Learning Objectives

- Understand the concept of warning lag
- Learn resilience fundamentals
- Understand how to prepare for disruptions
- Learn how to think to prioritize and to prepare for risks
- Distinguish between different causes of disruptions and effects

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4.5.3 Exogenous Factors

Exogenous Factors are external factors being forced on the supply chain that business has little control over. They may be expected or unexpected but can often shape the way that supply chains operate and may force different responses. In this lesson we will walk through exogenous factors, their sources, examples, and how companies respond.

STEEP FRAMEWORK

One way to organize the various factors impacting the supply chain is the STEEP Framework which consists of social, technological, economic, environmental, and political impacts.

- Social factors include demographics, religion, lifestyles and values
  - ex: aging population, crowdsourcing
- Technological factors include innovations, communication, R&D, and patents
  - Ex: machine learning, apps
- Economics factors include interest rates, global trade, taxes/fees/fines and inflation
  - Ex: trade between countries, financials

- Environmental factors include availability of resources, air & quality, food safety, pollution
  - Ex: drought, emissions, water pollution, product disclosure, labor conditions
- Political factors include regulatory stance, Government stability, tax policies, and trade unions
  - ex: disclosure, absolute requirements

Regulations

Regulation: a rule or directive made and maintained by an authority, typically governments. Economists view of regulations are that they are basically economic or social. **Economic** regulations set who can sell and what to charge such as market entry/exit, price controls. While **social** regulations prevent market failures by correcting for negative externalities like pollution, safety as all we information disclosure to consumers.

Types of regulation by mechanism:

Regulations can be applied in certain ways – as you can see in the following examples.

Type	**Description**	**Examples**
Absolute Mandates or Command & Control	Strict rules or "Bright Lines" about what is allowed, what is not and how to act.	Endangered Species Act, Electronic Logging Devices(ELD), Seatbelt Laws, Emission Standards, RoHS
Liability Regulations	Companies or other parties can be held responsible for actions locally or internationally, currently or in the past.	Foreign Direct Liability, CERCLA Act(Superfund), Product Liability Act
Performance Based or Incentive Regulations	Governments set a standard to achieve but firms have flexibility on how to achieve them	Clean Air and Clean Water Acts, Extended Producer Responsibility(EPR), Take-back Laws
Market Based Regulations	Regulations that encourage behavior through market signals(incentives and penalties) and allow for trading and credits	Greenhouse Gas Emissions, Ethanol, Cap & Trade
Disclosure & Transparency	Regulations that do not dictate behavior, but require specified disclosure about product content/performance and/or business processes/practices.	Modern Slavery Act, REACH, Conflict Minerals Act

TABLE 4.3: Example of Regulation

The objective of regulations is to impact behavior whether that be discouraging a behavior by banning or encouraging a behavior by requiring it. There are many challenges with regulations including:

- They affect almost every economic activity to some degree,
- They are constantly changing – deregulation or increase in regulation,
- They are often inconsistent (and sometime opposing) across regions and countries, and
- Firms are responsible for knowing and complying with all existing regulations and laws.

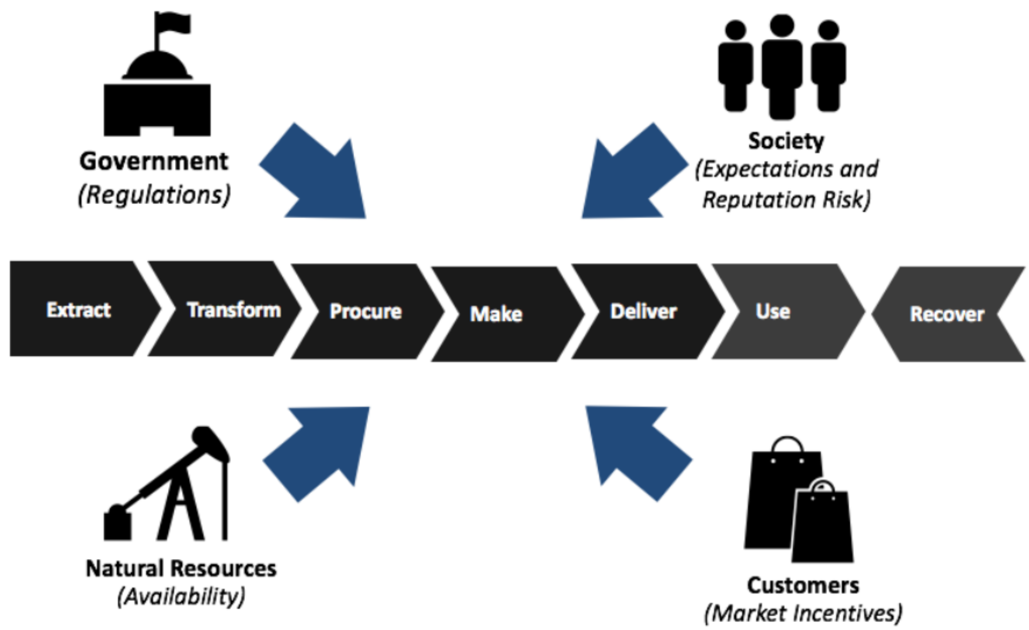

FIGURE 4.54: Impact of Regulation on the Supply Chain

Impacts on the Supply Chain

Impacts on the supply chain can be refined to government, society, natural resources, and customers. These factors are actively influencing how supply chains are designed and operate. Firm and supply chain behavior can be influenced by both "hard" (Government & Natural Resources) and "soft" (Society & Customers) factors.

For instance, how can an event or exogenous factors impact freight flows. It can have an impact on sourcing patterns, flow destination, routing, flow volume, and value density.

There are many types of regulations that are impacting the supply chain, some we will cover include factors related to products and processes.

- Products
  - Direct content (testable)
  - Lifecycle or pedigree (credence)
- Processes
  - End of life activities
  - Transportation, logistics, packaging
  - Sourcing practices

Regulations on Product Content

As far as consumers go, there is an increasing amount of attention and concern based on whether a product is ingested (food and pharmaceuticals), worn or in immediate proximity (clothes, cell phones), or in the general environment (metals in electronic goods, carbon).

Various Regulations across the globe focused on hazardous content:

Europe: regulations focused on chemicals and raw materials in electronics and recycling

- RoHS (Restriction of Hazardous Substances Directive)
- WEEE (Waste Electrical and Electronic Equipment Directive)
- REACH (Registration, Evaluation, Authorization and Restriction of Chemicals)

China: Order No. 39 Final Measures for the Administration of the Control and Electronic Information Products (sometimes called the "China RoHS")
South Korea: Act for Resource Recycling of Electrical and Electronic Equipment and Vehicles
California: Electronic Waste Recycling Act of 2003 (EWRA).

Monitoring Credence Attributes

Credence attributes are those that are not testable by a downstream company or the consumer. There are different attributes that make up a product. Some are testable and some are not. Types that are testable by a downstream consumer might include the color or texture or the experience of a product like the sweetness or consistency.

Credence attributes are NOT testable. Those are attributes that are practices in the supply chain but are not testable such as carbon footprint, child labor, or food safety adherence.

How to handle a problem like credence:

- We need to measure, monitor, track, and trace
- For environmental evaluation, requires Life Cycle Analysis (LCA)
  - Product-centric, cradle-to-grave analysis
  - Boundaries (breadth, depth, precision)
- Most Common Method to handle credence: Labels
  - Standard Setting: Need agreed upon targets, terminology & definitions
  - Testing – objective measure
    - For testable attributes – describe method for assessment
    - For credence attributes - need to Track and Trace to preserve the "identity" of the product (thus need for LCA!)
  - Certification Process
    - Self-certification versus 3rd party certification
    - Certification of the external certifiers!
  - Enforcement of Standards
    - Legal versus social versus market . . .

Disclosure Regulations

One of the most obvious regulatory impact on supply chains are disclosure regulations. These are that require companies to disclose certain information about practices and processes in the supply chain.

Examples include the California Transparency in Supply Chain Act and UK's Modern Slavery Act.

For instance, companies of a certain size must disclose whether they have forced labor in their supply chain. Forces labor is all work or service which is exacted from any person under the threat of a penalty and for which the person has not offered himself or herself voluntarily (ILO).

Most Common Forms in Supply Chains

- Bonded labor - working to pay off a never-ending debt
- Forced labor - forced to work, cannot escape
- Migrant workers - no alternatives to survive
- Domestic workers - cannot leave the house

Common Industries:

- Agriculture
- Manufacturing
- Textile & Garments
- Mining
- Fishing
- Warehousing

Children in employment – all children engaged in any economic activity for $\geq$ 1 hour/week, for pay or not, part time or full time, inside or outside the family setting.
Children in child labor – subset of above – workers below the minimum age engaged for many hours/weeks not doing permitted "light work."
Hazardous work – subset of above – children engaged in work that is harmful to their safety, health and moral development. Includes night work, long hours, exposure to physical, psychological or sexual abuse; work underground, under water, at dangerous heights, in confined spaces, heavy loads, dangerous machinery, dangerous substances.

Comparing the Disclosure regulations

California Transparency in Supply Chains Act -

a company must disclose to what extent, if any:

- Engages in verification of product supply chains to evaluate and address risks of human trafficking and slavery. The disclosure shall specify if the verification was not conducted by a third party.
- Conducts audits of suppliers to evaluate supplier compliance with company standards for trafficking and slavery in supply chains. The disclosure shall specify if the verification was not an independent, unannounced audit.
- Requires direct suppliers to certify that materials incorporated into the product comply with the laws regarding slavery and human trafficking of the country or countries in which they are doing business.
- Maintains internal accountability standards and procedures for employees or contractors failing to meet company standards regarding slavery and trafficking.
- Provides company employees and management, who have direct responsibility for supply chain management, training on human trafficking and slavery, particularly with respect to mitigating risks within the supply chains of products.

Modern Slavery Act -

An organization's slavery and human trafficking statement may include information about:

- the organization's structure, its business and its supply chains;
- its policies in relation to slavery and human trafficking;
- its due diligence processes in relation to slavery and human trafficking in its business and supply chains;
- the parts of its business and supply chains where there is a risk of slavery and human trafficking taking place, and the steps it has taken to assess and manage that risk;
- its effectiveness in ensuring that slavery and human trafficking is not taking place in its business or supply chains, measured against such performance indicators as it considers appropriate;
- the training about slavery and human trafficking available to its staff.

Takeaways for supply chain managers:

- Requires mapping of supply chains
- Added vendor/supplier compliance vetting – inclusion of a "Supplier Code of Conduct"
- Risk assessment includes not only legal compliance, but for reputation and market risks
- Contracting and use of 3rd party auditing firms/organizations
- But labor is just one attribute of many that are becoming scrutinized
- Should firms treat them all equally or differ our response based on other factors?

Unintended Consequences – A cautionary Tale

As regulations are well meaning and are designed to improve conditions by controlling, practices and processes, they can sometimes have unintended consequences. For instance, the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010: *"An Act to promote the financial stability of the United States by improving accountability and transparency in the financial system, to end "too big to fail", to protect the American taxpayer by ending bailouts, to protect consumers from abusive financial services practices, and for other purposes."*

In Section 1502 – it required the disclosure of the use of conflict minerals. The country of origin is essentially a **credence** attribute for these minerals.

Supply Chain response for Dodd-Frank

- Where are the challenges?
  - Manufactured items can contain hundreds to thousands of parts/components and suppliers/vendors
  - Mineral content is usually not contained in BOM or ERP
  - Sourcing of materials is global and suppliers from other countries are not required to assist
  - Any 3TG found to be from the covered area requires much higher scrutiny and detailed reporting
- What could possibly go wrong?
  - Cost of compliance for manufacturer can be very high
    * Estimated ¿10 million paperwork hours annually spent by firms
    * Over $10 billion in additional costs annually (2016)
    * In 2014, Intel sent 90 employees to investigate smelters globally

- This has led to an initial *de facto* embargo of minerals from the DRC resulting in loss of revenue to legitimate mines in the area
- Continuing (or increased) sales of conflict minerals to firms in countries not operating under these regulations

Similar unintended consequences in Endangered Species Act of 1973 and Comprehensive Environmental Response, Compensation, and Liability Act of 1980 (superfund).

Product Oriented Regulations/Pressures

- **Safety** – ensure safe operations across supply chain
  - Examples: Hours of Service (HOS) Rules, Truck Size and Weight Restrictions, Driver Licensing, Electronic Logbooks (ELD)
  - Can influence design and operation of networks
  - Typically treated as absolute mandates or constraints
- **Waste Reduction** – promotes more efficient use of resources
  - Examples: Truck fuel efficiency, GHG emissions by trucks, Smart package sizing, Efficient mode selection (TL to IM conversion), Zero-waste facilities
  - Happy coincidence that reduction of waste (miles, cardboard, fuel) improves efficiency and usually reduces total costs
  - Typically treated as cost reductions in making better trade-off decisions
- **End-of-Life** – encourage companies to design products for reuse, recyclability, and materials reduction
  - Examples: EU directives and regulations on Packaging and packaging waste (1994), End of Life Vehicles (2000), Waste Electrical and Electronic Equipment (2003), Batteries and Accumulators (2006) etc.
  - Takes many different forms involving governments, manufacturers, and others
  - Introduces (imposes) entirely new processes to supply chains

For **End of Life Processes** there are two main approaches: **Product Stewardship** and **Extended Producer Responsibility** (EPR). Product-centric strategy is where everyone involved in the lifespan of the product is responsible for all environmental, health, and other impacts. EPR is Manufacturer centric strategy designed to promote the integration of environmental costs associated with goods throughout their life cycles into the market price of the products.

In relation to this there are **take-back laws** to prevent waste. Their primary goal is to prevent waste encourage companies to design products that are easier to reuse or recycle, contain safer materials, reduce their environmental impact, and minimize waste management costs. As well as Promote innovation in recycling technology & collection systems

Firm Response and Approach

A company needs to segment its response to these issues for strategy. They must segment potential responses and action based on pressure and importance. Responses are on a continuum from "Do nothing" to "Focus all Activities". A way of identifying what is important to the company and to the stakeholders is a materiality assessment, there are other similar frameworks for doing this.

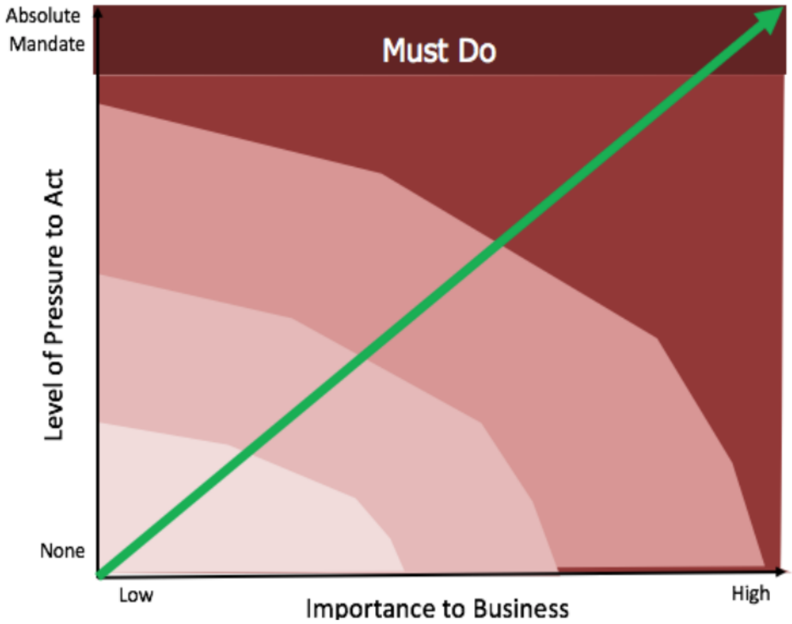

FIGURE 4.55: Pressure to Act Vs. Importance to Business

Do Nothing • No change to practices • Exogenous factor is not considered in design or operations
Avoid • Change in design or operations to avoid conflict, issue, or reporting need
Comply • Level of compliance can range from *bare minimum* to *exceeding standards* • Segment factors to identify level of compliance and focus - does not need to be same for all factors within a firm • Priority is based on strategy, brand-awareness, industry, product profile, customer markets, etc.
Embrace • Refocus supply chain to fully respond to exogenous factor • Treat these capabilities as a competitive advantage

TABLE 4.4: Company Action Continuum

Warning!

This lesson barely scrapes the surface of an ever-changing and increasingly important topic! It is not intended to make you an expert, just provide you with a framework to think about handling exogenous forces in your supply chain.

Learning Objectives

- Recognize there are many exogenous factors being placed on the supply chain including social, market, government, and natural.
- Identify types of regulations supply chains are held responsible and their impact on the supply chain.
- Become familiar with different product attributes like direct and credence.
- Review supply chain processes like safety, waste reduction, and end-of-life.
- Understand that firms can respond in different ways and are segmented by factor, market, etc.

4.5.4 Palm Oil Case Study

Palm Oil is a widely used oil in a large range of products. It is common in most CPG products because of its properties as well as packaged food products because of its shelf life and attributes as a non-trans fat. The reasons we spend so much time focusing on palm oil is because it brings together many of the exogenous factors we reviewed in Lesson 1 and culminates in different responses from different companies based on their stakeholders' demands and business interests.

Palm oil is with you all day long. See Figure 4.57.

The Rise of Palm Oil

- 1848 – oil palm, a native of West Africa was brought by Dutch Traders to SE Asia
- Found useful in soap and lubricant for steam engines, grew well in SE Asia
- Grew rapidly in 1960s in SE Asia, esp. in Malaysia where government sought to reduce the country's dependence on rubber and tin. Demand grew and plantations took over in Malaysia and Indonesia, today 85% of palm oil from these two countries

Together Malaysia and Indonesia make up over 85% of the market!

At the same time Palm oil has grown rapidly in demand, so has areas of forest cleared. Its clear that there are many reasons for forest cleared, but this shows some correlation between the two processes.

And in 2010 – Kit Kat, Nestle was attacked for their use of palm oil by Greenpeace. This propelled this issue into the public eye.

Palm Oil End to End

The Problem with Palm Oil

The rapid demand of palm oil has resulted in both positives and negatives. The positive impact of palm oil is that it provides economic growth to countries like Malaysia and Indonesia, it bring livelihoods to some that did not have viable livelihood previous to palm oil, it is more productive per tree than its nearest competitor, in fact 10x more productive meaning it takes up less space to produce the same amount of oil, and it is also not a trans fat.

FIGURE 4.56: Uses of Palm Oil

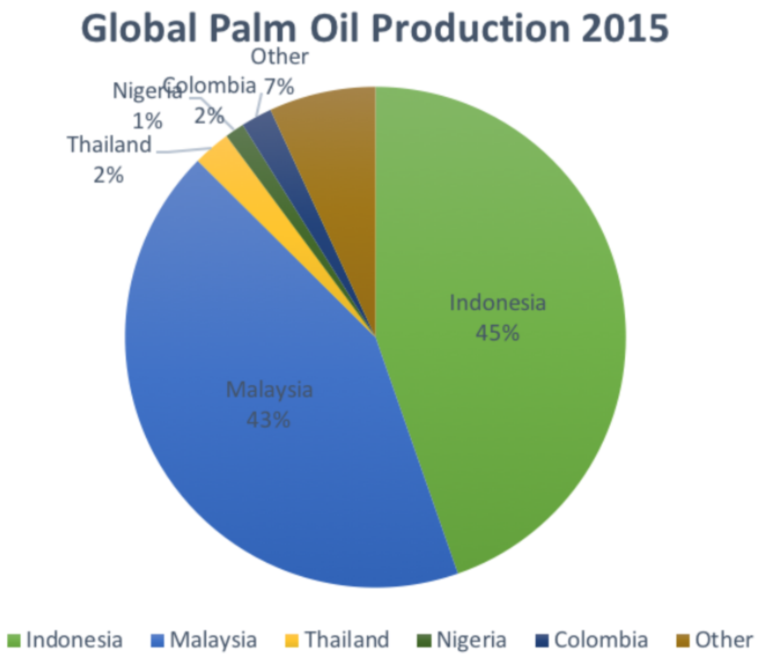

FIGURE 4.57: Palm Oil Production

Social and environmental Impacts

However, the rapid demand beginning especially in the 2000s has caused rapid deforestation that results in carbon emissions as a result of rainforest clearing, haze from burning of rainforest, loss of biodiversity as a result of rainforest clearing and more. There are also extensive social implications such as:

- Child and family labor and insufficient protection
- Frequent use of forced, bonded labor
- Use of fixed short term contracts
- Land grabs
- Complex and opaque supply chain

Types of Plantations

Palm oil is largely grown on 3 types of palm oil plantations including estates, associated or scheme smallholders and smallholders. Both estates and associated or scheme smallholders are generally more organized and easier to work with. Estates are often owned by larger corporations while associated or scheme smallholders are structurally bound by contract or credit agreement to a particular mill. Smallholders are generally self financed, managed, equipped, and not bound to any one mill.

Organization for Sustainability

There are many different types or organizations working towards more sustainable palm oil. They have different objectives, strategies, and approaches and often have conflict over what sustainable palm oil means. Some are advocating for the cause, some are catalysts for change, and some support business in their journey to achieve more sustainable palm oil.

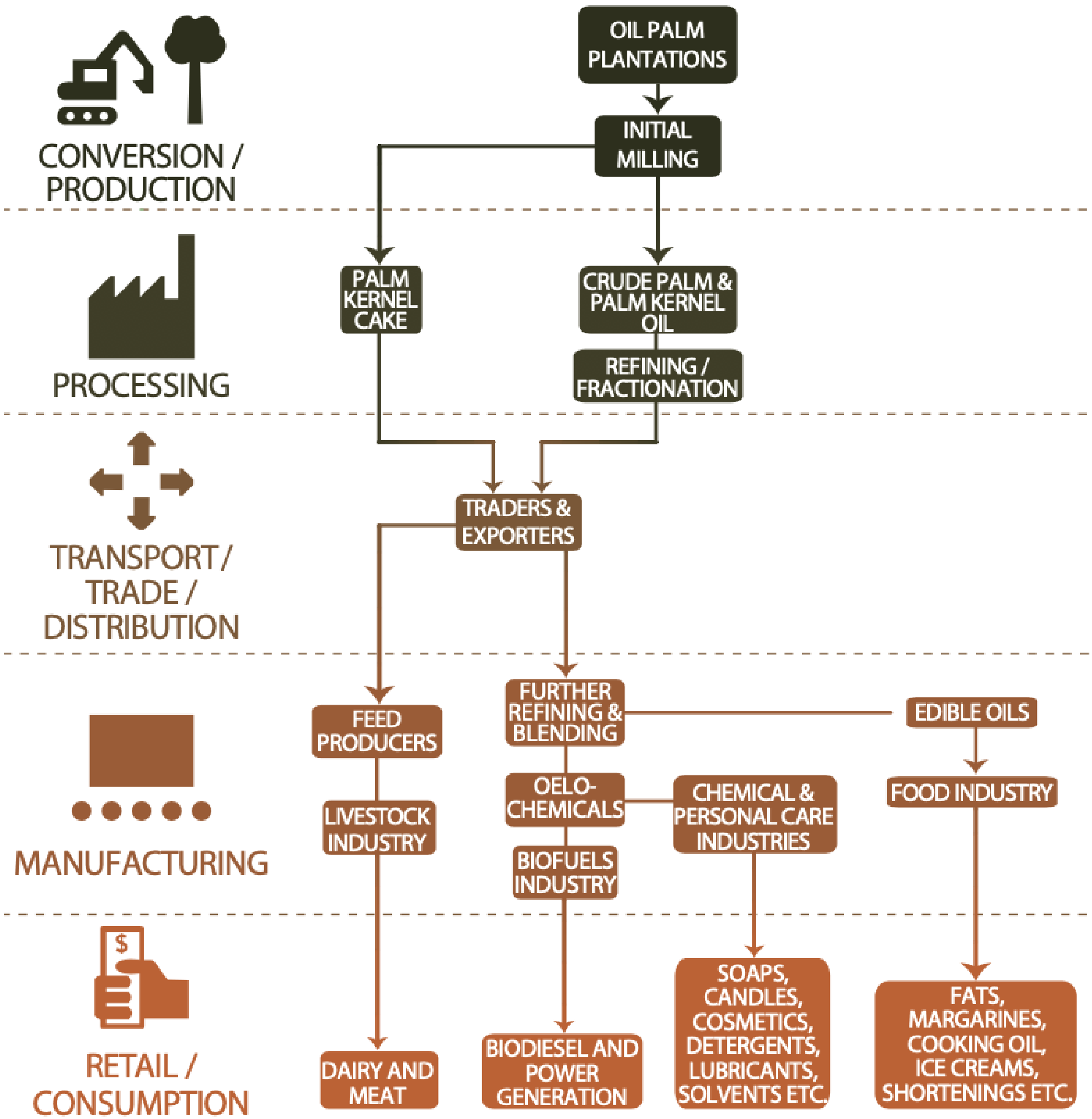

FIGURE 4.58: Palm Oil End to End Logistics

- NGOs
  - The Forest Trust
  - World Resources Institute
  - Wild Asia
  - Rainforest Action Network
- Industry associations
  - Roundtable on Sustainable Palm Oil (RSPO)
  - Palm Oil Innovation Group (POIG)
- And many more...

Different Scenarios for Palm Oil

- Many different scenarios as we can imagine (Scenario Planning)
  - All palm oil becomes sustainable, sustainable palm oil becomes cost competitive
  - Companies move away from palm oil as a key oil source
  - Continue on path with incremental changes
- This problem is not isolated, similar challenges with other materials:
  - Social pressure
  - Finite amounts
  - Regulation
  - Market Demand

Learning Objectives

- Recognize the different exogenous factors apparent in the palm oil supply chain
- Become familiar with the different forces being placed on the supply chain and the new expectations by society and consumers
- Become familiar with commodity supply chain challenges
- Recognize companies will respond in different ways based on their orientation, capacity, and long-term goals
- Identify different scenarios and how they might help company strategy

Part V

SC4x - Supply Chain Technology and Systems

5

Supply Chain Technology and Systems

CONTENTS

5.1 Looking to Data for Answers

5.1.1 Summary

Supply chains are moving at ever-faster rates with technology and systems supporting this movement. Data management plays a critical role in enabling supply chains to move at the speed and precision they do today, and the need for advanced data management will only continue. In recent years, there has been an explosion of information and this is especially true in supply chains. A few examples introduced include Amazon's massive supply chains selling 480 million unique items to 244 million customers while UPS is delivering 20 million packages to 8.4 million delivery points. This information is coming from multiple sources, in addition to sensors, the "internet of things", and regulations requiring increasing amounts of information. All of this information is commonly referred to as the "Big Data" challenge. Data is driving our modern world, but how can we be sure of it and use it most effectively? As we will review – data is messy, it requires cleaning and programming. Data is frequently trapped in silos coming from different sources, which makes working with it more challenging. In addition, data is big and getting even bigger daily. The tools we have all become comfortable with (spreadsheets) can no longer handle that amount of data, so we must use different tools to enable greater analysis.

5.1.2 The Cross Industry Standard Process for Data Mining (CRISP-DM)

One way to approach data from a business perspective is the "Cross Industry Standard Process for Data Mining (CRISP-DM). CRISP-DM is a framework for understanding the typical steps and workflow when working with real businesses, and their data, to solve problems and answer questions. CRISP-DM was developed collaboratively, and is intended to be a generalizable guide for planning successful data projects.

CRISP-DM outlines six unique steps.

- Business Understanding: Learning the project goals from the business perspective and translating these goals into technical questions that we can investigate.
- Data Understanding: Looking at the available to data to understand they type of records that were kept, the quantity, and any data quality problems.
- Data Preparation: Everything necessary to create the final data set with which you will execute your analyses. This can include cleaning, joining and transforming.
- Modeling: Choosing and calibrating different models in order to get insights from the data.
- Evaluation: Thoroughly reviewing the model construction, assumptions, and outputs in order to see if your work truly captures the necessary business rules and answers the business's questions.

- Deployment: Taking your analysis back to the company to act upon it. Deployment can take many different forms depending on the project and the technology available.

The CRISP-DM framework suggests a workflow with many opportunities for back-and-forth between steps as well as internal loops back to the beginning. Typically, a project will start at the Business Understanding step, and move to the Data Understanding step. However, back and forth between these two steps is common. Next comes Data Preparation, then Modeling. However, here back and forth is common as well. After a model has been selected, the next step in the process is Evaluation. After a successful evaluation, comes deployment. An unsuccessful evaluation could cause the analyst to start over again.

Learning Objectives

- When looking to data for answers, we can look to the CRISP-DM model as a guide to help us outline a potential workflow.
- The CRISP-DM framework is industry- and application-neutral. Not every project uses it, but it is generally representative.
- CRISP-DM outlines 6 basic steps.
- The CRISP-DM framework shows back and forth and between many of the six steps, as well as internal loops back to the beginning.

5.2 Data Modeling

5.2.1 Summary

Now that we have been introduced to data management and the issue of big data, we now deep drive into data modeling and learn how to design databases. Data modeling is the first step in database. Data modeling is the process of transitioning a logical model into a physical schema.

To understand the process of data modeling, we review several components including relational databases, data organization, data models for designing databases, and what constitutes a good data model. A data model consists of several parts including: entities and attributes, primary keys, foreign keys, and relationships and cardinality.

5.2.2 Relational Models

The relational model is an approach to managing data that uses structure and language where all data is grouped into relations. A relational model provides a method for specifying data and queries. It is based on first-order predicate logic, which was described by Edgar F. Codd in 1969. This logic defines that all data is represented in terms of tuples, grouped into relations. There are several definitions to be familiar with as we reviewed previously with relational models:

- **Entity:** object, concept or event
- **Attribute (column):** a characteristic of an entity
- **Record or tuple (row):** the specific characteristics or attribute values for one example of an entity
- **Entry:** the value of an attribute for a specific record
- **Table:** a collection of records
- **Database:** a collection of tables

Tables and Attributes

Data in relational tables are organized into tables, which represent entities. Single tables within a database can be seen as similar to a spreadsheet. However, we use different words to refer to "rows" and "columns". Attributes are the characteristics of an entity.
Tables

- Tables represent entities, which are usually plural nouns
- Tables are often named as what they represent (typically plural nouns, without spaces): e.g. Companies, Customers, Vehicles, Orders, etc.

Attributes

- Characteristics of an entity (table), typically nouns
- Examples in the form of: Table (Attr1, Attr2, ... AttrN), Vehicles (VIN, Color, Make, Model, Mileage)

Entity Types and Entity occurrence: an entity is any object in the system we want to model and store. An entity occurrence is a uniquely identifiable object belonging to an entity type.

5.2.3 Designing Data Models

There are several steps to designing a database to store and analyze data.

1. Develop a data model that describes the data in the database and how to access it
2. Define tables and attributes in the database (each important concept/noun in the data is defined as a table in the database)

Data models help specify each entity in a table in a standardized way. They allow the user to impose rules, constraints and relationships on the data that is stored. It also allows users to understand business rules and process and analyze data.

Rules for a Relational Data Model

- Acts as a schematic for building the database
- Each attribute (column) has a unique name within a table
- All entries or values in the attribute are examples of that attribute
- Each record (row) is unique in a good database
- Ordering of records and attributes is unimportant

What makes a good relational data model? A good relational model should be complete with all the necessary data represented. There should be no redundancy. Business rules should be effectively enforced. Models should also be reusable for different applications. And finally, it should be flexible and be able to cope with changes to business rules or data requirements.

5.2.4 Relationships and Cardinality

When we begin to work with the data, we have to understand how data relates to each other and how unique are the attributes. Some of this can be managed through entity types and attributes. Relationships + cardinality = business rules.

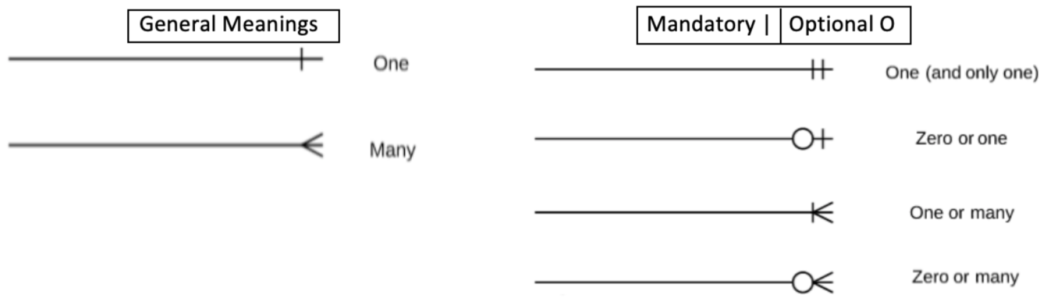

FIGURE 5.1: Cardinality – crow's foot notation

Entity and Attributes

An entity is a person, place, thing, or concept that can be described by different data. Each entity is made of a number of attributes. Entity types should be described as part of the data modeling process, this will help with the documentation and determination of business rules.

How to draw and entity-relationship diagram: An ERD is a graphical representation of an information system that visualizes the relationship between the entities within that system.

- ERD or entity-relationship diagram is a schematic of the database
- Entities are drawn as boxes
- Relationships between entities are indicated by lines between these entities
- Cardinality describes the expected number of related occurrences between the two entities in a relationship and is shown using crow's foot notation (see figures below)

Domain Validation Entities: Also known as "pick lists" or "validation lists". Domain validation entities are used to standardize data in a database, they restrict entries to a set of specified values. They are tables with a single attribute that enforces values of attribute in related table(s).

5.2.5 Keys

Primary keys are attributes used to uniquely identify a record while foreign keys are attributes stored in a dependent entity, which show how records in the dependent entity are related to an independent entity.

Primary key: one or more attributes that uniquely identify a record. The attribute has been uniquely suited.

Foreign Key: Primary key of the independent or parent entity type is maintained as a non-key attribute in the related, dependent or child entity type, this is known as the foreign key.

Composite key: is a primary key that consists of more than one attribute, ex: charter airline, every flight has a different number.

Many to Many Relationships: A many-to-many relationship refers to a relationship between tables in a database when a parent entity contains several child entity types in the second table. ex- Vehicle can be driven by many drivers, drivers can drive many vehicles. In this case an associative table (entity), aka junction table is appropriate where the primary key of parent is used in primary key of child.

Referential integrity

Referential integrity maintains the validity of foreign keys when the primary key in the parent table changes. Every foreign key either matches a primary key (or is null).

Cascade rules: choose among delete options

- Cascade restrict: Rows in the primary key table can't be deleted unless all corresponding rows in the foreign key tables have been deleted
- Cascade delete: When rows in the primary key table are deleted, associated rows in foreign key tables are also deleted

Learning Objectives

- The data model describes the data that is stored in the database and how to access it.
- Data models enable users to understand business rules and effectively process and analyze data.
- Understand that business rules are imposed on the database through relationships and cardinality.
- Recognize that data models may vary for a given dataset as business logic evolves.
- Remember that the data modeling process may reveal inconsistencies or errors in the data, which will have to be corrected before importing into a database.
- Selection of entities and associated attributes from a flat file is not always obvious.

5.3 Database Queries

5.3.1 Summary

As we continue our discussion of database management, we dive into the issue of database queries. The ability to make effective queries in a large database enables us to harness the power of big data sets. SQL (Structured Query Language) is a language and contains the commands we use to create, manage, and query relational databases. As in all technology and systems applications, there are a multitude of vendors who offer SQL variations, but in general they have a common set of data types and commands. SQL is portable across operating systems and in general, portable among vendors. Having covered the commonly used data types in previous lessons, in this next section we will cover very commonly used queries.

5.3.2 Structured Query Language

SQL is used to query, insert, update, and modify data. Unlike Java, Visual Basic, or C++, SQL is not a complete programming language; it is a sub-language of approximately 30 statement types. It is generally embedded in another language or tool to enable database access. A few definitions we need to be aware of as we explore SQL are:

- **Data definition:** Operations to build tables and views (virtual tables)
- **Data manipulation:** INSERT, DELETE, UPDATE or retrieve (SELECT) data
- **Data integrity:** Referential integrity and transactions enforce primary and foreign keys
- **Access control:** Security for multiple types of users
- **Data sharing:** Database accessed by concurrent users

Numeric Type	Description
INT	A standard integer
BIGINT	A large Integer
DECIMAL	A fixed-point number
FLOAT	A single-Precision, floating-point number
DOUBLE	A double-Precision, floating-point number
BIT	A bit field

TABLE 5.1: Core My SQL Data Types - Numeric

String Type	Description
CHAR	A fixed-length, non-binary string (character)
VARCHAR	A variable-length, non binary string
NCHAR	Same as above + Unicode Support
NVARCHAR	Same as above + Unicode Support
BINARY	A fixed-length, binary string
VARBINARY	A variable-length, binary string
TINYBLOB	A very small BLOB (binary large object)
BLOB	A small BLOB
TEXT	A small, non-binary string

TABLE 5.2: Core MySQL Data Types – Strings (Text)

A few issues to make note of as you work with SQL is that it has several inconsistencies. For example, NULLs can be problematic and we will explore that later. In addition, when working with SQL it is important to recognize that it is a declarative language, not procedural language. This means that you write the command in such a way that describes what you want to do, not HOW you want to do it. It is left up to the application to figure it out.

Variations among SQL Implementation

Because different databases use SQL, there can be variation in how SQL is implemented. The variations include:

- Error codes
- Data types supported (dates/times, currency, string/text variations)
- Whether case matters (upper, lower case)
- System tables (the structure of the database itself)
- Programming interface (no vendor follows the standard)
- Report and query generation tools
- Implementer-defined variations within the standard
- Database initialization, opening and connection

5.3.3 Creating Databases and Tables

To get started, we will need to know how to create databases and tables. While MySQL can be an intimidating program, but once you master some of the basics, you will be able to work effectively with large data sets.

Date / Time Type	**Description**
DATE	A date value in (CCYY-MM-DD) format
TIME	A time value in (hh:mm:ss) format
DATETIME	Date/Time in (CCYY-MM-DD hh:mm:ss) format
TIMESTAMP	Timestamp in (CCYY-MM-DD hh:mm:ss) format
YEAR	A year value in (CCYY or YY) format

TABLE 5.3: Core MySQL Data Types – Dates/Times

- To create a Database, we use the CREATE DATABASE command
- Once you have created the database, you will now apply the USE command to tell the system which database to use

Once you have created a database, you will want to create tables within the larger database:

- New tables are declared using the CREATE TABLE command
- We can also set the name and data type of each attribute
- When creating new tables, we can specify primary keys and foreign key relationships
- We can also decide whether or not NULL or empty values are allowed

Inserting Data into a new Database

Once you have created a new database, you are ready to insert data. The data model will act a guide to load data into a new database. If the database builds well, it may mean that you have found the real business rules. Or, if you have some errors, you may have the real business rules, but the data may be messy. Finally, if it builds with many errors – this may be the case that the business rules are not accurate. In many cases, it is useful to get sample data and browse it during the process of building the model.

SQL Select Queries

SQL SELECT query is used to fetch the data from a database table, which returns data in the form of a result table. SELECT returns a set of attributes in a query. In most applications, SELECT is the most commonly used data manipulation language command. SELECT statements are constructed from a series clause to get records from one or more tables or views.

Clauses must be in order; only SELECT and FROM are required:

- SELECT *attributes/columns*
- INTO *new table*
- FROM *table or view*
- WHERE *specific records or a join is created*
- GROUP BY *grouping conditions (attributes)*
- HAVING *group-property (specific records)*
- ORDER BY *ordering criterion* ASC — DESC
- DISTINCT *return distinct values*

Wildcards in SQL

Most database implementations offer additional regular expressions – wildcards. A wildcard character can be used to substitute for any other character(s) in a string. Regular expressions can be used to find records, which match complex string patterns. For instance, MySQL has:

- [*list*] match any single character in list, e.g. [a-f]
- [*list*] match any single character not in list, e.g. [^h-m]

Editing a Table

In some cases, you will be faced with the need to edit a table. In this case you will use the following:

- INSERT is used to add a new record to a table that contains specific values for a set of attributes in that table
- The UPDATE keyword is used to modify a specific value or set of values for a set of records in a table
- DELETE is used to remove records from a table that meet a specific condition

Learning Objectives

- Become more familiar with SQL.
- Recognize different implementations of SQL have differences of which to be aware.
- Review the different data types.
- Learn how to create new databases and tables.
- Understand how to use a SELECT query.
- Be familiar with wildcards and when to use them.
- Review how to edit a table.

5.4 Database Conditional, Grouping, and Joins

5.4.1 Summary

In the next section, we examine how to deal with database conditional, grouping, and joins. As we get further into SQL, we will need to refine our approach to make our actions more effective. For example, we will need to narrow the set of records that get returned from a query. We will also need to make statistical queries across different groupings or records. In addition, we will need to sample or order our output results. Another challenge will include integrating data from other sources within our database. The following review will cover these challenges and others as we continue to work with SQL.

5.4.2 Database Conditional Clauses

A conditional clause is a part of a query that restricts rows matched by certain conditions. You can narrow SELECT statements with conditional clauses such as WHERE IN, or the BETWEEN keyword. WHERE IN statements are used to identify records in a table with an attribute matching a value from a specified set of values. The BETWEEN keywords are used to identify records that have values for a particular attribute that fall within a specified range

WHERE IN: WHERE *attribute* IN is used to select rows that are satisfied by a set of WHERE conditions on the same attribute. Example:

```
SELECT *
FROM Offices
WHERE State IN ('CO', 'UT', 'TX');
```

is equivalent to:

```
SELECT *
FROM Offices
WHERE State = 'CO' OR State = 'UT'
OR State = 'TX'
```

BETWEEN Keyword: Select records where the attribute value is between two numbers using BETWEEN, range is inclusive and also works with time and date data.

Null Values

Null values are treated differently from other values; they are used as a placeholder for unknown or inapplicable values. If values are empty or missing, they are stored as NULL. A few issues to be aware of for NULL values:

- NULL values evaluate to NOT TRUE in all cases
- Check for NULLS using IS and IS NOT
- When a specific attribute may contain NULL or missing values, special care must be taken when using conditional clauses

Grouping Data and Statistical Functions

Once you are a bit more comfortable working with SQL, you can start to explore some of the statistical functions that are included in many implementations of SQL. These functions can operate on a group of records. Using the GROUP BY clause will return a single value for each group of records. To further restrict the output of the GROUP BY clause to results with certain conditions, use the HAVING key words (analogous to the WHERE clause).

Aggregate Statistical Functions in SQL

Commonly used functions include:

Name	Description
AVG()	Return the average value of the argument
COUNT()	Return a count of the number of rows returned
COUNT(DISTINCT)	Return the count of the number of different values
MAX()	Return the maximum value
MIN()	Return the minimum value
STD()	Return the population standard deviation
STDDEV_SAMP()	Return the sample standard deviation
SUM()	Return the sum
VAR_POP()	Return the population standard variance
VAR_SAMP()	Return the sample variance
VARIANCE()	Return the population standard variance

TABLE 5.4: Commonly Used SQL Functions

More advanced statistical functions can be created using the basic statistical functions built into SQL such as calculating the weighted average or getting the z-score values by combining different functions.

5.4.3 Sorting and Sampling Data

You will also be faced with the need to sort and sample the data. Several clauses will help you will that including ORDER BY, LIMIT, and RAND.

ORDER BY: The ORDER BY clause specifies that the results from a query should be returned in ascending or descending order

LIMIT the number of returned records: A LIMIT clause restricts the number of records that would be returned to a subset, which can be convenient for inspection or efficiency

Randomly select and order records: The RAND() function can be used to generate random values in the output or to randomly sample or randomly order the results of a query. For instance:

Reorder the entire table:

```
SELECT *
FROM table
ORDER BY RAND();
```

Randomly select a single record:

```
SELECT *
FROM table
ORDER BY RAND()
LIMIT 1;
```

Generate a random number in the output results:

```
SELECT id, RAND()
FROM table;
```

Creating New Tables and Aliases

AS Keyword (Aliases): The AS keyword creates an alias for an attribute or result of a function that is returned in a query

CREATE TABLE AS: Use CREATE TABLE with AS to create a new table in the database using a select query. It matches columns and data types based on the results in the select statement.

Results from a query can be inserted into a new table using the CREATE TABLE with the AS keyword. As seen in the following:

CREATE TABLE new_table
AS (SELECT column_name(s)
FROM old_table);

SELECT INTO: Results from a query can be inserted into an existing table using a SELECT INTO clause if the table with the appropriate structure already exists.Take the results of a select statement and put them in an existing table or database:

SELECT column_name(s)
INTO newtable [IN externaldb]
FROM table1;

5.4.4 Joining Multiple Tables

The relational database model allows us join multiple tables to build new and unanticipated relationships. The columns in a join must be of matching types and also must represent the same concept in two different tables. This can help us to contextualize or integrate a table in our database with data from an external source.

We want to learn how to take data from different tables and combine it together. This may include data from other data sources that complement our own, such as demographic information for a zip code or price structure for shipping zones for a carrier. The process of merging two separate tables is called "joining". Joins may be done on any columns in two tables, as long as the merge operation makes logical sense. See below for a visual representation of joining:

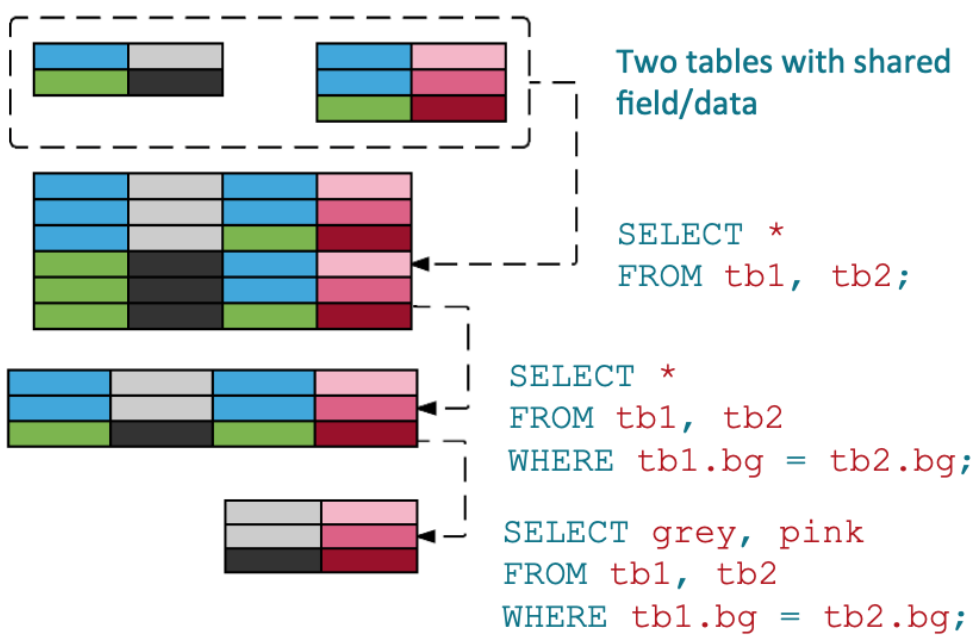

FIGURE 5.2: Joining tables using SQL

Columns in a JOIN

- They don't need to be keys, though they usually are
- Join columns must have compatible data types
- Join column is usually key column: Either primary or foreign
- NULLs will never join

Types of Joins and Views

Join from 3 Tables: Joining three tables together just involves one additional join between two already joined tables and a third table.

Join Types
Different types of joins can be used to merge two tables together that always include every row in the left table, right table, or in both tables. The following are different types of JOIN:

- INNER JOIN: returns only the records with matching keys (joins common column values)
- LEFT JOIN: returns all rows from LEFT (first) table, whether or not they match a record in the second table
- RIGHT JOIN: returns all rows from RIGHT (second) table, whether or not they match a record in the first table
- OUTER JOIN: Returns all rows from both tables, whether or not they match (Microsoft SQL, not MySQL)
- In MySQL, JOIN and INNER JOIN are equivalent

Views

Views are virtual tables that do not change the underlying data but can be helpful to generate reports and simplify complicated queries. They are virtual tables that present data in a denormalized form to users. They do not create separate copies of the data (they reference the data in the underlying tables). The database stores a definition of a view and the data is updated each time the VIEW is invoked.

There are several advantages to VIEWS. User queries are simpler on views constructed for them. They offer a layer of security that can restrict access to data in views for users. They also provide greater independence, meaning that the user or program is not affected by small changes in underlying tables.

Learning Objectives

- Learn how to work with SELECT for conditional clauses.
- Recognize the role and use of NULL values.
- Review how to group data with the GROUP BY clause.
- Introduce the existence of statistical functions in all SQL functions.
- Recognize how to apply aggregate statistical functions.
- Review sorting and sampling techniques such as ORDER BY, LIMIT and RAND.
- Learn how to create new tables and aliases using the AS keyword.
- Becoming familiar with joining multiple tables.

- Recognize the types of JOINs.
- Identify when and how to use VIEWs.

5.5 Topics in Databases

5.5.1 Summary

In this section, we will be covering some additional database topics. These topics are going to help SC professional apply databases in the real world. To learn on how to optimize performance in relationship database, this section will introduce you to indexing and its impact on performance. Then, you will learn another type of database that is recommended for cases where analytics and reporting need to be performed to help make data driven decisions. We will then move away from relationship database and explore NoSQL database. NoSQL database can offer flexibility and superiors performance for some type of applications especially in cases where data is unstructured. The next topic that we'll look at is what are the benefits of storing data and processing data on the cloud. Finally, we'll think about how to pre-process and clean our data sets before we put them into databases. This skill will be essential for us to make sure that we actually store and retrieve reasonable data every time we use our database.

5.5.2 Indexes and Performance

The best way to improve the performance of SELECT operations is to create indexes on one or more of the columns that are tested in the query. The index entries act like pointers to the table rows, allowing the query to quickly determine which rows match a condition in the WHERE clause, and retrieve the other column values for those rows. All MySQL data types can be indexed. Primary and Foreign Keys are automatically indexed by SQL to allow them to be searched faster.

Although it can be tempting to create an index for every possible column used in a query, unnecessary indexes waste space and waste time for MySQL to determine which indexes to use. Indexes also add to the cost of inserts, updates, and deletes because each index must be updated. You must find the right balance to achieve fast queries using the optimal set of indexes.

Example of Indexing

Consider a Customer table that contains the following columns: Customer ID, Customer name, City, state, Postal (zip) code and Address. Customer_ID, here, is the primary key. Let us assume that we want to index State and City so that one can quickly narrow down the lists of customers in a particular state and city. This can be done by using the below syntax where IX_City_State is just the name of the index that will be created on the table "Customer" and the attributes are "State" and "City"
CREATE INDEX IX_City_State ON Customer (State, City)

5.5.3 Databases and Data Warehouses

So far, we have looked at one type of databases that are known as online transaction processing, or OLTP. There is another class of database that you should be familiar with. These are known as online analytical processing or OLAP. OLTP and OLAP, each have different use case and purpose. The below tables list down the common use case and key differences between these databases:

5.5.4 NoSQL Database

A NoSQL database provides a mechanism for storage and retrieval of data that is modeled in means other than the tabular relations used in relational databases. Unlike SQL, the data in NoSQL database

OLTP	OLAP
Manage real-time business operations	Perform analytics and reporting
Support implementation of business tasks	Support data driven decision making
e.g. Transaction in an online store	e.g. Data Mining and Machine Learning
e.g. Dashboard showing health of business over the last few days/hours	e.g. Forecasting based on historic data
Concurrency is paramount	Concurrency might not be important

TABLE 5.5: Commonly Used SQL Functions

OLTP	OLAP
Optimized for writers (updates and inserts)	Optimized for reads
Normalized using normal forms, few duplicates	Denormalized, many duplicates
Few index, many tables	Many indexes, fewer tables
Optimized for simple queries	e.g. Used for complex queries
Uncommon to store metrics which can be derived from existing data	Common to store derived metrics
Overall performance: Designed to be very fast for small writes and simple reads	Overall performance: Designed to be very fast for reads and large writes, however relatively slower because data tends to be much larger

TABLE 5.6: Commonly Used SQL Functions

is unstructured and is not stored in tables. NoSQL databases uses other mechanisms to store data, some of which are discussed in the next section.

A NoSQL database offers the following benefits over a SQL database:

- NoSQL may be faster in some use cases. Simple reads and writes are very fast with NoSQL solutions
- NoSQL databases are flexible compared to SQL database sand allow storage of data when the relationship between many of entities is not clear.
- NoSQL databases are easy to scale because of their simplified structure.

Transactional business data is still commonly stored in relational databases due to consistency and how reads and writes are handled

Types of NoSQL database

Please see the figure 5.3

5.5.5 Cloud Computing

Cloud computing enables users and enterprises to access shared pools of resources (such as computer networks, servers, storage, applications and services) over the internet. Cloud computing offers various benefits over traditional systems, including:

Key-value database

- The least complex NoSQL option, which stores data in a schema-less way that consists of indexed keys and values.
- Each record may have different data fields or attributes, which are stored together with a unique key
- Data model is not predefined, empty fields are not stored

Key	Value
1000	Chris
1001	Julie
1002	Clark

Key	Value
1000	{name: "Chris", language: "English", city: "Boston", state: "MA"}
1001	{name: "Julie", state: "NY"}
1002	{name: "Clark", language: "French", language: "Spanish"}

Document-Oriented database

- Similar to key-value concept but adds more complexity.
- search document (grouping of key value pairs, also known as collection) in this type of database has its own data, and its own unique key, which is used to retrieve it.

Employees	
Key	Value
1000	{name: "Chris", language: "English", city: "Boston", state: "MA"}
1001	{name: "Julie", state: "NY"}
1002	{name: "Clark", language: "French", language: "Spanish"}

Offices	
Key	Value
10000	{name: "Boston Office", city: "Boston", state: "MA", employee: "Chris", employee: "Clark"}
10001	{name: "New York Office", state: "NY", employee: "Julie"}

Column-oriented database

- Stores data tables as columns rather than rows. Each row is indexed and thus has "RowID"
- "RowID" acts as the "key" and the attribute acts as the "value"
- Now information about each column is stored more efficiently for some queries, allowing for excellent scalability and high performance.

Employees		
RowID	First Name	State
1	Christopher	Massachusetts
2	Julie	New York
3	Christopher	New York

First Name	RowID
Christopher	1, 3
Julie	2

State	RowID
Massachusetts	1
New York	2, 3

Graph database

- Stores data that's interconnected and best represented as a graph
- This method is capable of lots of complexity

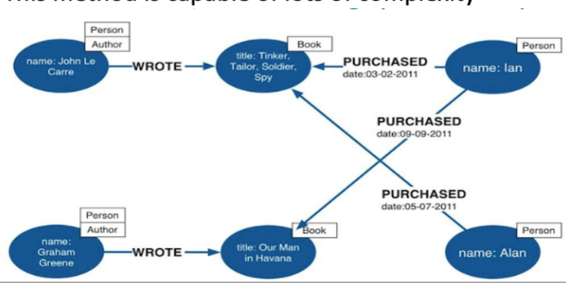

FIGURE 5.3: Types of NoSQL Databases

Infrastructure as a service (IaaS)	• Outsource hardware • User provides operating system, database, app, etc. • Most flexible, higher upfront total IT cost • Example: Rackspace
Platform as a service (PaaS)	• Outsource operating environment • Cloud platform provides OS, database, etc. • Examples: Amazon, Microsoft, Google
Software as a service (SaaS)	• Outsource software • Configure the third-party app • Examples: Salesforce, HubSpot • Least flexible, lower upfront total IT cost

FIGURE 5.4: Types of Cloud Computing

- Low start-up cost
- Low risk development and testing
- Managed hardware (and software)
- Global reach
- Highly available and highly durable
- Scale on demand, pay for what you use
- In some cases, can work with local infrastructure if needed

Types of Cloud Computing

5.5.6 Data Cleaning

Data must be cleaned or pre-processed before it can be inserted into a database. This is a time consuming but a mandatory exercise, as the quality of the database depends on the integrity of the data.

Types of data cleaning solutions

Data cleaning can be performed with free software and tools, however the learning curve for these can be steeper than that of commercial software. These options are listed below:

Learning Objectives

- Understand Indexing and its impact on performance.
- Introduce online transaction Processing(OLTP) and online analytical processing (OLAP)
- Review use cases and key differences of OLTP and OLAP
- Introduce NoSQL database
- Review various types of NoSQL databases.
- Introduce Cloud computing and review types of services offered through cloud.
- Become familiar with advantages of cloud computing.
- Recognize importance of data cleaning

Off the shelf software	• Graphical user interfaces, no programming required, enables collaboration with non-programmers • Offer various advance features including capability to operate on large data set, reproducible workflow and version control • Software is not free • Examples: Trifacta, Paxata, Alteryx, SAS
Open-source programming languages	• Working in data frames and data dictionaries, requires some programming skills, but languages are relatively google-friendly • Offers the same benefits as "Off the shelf software" but requires programming experience and can be sometime time consuming. • Software is free
Unix command line tools	• Not as user friendly as previous options, but very fast and versatile • Excellent for breaking up large datasets that would crash other software • Software is free and often built in the OS.

FIGURE 5.5: Types of Data Cleaning Solutions

5.6 Introduction to Machine Learning

5.6.1 Summary

In this lesson, we explore machine learning. This includes identifying when we need machine learning instead of other techniques such as regression. We break down the different classes of machine learning algorithms. In addition, we identify how to use machine-learning approaches to make inferences about new data.

Review of Regression

Linear regression uses the value of one or more variables to make a prediction about the value of an outcome variable. Input variables are called independent variables and the output variable is known as the dependent variable.

- Linear regression output includes coefficients for each independent variable.
  - This is a measure of how much an independent variable contributes to the prediction of the dependent variable.
  - The output also includes metrics to be able to assess how the model fits the data. The better fit of the model, the better you are able to make accurate predictions about new data.
- Using coefficients calculated from historic data, a regression model can be used to make predictions about the value of the outcome variable for new records.

5.6.2 Overview of Machine Learning Algorithms

Machine learning algorithms are primarily use to make predictions or learn about new, unlabeled data. There are several classes of algorithms:

- **Classification:** assigning records to pre-defined discrete groups

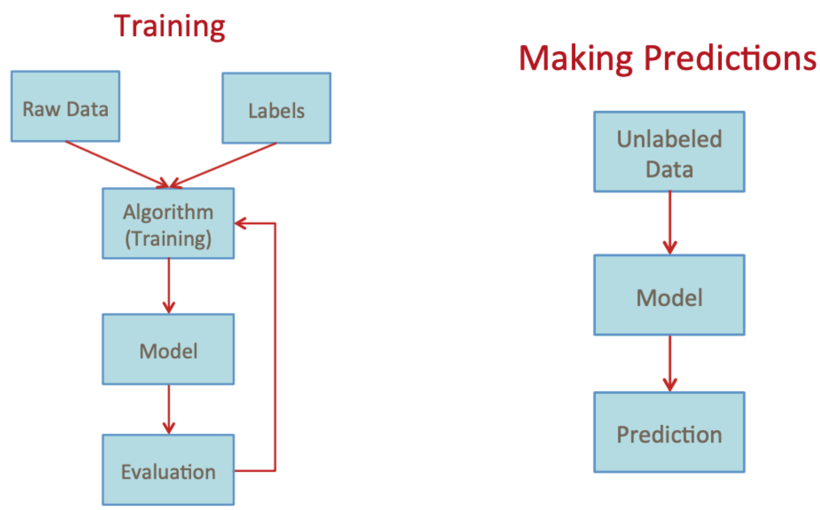

FIGURE 5.6: Learning Flow for supervised machine learning

- **Clustering:** splitting records into discrete groups based on similarity; groups are not known *a priori*
- **Regression:** predicting value of a continuous or discrete variable
- **Associate learning:** observing which values appear together frequently

Supervised vs. Unsupervised Machine Learning

Supervised learning uses outcome variables, known as labels, for each record to identify patterns in the input variables or features related to the outcome variable.

- Correct answer, label is known in the training data
- Label is applied by a person or already exists
- Labeled data are used to train an algorithm using feedback
- Apply or test the trained model on new, unseen data to predict the label

In **unsupervised learning**, the outcome variable values are unknown, therefore relationships among the input variables are used to identify patterns of clusters of records.

- Finds previously unknown patterns in the data without labels or guidance
- No training/testing/validating process because correct answer is unknown

5.6.3 Model Quality

Machine learning models should be trained on an unbiased set of data that is representative of the variance in the overall dataset. Bias quantifies the lack of ability of a model to capture underlying trend in the data. More complex models decrease bias but tend to increase variance. Variance quantifies a model's sensitivity to small changes in the underlying dataset.

- Ideally, we want low bias and low variance, but there is a trade-off between the two quantities
- If there is a bias in the training data or if too many features are included in a model, the model is at risk of being overfit. In overfit models, the coefficients, known as parameters, will not be generalizable enough to make good predictions for new records.

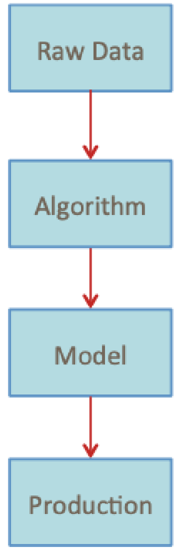

FIGURE 5.7: Learning Flow for unsupervised machine learning

- A large and representative sample of the labeled data should be used to train the model, the remainder is used for testing.

Overfitting vs underfitting

- Underfitting – model is too simple, high bias and low variance
- Overfitting – model is too complex, low bias and high variance
  - Overfitting is a more common pitfall
  - Tempting to make models fit better by adding more features
  - Results in a model that is incapable of generalizing beyond the training data

Learning Objectives

- Be introduced to machine learning
- Become familiar with different types of machine learning algorithms
- Be able to differentiate supervised and unsupervised learning and their processes
- Recognize model quality and the tradeoffs between bias and variance
- Learn how to identify when a model is over or underfit

5.7 Machine Learning Algorithms

5.7.1 Summary

In this lesson, we are going to dive deeper into machine learning algorithms. Each model has different properties and is best for different types of tasks. We review how to compare them with performance metrics. We need to be able to group records together without labels to inform prediction using unsupervised classification. In addition, we review capability to confidently reduce the number of features included in an analysis without losing information. The lesson also introduces how to compare predictor accuracy and test for sensitivity and specificity.

5.7.2 Dimensionality reduction

Dimensionality reduction is a term for reducing features included in analysis. It is often needed for analysis with many features. Trying to reduce dimensionality randomly or manually leads to poor results

- Results need to be interpreted by humans, should be tractable
- Increasing the number of features included increases the required sample size
- Features should not be included or discarded from analysis based on instinct
  - Dimensionality reduction techniques should be employed, such as principal component analysis.
- Summary statistics are a means of dimensionality reduction

5.7.3 Principal component analysis (PCA)

PCA is a mathematical approach to reduce dimensionality for analysis or visualization. It exploits correlations to transform the data such that the first few dimension or features contain a majority of the information of variance in the dataset. PCA determines which variables are most informative based on the distribution of data and calculates the most informative combinations of existing variables within the dataset. PCA works well for datasets with high dimensionality.

- No information is lost, first few components hold much of the information
- Same premise as linear regression except without a dependent variable
  - Linear regression solution is the first principal component
  - Disregarding the information describing the principal component, PCA calculates the second most informative component, then the third, and so on
- Linear combinations form a set of variables that can be used to view the data – new axes
- Components are ranked by importance, so all but the first few can be discarded, leaving only the most important information with very few components
- The coefficients in the table give the proportion of each of the original variables that went into each component
- Relative signs +/- indicate that two variables are positively negatively correlated in that particular component
- The components are difficult to interpret using only the coefficient values, plotting often improves understanding

$$PC1 = (a * var1 + b * var2 + c * var3 + \dots)$$
$$PC2 = (d * var1 + e * var2 + f * var3 + \dots)$$
$$PC3 = (g * var1 + h * var2 + i * var3 + \dots)$$

5.7.4 Clustering

Another way of thinking about dimensionality reduction is how close each point is to other points. The idea is to separate data points into a number of clusters that have less distance between the points internally than to other clusters. Clustering can be helpful to identify groups of records that have similar characteristics to one another. When data is unlabeled, clustering can be used to group records together for deeper inspection. Upon deeper inspection of the records in each cluster, users can understand the patterns that lead to records being grouped together, and also identify reasons for records being grouped separately.

K-means clustering

k-means clustering starts with selecting the number of clusters, k. k cluster-centers are placed randomly in the data space and then the following stages are performed repeatedly until convergence. K-means does not determine the appropriate number of clusters, this is set by the user based on intuition or previous knowledge of the data The algorithm can terminate with multiple solutions depending on initial random positions of cluster-centers and some solutions are better than others.

- Data points are classified by the center to which they are nearest
- The centroid of each cluster is calculated
- Centers are updated to the centroid location

5.7.5 Classifications

- Clustering and PCA allow users to see patterns in the data, which is the best that can be done because there are no labels to guide the analysis
- With supervised learning, the label is included in the learning process:
  - Unsupervised: what features are most important or interesting?
  - Supervised: what features are most informative about the differences between these groups?
- Classification methods: each record falls into some category or class, predict the category of a new record based on values of other features in the record
- Regression methods: one variable depends on some or all of others, predict the value of the dependent variable based on the values of the independent variables

Classification Trees

Classification trees split data to find optimal values for features, used to split data by class. Tree diagrams show the class makeup of each node, and the relative number of data points that reach each node

- Tree pruning
  - Tree pruning removes rules associated with overfitting from the tree
  - The new tree misses a few points classified correctly, but contains only meaningful rules, more generalizable to new data

The Naïve Bayes classifier

- The Naïve Bayes algorithm considers the value of each feature independently, for each record, and computes the probability that a record falls into each category
- Next, the probabilities associated with each feature are combined for each class according to Bayes' rule to determine the most likely category for each new record
- Almost completely immune to overfitting - Individual points have minimal influence; Very few assumptions are made about the data

Random forest

Random forest is an ensemble classifier that uses multiple different classification trees. Trees are generated using random samples of records in the original training set. Accuracy and information about variable importance is provided with the result.

- No pruning necessary
- Trees can be grown until each node contains very few observations
- Better prediction than classification
- No parameter tuning necessary

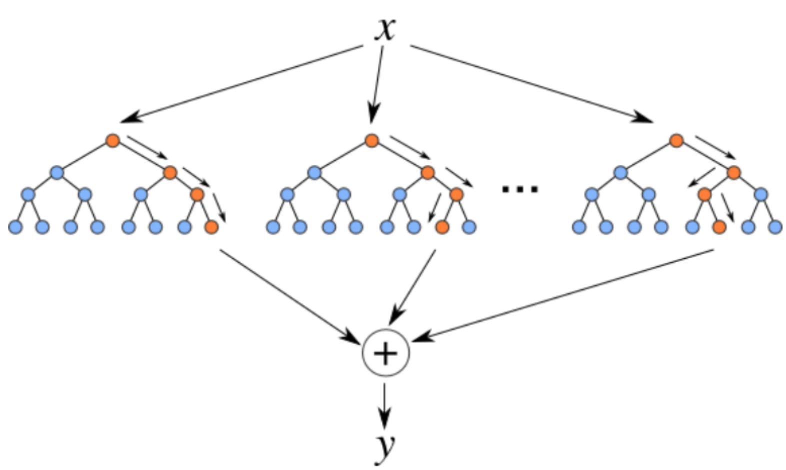

FIGURE 5.8: Random Forest

5.7.6 Comparing predictor accuracy

Cross - validation

- Models should be good at making classifications of unlabeled data, not describing data that is already classified.
- Randomly divide data into a training set and a test set
  - Hide test set while building the tree
  - Hide training set while calculating accuracy
  - Computed accuracy represents accuracy on unseen data
- Techniques are available to do this multiple time, ensuring each record is in the test set exactly once, e.g. k-folds

Comparing models

- Several standard measures of performance exist, can run multiple models and compare metrics:
  - Accuracy
  - Precision
  - Recall
  - And more
- Application drives which performance metrics are most important for a given task

5.7.7 Sensitivity and specificity

Sensitivity and specificity are statistical measures of the performance of a classification test. Sensitivity measures the proportion of positives results that are categorized correctly. Specificity measures the proportion of negatives results that are categorized correctly. So for example, if you needed to measure, say, the ability of a test to always detect a problem, you would look at sensitivity. Or if you need to look at the performance of those classifiers at not generating false positive classifications, then specificity. See example below:

	Disease				
Test	**Present**	**n**	**Absent**	**n**	**Total**
Positive	True Positive (TP)	*a*	False Positive (FP)	*c*	*a* + *c*
Negative	False Negative (FN)	*b*	True Negative (TN)	*d*	*b* + *d*
Total		*a* + *b*		*c* + *d*	

Sensitivity	$\frac{a}{a+b}$	**Specificity**	$\frac{d}{c+d}$
Positive Likelihood Ratio	$\frac{\text{Sensitivity}}{1 - \text{Specificity}}$	**Negative Likelihood Ratio**	$\frac{1 - \text{Sensitivity}}{\text{Specificity}}$
Positive Predictive Value	$\frac{a}{a+c}$	**Negative Predictive Value**	$\frac{d}{b+d}$

FIGURE 5.9: Performance Metrics for Machine Learning

The ROC Curve

- The Receiver Operating Characteristic (ROC) curve plots the true positive rate (Sensitivity) versus the false positive rate (100 - Specificity) for different cut-off points
- Each point on the curve represents a pair of sensitivity/specificity values corresponding to a particular decision threshold

- A test with perfect discrimination (no overlap in the two distributions) has an ROC curve that passes through the upper left corner (100% sensitivity, 100% specificity)
- The closer the ROC curve is to the upper left corner, the higher the overall accuracy of the test

Confusion Matrix

A confusion matrix is a table that is often used to describe the performance of a machine learning algorithms on a set of test data for which the true values are known. Below is an example confusion matrix:

	Predicted High	**Predicted Low**
Actual High	100	20
Actual Low	10	200

TABLE 5.7: Confusion matrix example

The table tells us the below about the classifier:

- There are two possible predicted classes: ”high” and ”low”. If we were predicting the average spending by a user then a “high” will imply these customers are high spender, and “low” will mean they are low spender.
- The classifier made a total of 330 predictions
- Out of those 330 cases, the classifier predicted ”high” 110 times, and ”low” 220 times.
- In reality, 120 patients in the sample are high spenders, and 210 are low spenders.

Learning Objectives

- Be further introduced to machine learning algorithms and how to work with them
- Become familiar with dimensionality reduction and when and how to use it
- Recognize when to use clustering as an approach to dimensionality reduction
- Review different classification methods such as classification tress and random forest
- Learn how to compare predictor accuracy
- Become familiar with sensitivity and specificity as indicators of a binary classification test

5.8 Supply Chain Systems - ERP

5.8.1 Summary

In this next segment, we explore supply chain IT systems. Supply chains are essentially made up of three flows: information, material, and money. IT systems support the information flow. For example, in a supermarket, the systems have to deal with different types of supply chain data such as supplier inventory, facility management and payroll, sales, and expired and obsolete inventory. There are many daily transactions in a supermarket that need to be captured and ensured for consistency and timeliness. That information needs to be somehow translated into usable information for business decisions, and then these objectives need to be efficiently achieved. The amount of information for transactions per week in a single supermarket can number in the millions. This is for a single store.

On an enterprise level, companies need systems that help them manage and organize this information for use. While supply chains are always portrayed as neat and linear systems, the reality is much different, as we have learned over the previous courses. Flows move up and down the chain and through many partners until they reach their final destination. Supply chains need IT systems because while teams may sit in different functional units they frequently need to share information. In addition, many different firms interact in the supply chain, they need systems to carry that information between them, this helps de-silo the supply chain. There needs to be coordination across functions, which is the essence of supply chain management and can be facilitated with systems like Enterprise Resource Planning (ERP).

5.8.2 Supply Chain IT Systems

Supply chains need IT systems because they are large, complex and involve many players. They often become intertwined and individual actors impact others. Decision-making is based on common data and interaction with other functions in a firm. IT systems enable seamless and instantaneous communication for B2B, B2C, M2M and many other interactions. (B2B = business to business, B2C = Business to Consumer, M2M = machine to machine)

Enterprise Resource Planning (ERP) systems serve as a general ledger and central database for all firm activity. The next are **Supply Chain Planning Systems.** These systems are primarily for production planning and scheduling, demand planning and product lifecycle management. The last are for **Supply Chain Execution;** which are transportation and warehouse management systems and manufacturing systems. The first we will tackle are Enterprise Resource Planning systems.

5.8.3 Enterprise Resource Planning

In the following section we cover why firms use ERPs; the core functions of ERPs; data needed; communications of the systems; and strategic benefits of an ERP. Most firms have an ERP because many functions in a firm such as sales, inventory, production, finance, distribution, and human resources have siloed databases. With a centralized ERP, these databases can more easily be exchanged and shared.

Benefits of ERPs allow enterprises to organize processes and data structure, integrate information into a unified repository, make data available for many users, eliminate redundant systems and data, reduce non-value-added tasks, standardize process designs, as well as be more flexible. But there are significant drawbacks to using ERP too. These include: significant implementation time and maintenance that come at a cost, data errors ripple through systems, competitive advantage can be dampened, firm reliance on a single vendor, shortage of personnel with technical knowledge of system, and high impact of down time of said system.

ERP Core Functions

Most ERP Systems share the same core functions. They tie together and automate enterprise-wide basic business processes:

Customer Management is the face to consumers and serves the following functions:

- enables order entry, order promising, open order status
- allows marketing to set pricing schemes, promotions, and discounts
- provides real-time profitability analysis,
- permits order configuration, customer delivery schedules, customer returns, tax management, currency conversion, etc.

Manufacturing is the face to production and serves the following functions:

- includes MRP processing, manufacturing order release, WIP management, cost reporting, shop floor control etc.,

- provides real time linkage of demand to supply management enabling real time Available-to-Promise (ATP) & Capable-to-Promise (CTP),
- serves as primary interface to “bolt-on” advanced planning and scheduling optimization modules.

Procurement is the face to suppliers and serves the following functions:

- integrates procurement with supplier management,
- facilitates purchase order processing, delivery scheduling, open order tracking, receiving, inspection, and supplier performance reporting, and
- creates requests for quotation (RFQ)
- manages negotiation and pricing capabilities.

Logistics is the face to internal and external supply chain and serves the following functions:

- runs the internal supply chain for enterprise,
- provides connectivity to remote trading partners (3PLs, carriers, etc.),
- handles distribution channel configuration, warehouse activity management, channel replenishment, planning, distribution order management, etc., and
- serves as primary interface to “bolt-on” warehouse and transportation management systems (WMS and TMS).

Product Data is the face to all material and serves the following functions:

- describes products manufacturer and/or distributor,
- contains proprietary data on costs, sources, engineering details, dimensions, weight, packaging, etc.,
- interfaces with inventory, manufacturing, and product lifecycle management, and
- sometimes included in partner collaborations in order to compress time to market of new products.

Finance is the face to the CFO and serves the following functions:

- provides real-time reporting of all transactions resulting from inventory movement, accounts receivable, accounts payable, taxes, foreign currency conversions, and any other log entries, and
- supports detailed reporting and budgeting capabilities.

Asset Management is the component that controls key assets.

- controls enterprise’s fixed assets
- established and maintains equipment profiles, diagnostics and preventive maintenance activities, and depreciation activities.

Human Resources is the face to employees.

- manages all aspects of human capital with enterprise
- monitors performance of transaction activities to include time, payroll, compensation, expenses, recruitment, etc.
- supports employee profiles, skill development, career planning, performance evaluations, and retention.

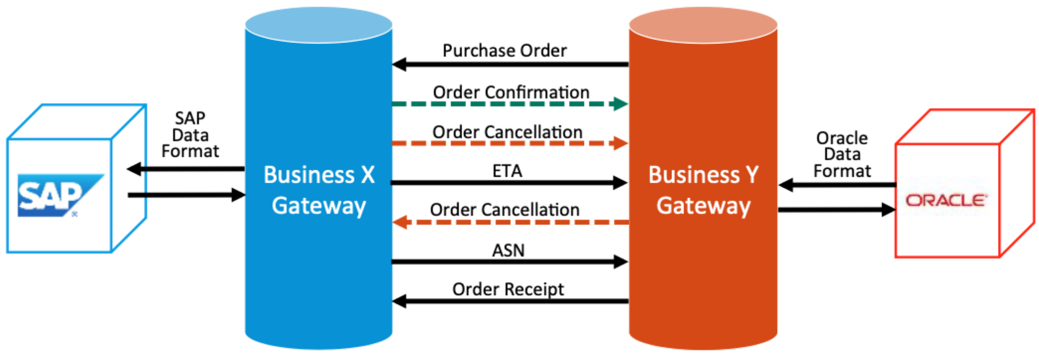

FIGURE 5.10: ERP Communication

ERP Data

There are three types of ERP Data:
Organization data: represents and captures the structure of an enterprise.
Master data: represents entities (customers, vendors) with processes. It is the most commonly used. But because specific processes use materials differently and specific data needs differ by processes – this adds to complexity of master data needs. Material types can be in different states and can be grouped differently based on firm needs.
Transaction data: reflects the outcome of executing process steps. It comes in organizational, master and situational data. Transaction documents include purchase orders, invoices, etc.

5.8.4 ERP Communication

Business-to-Business (B2B): Commerce transactions between manufacturers, wholesalers, retailers. Each business represents a link in the supply chain.

Business-to-Consumer (B2C): Sale transactions between firms and end-consumers. The volume of B2B transactions is much greater than B2C.

Accelerating and validating B2B and B2C transactions: For B2B this is achieved through Electronic Data Interchange (EDI). For B2C this is achieved through a website and email.

Electronic Data Interchange (EDI): "The computer-to-computer interchange of strictly formatted messages that represent documents other than monetary instruments." There is no human intervention in the process.

ERP systems can "communicate" via EDI, sharing near real-time information. See figure 5.10.

5.8.5 The Value of ERP for SCM

There are three important values of ERP for supply chain management: reduction of the bullwhip effect, enabling widespread analytics, and extending the enterprise.

Reducing the Impact of Bullwhip Effect

One of the key values of an ERP system is reducing or preventing the Bullwhip Effect. **The Bullwhip Effect** is phenomenon where information distortion leads to increasing order fluctuations in the upstream supply chain (forecast-driven supply chains). It is driven by several behavioral causes like overreaction to backlogs and lack of transparency. There are also many operational errors such as forecasting errors, lead-time variability, and promotions.

ERP can reduce the impact of the Bullwhip Effect by extending visibility downstream to customer demand and upstream to participants enabling collaboration and information sharing. It also facilitates point of sale capturing and helps reduce batch size and demand variability through smaller and more frequent orders.

Enabling Analytics

ERP systems play a key role in enabling analytics. They are primarily retrospective, serve as the ledger of the firm, and provide the CFO with financial snapshots. They also enable other forms of analytics for **Business Intelligence (BI):** which transforms raw data into meaningful information to improve business decision-making. These can be descriptive, predictive, and prescriptive.

Extending Enterprise

While ERP systems are primarily used in intra-firm process management to connect various departments and provide access to data, they also serve an extending function for better connection with partners. ERPs serve a value in connecting End to End Supply Chains with better connections across SC participants, providing shared understanding, reducing coordination and monitoring costs, and responding quickly to market feedback.

Learning Objectives

- Introduction to supply chain IT systems, their value, application, and constraints.
- Review ERP, its setup functionality, and applications.
- Recognize core functions of ERP.
- Review how that data is used to facilitate communication.
- Understand some of the value of ERP systems for supply chains.

5.9 Supply Chain Systems - Supply Chain Modules

5.9.1 Summary

In this next segment we review different supply chain modules as a subset of supply chain IT systems. To understand where we are now with supply chain IT systems we need to review the evolution of supply chain tools. We journey from the 1960-70's with the Bill of Materials Processor, mainframes-based database systems, and material requirements planning (MRP) to the 1980's with the second coming of MRP that added finance and marketing, Just-In-Time manufacturing methodology, expansion to other functions, and the precursor to the ERP systems of today. In the 1990s, most MRP systems were absorbed into ERP suites; there was the introduction of Advance Planning Systems (APS), and wider adoption of ERP systems. In the 2000s, many of these systems adopted web-based interfaces, improved communication, and adopted shared or cloud-based solutions. There was also a major consolidation of supply chain software vendors and expansion of ERP systems to include SCM.

Now as we explore how to further extend the enterprise and its ability to adequately manage its information on its own and together with other companies, many firms use a series of IT modules. These systems are sometimes a part of ERP, may be standalone applications, or can be part of a larger supply chain ecosystem. We will review two main functionalities including Advance Planning Systems (APS) and Execution Applications. Advanced Planning Systems (APS) are long range, optimization-based decision support tools while execution applications include Warehouse Management Systems (WMS), Transportation Management Systems (TMS), and Manufacturing Execution Systems (MES).

Planning vs. Execution

Although planning modules seek to enable future planning and enabling efficient processes, there is often a gap between the planning and execution tasks. The figure below illustrates this gap.

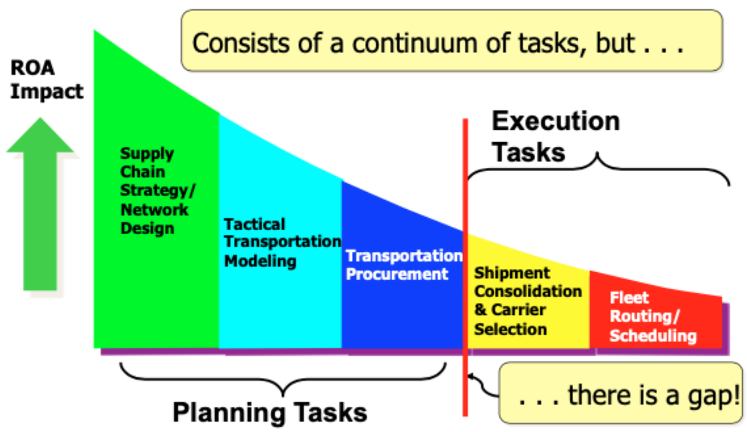

FIGURE 5.11: Planning vs. Execution

Questions, Approaches, and Technologies Change Based on Time Frame

Questions can be **strategic** such as: "What carriers should I partner with and how?" "How should I flow my products?" or they can be **tactical** such as: "How can I quickly secure rates for a new DC/plant/lane?" "What lanes are having performance problems?" or **operational**: "Which carrier should I tender this load to?" "How can I collaboratively source this weeks' loads?"

The time frame also drives the **approach**. For instance, in the strategic face – a company will be establishing a plan and strategy, have event-based enablement and complete non-routine analysis. In an operational time frame they will be executing the strategic plan, operate on transaction-based rules and processes, and have automated actions.

And **technologies** also align with time frames. For instance, strategic timeline will allow for analysis engines tools like optimization, simulation and data analysis and communication via the web, file exchange and remote access. The tactical timeline allows for the same analysis and communication technologies while the operational timeline allows for communication but also workflow software such as compliance tracking, rules, and transaction processing.

5.9.2 Advanced Planning Systems

We now take a closer look at advanced planning systems that are primarily used as decision support systems. They typically include functionality for network design, demand planning, production planning, production scheduling, distribution planning, and transportation planning. Advanced Planning Systems utilize large scale mixed integer linear programs (MILPs) and sometimes simulation.

Planning Horizons

Advance Planning Systems help with planning horizons. The following provide a rough guideline but each firm differs and it is unique to specific industries.:

- 3 months out – Master Production Schedule (MRP, DRP)
  - < 4 weeks out - Frozen MPS
  - 5 to 8 weeks out – Slush MPS – some changes allowed (+- 10%)
  - > 8 weeks out – Water MPS – more changes are allowed (+- 30%)
- 3-18 months out – Aggregated Planning
- > 18 months out – Long Range Planning – Network Design, etc.

Flow

Inputs (from ERP or other systems): Current costs, manufacturing and storage capacities, consensus forecast, sales orders, production status, purchase orders, and inventory policy recommendations, etc.

Decision Process: Large scale optimization (MILP) across multiple facilities and time horizons in a single planning run; unconstrained, constrained, and optimal plans

Outputs: Demand forecast and plan for meeting demand; a feasible production plan for future periods to include allocation of production to plants; allocation of orders to suppliers; identification of bottlenecks; Root Cause Analysis.

5.9.3 Transportation Management Systems (TMS)

TMS is software that facilitates procurement of transportation services, short-term planning and optimization of transportation activities, assets, and resources, and execution of transportation plans (Gonzalez 2009). It is often geographic and mode specific. The core functions of TMS are: transportation procurement; mode and carrier selection; carrier communication; routing guide generation and maintenance; fleet management; audit, payment, and claims; appointment scheduling; yard management; and route planning.

Transportation Execution

The TMS serves as the interface to the carriers while connecting the Order Management System (OMS), Payment Systems, and the ERP. Its main objective is to: move products from initial origin to final destination cost effectively while meeting the level of service standards and executing the plan using the procured carriers. This is broken down in phases below:

PLAN: Create Shipments from Orders
EXECUTE: Select and tender to Carriers
MONITOR: Visibility of the status of Shipments
RECONCILE: Audit invoices and pay for Transportation

There are many considerations to be made in TMS such as:

- How do orders drop? Batched vs Continuous?
- How much time is allowed between drop and must-ship? Weeks? Days? Hours? Minutes?
- What percentage of orders change after release?

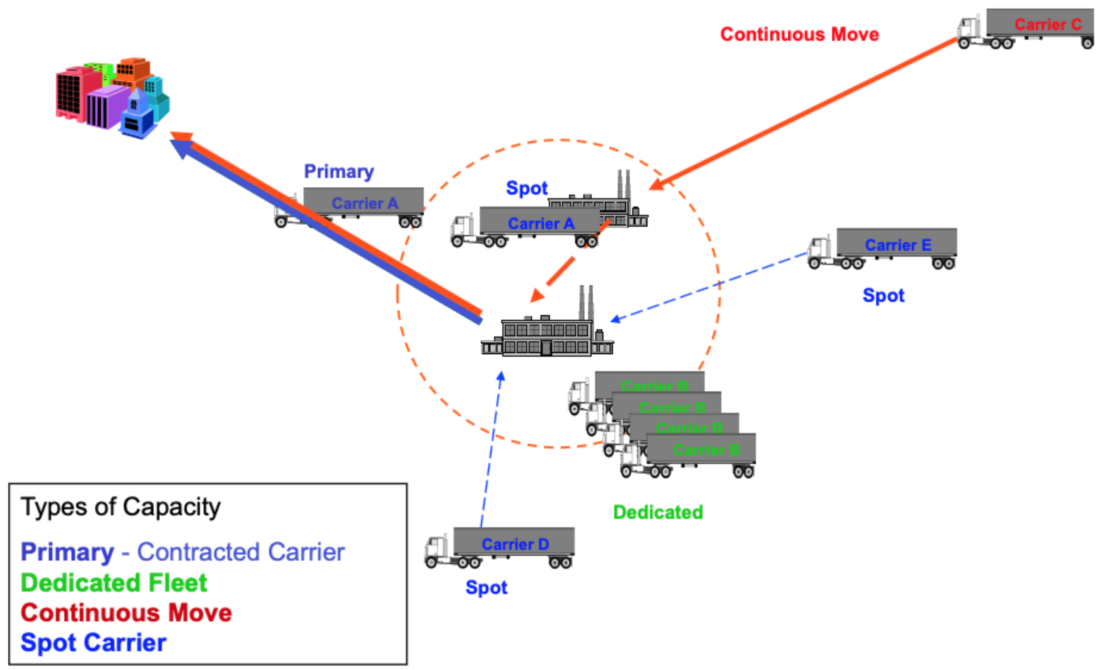

FIGURE 5.12: Carrier selection example

- How do they change? Quantity? Mix? Destinations? Timing?
- What is the length of haul?
- How many orders are "in play" at any time?

There are also key decisions like carrier selection and load building.

TMS Carrier Communication & Selection

Useful EDI Transaction Sets

- 204 – Motor Carrier Load Tender: Used by shippers to tender an offer for a shipment to a full truckload motor carrier. It may be used for creating, updating or replacing, or canceling a shipment.
- 990 - Response to a Load Tender: Used by motor carriers to indicate whether it will pick up a shipment offered by the shipper
- 214 - Transportation Carrier Shipment Status Message: Used by carriers to provide shippers and consignees with the status of their shipments.

Warehouse Management Systems & Automation

WMS is a software system that facilitates all aspects of operations within a warehouse or distribution center and integrates with other systems. It is not the same as inventory management systems; WMS complements IMS. Examples of the benefits of a WMS include: real-time stock visibility and traceability, improved labor productivity, and improved customer service. Some of these benefits are closely tied to automation of material handling and paperless device interfaces.

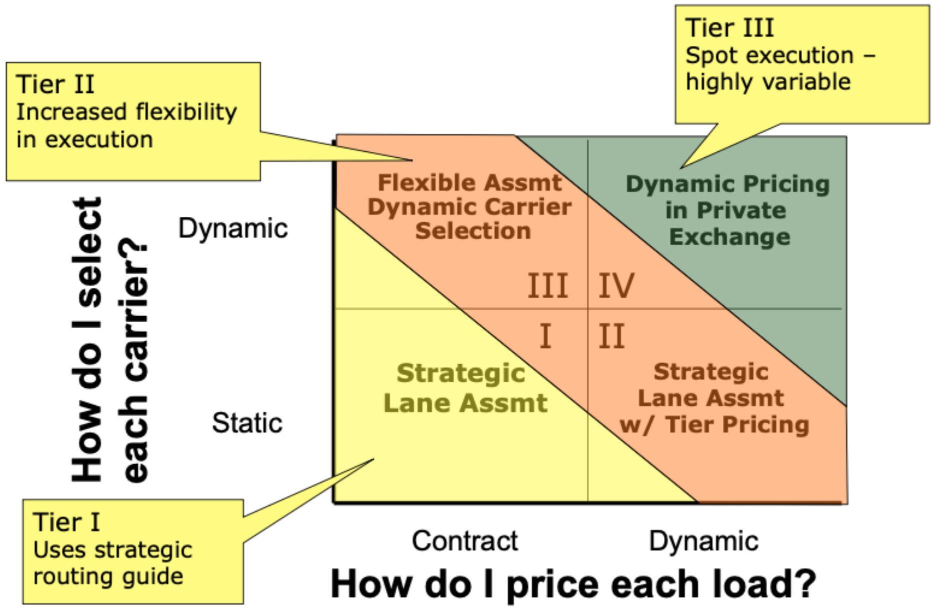

FIGURE 5.13: Linking approaches to carrier selection

Examples of Warehouse Automation include:
Automatic identification technologies: Bar codes and bar code scanners, radio frequency tags (RFID) and antennae, smart cards and magnetic stripes, vision systems.
Automatic communication technologies: Radio frequency data communications, synthesized voice, virtual displays, pick to light / voice systems
Automated material handling technologies: Carousels, conveyors/robotics, flow racks, AS/AR Systems

WMS Software Components

Order Processing

- Order checking & batching
- Allocation
- Auto-replenishment

Receiving

- ASN planning
- In-bound tracking
- Delivery appointment scheduling
- PO verification
- Returns processing

Put-Away

- Palletizing

- Zoning and slotting
- Random/directed put away
- Routing for putaway & replenishment

Picking

- Batch/Wave/Zone/Directed picking
- Carton/pallet select
- Assembly/kitting
- Pick-to-light/voice

Shipping

- Pallet sequencing & Load planning
- Pallet layering
- Trailer management

Labor Management

- Individual/team performance mgmt
- Labor scheduling
- Time standards

Equipment Support

- Interface to automated equipment
- Equipment maintenance

5.9.4 Manufacturing Execution Systems

MES is a software system that manages and monitors all work-in-process (WIP) in the production process. It is integrated with the ERP to manage the execution of release of production orders to finished goods delivery, trigger supply chain replenishments, and enhance product traceability through manufacturing.

The functionality of a MES includes:

- Machine scheduling
- Process management
- Document control
- Labor management
- Inventory management
- Product (WIP) tracking
- Performance analysis
- Labor management
- Quality management
- Production reporting

Learning Objectives

- Become familiar with systems that are common in supply chains that extend the enterprise.
- Differentiate between Advanced Planning Systems and Execution Systems.
- Recognize the gaps in planning vs. execution and the time frames embedded in both.
- Review Advanced Planning Systems, their use and application.
- Become familiar with the main execution systems in SC such as TMS, WMS, and MES.

5.10 Supply Chain Systems - Software Selection & Implementation

5.10.1 Summary

As we have journeyed through various types of supply chain systems, we will now cover the process of software selection and implementation. As firms are in the process of selecting their supply chain systems they will need to be aware of various factors that will guide their decision of whether to select the system or not. So, we now discuss the process of software selection and evaluation criteria. While selecting the appropriate software system can be challenging, implementation is far more difficult. The process of implementation can be long and costly, using additional resources. In this lesson, we cover general guidelines on what to be prepared for when implementing software systems.

5.10.2 Architecture

Evolution of Architecture

To understand where supply chain systems stand now, it is helpful to understand the evolution of the architecture starting in the 1970's. The following are the various forms of architecture for the last fifty years:

- Mainframe (1970s)
- Personal Computers (mid-1980s)
- Client-Server (late 80s to early 90s)
- Wide Web and Web 2.0 (mid-90s to present)
- Cloud or Post-PC (today and beyond)

Today there are a variety of software systems available to businesses. In terms of architectural format, they can choose between "On Location" or "On Premise" – meaning that the firms host the software in their own facilities or on their hardware and within their own firewall. However, companies are increasingly opting for cloud computing options. This means that they have several deployment models available to them.

5.10.3 Cloud Computing

As cloud computing becomes increasingly more popular, there are a variety of offerings that can be tailored to firm needs. They are Infrastructure as a Service (IaaS), Platform as a Service (PaaS), and Software as a Service (SaaS), third party management increases from IaaS to SaaS. We discuss each format as well as benefits below:

Infrastructure as a Service (IaaS): In this format, the third party provides the firm with the computing infrastructure, physical or virtual machines and other resources. Firm owns and manages the software application. The benefits of this are:

- No need to invest in your own hardware
- Infrastructure scales on demand to support dynamic workloads
- Flexible, innovative services available on demand

Platform as a Service (PaaS): In this format, the third party provides firm computing platforms to include operating system, database, web server etc. Firm owns and manages the software application. The benefits of this are:

- Develop applications and get to market faster
- Deploy new web applications to the cloud in minutes
- Reduce complexity with middleware as a service

Software as a Service (SaaS): In this case, the third party provides firm with access to the application software and handles installation, setup, maintenance, and running. Firm is charged by use. Benefits include:

- You can sign up and rapidly start using innovative business apps
- Apps and data are accessible from any connected computer
- No data is lost if your computer breaks, as data is in the cloud
- The service is able to dynamically scale to usage needs

While there are many benefits to cloud computing, there are widespread disadvantages of cloud computing that include but are not limited to: vendor outages, unrestricted government access, security & privacy risks, and key data and processes require network access.

Software Selection Sources

There are different sources of software that firms need to be aware of. These include a customized in-house system designed for a business, an ERP expanded system with additional bells and whistles tailored for a company, best of breed solutions (of market solutions), and best of breed platforms. These are discussed in further detail in the chart below:

Outsourcing

There is also the option of outsourcing some of these systems to different providers. For instance, in logistics, 3PLs or Third-Party Logistics Providers, serve as an organization that can run the software as well as perform all of the business processes. Having a 3PL run your logistics eliminates the need for hardware and software. 3PLs can possibly replace personnel within the firm. The use of 3PLs is most common with smaller firms.

The main reasons to outsource are to reduce capital expenditure for software and hardware. It may also reduce costs as a result of partner's economies of scale; they often have the ability to do it faster and better as well as be more flexible and agile. It may also be an opportunity to increase levels of service at reasonable costs. Firm can focus on core business and bring in expertise that is not affordable in-house. There are myriad other reasons to outsource, but there are also many not to, discussed below.

At the top of the reasons not to outsource are security and privacy concerns; someone else has access to the firm's data. There are also worries of vendor dependency and lock-in. The firm may lose in-house expertise to a core function. There are also high-migration costs as well as concerns over availability, performance, and reliability. There are additional reasons not to outsource. Firms need to weigh the pros and cons of outsourcing matched with business objectives to decide which is suitable for them.

Source	Advantages	Disadvantages
Customized In-House System	• Best fit to the firm and its processes.	• Exceptionally difficult and time consuming to develop • Most expensive total cost of ownership • Difficult to maintain • Can result in "inward looking" solution
ERP Expanded Systems	• Relatively fast implementation • Less expensive than in-house customization • Efficient from IT perspective • Easier to upgrade with ERP enhancements	• Tends to be inflexible in terms of process • Could require change in business processes • Not guaranteed to be best solution approach
Best of Breed Solutions	• Best performing market solution for each function	• Difficult to integrate different systems • Can have slow performance • Requires the use of middleware between the applications • Upgrading individual components can cause ripple effect problems
Best of Breed Platforms	• Very good, if not best, solution for each function with easier integration between individual modules	• Requires the use of middleware between the applications

FIGURE 5.14: Software Selection Sources

5.10.4 Software Vendor Selection

In the end, a firm must select its vendor. Typically, an organized and formalized fashion to select a vendor and in general it goes as follows:

1. **Form a Project Team (Internal and/or External) & Objectives**
2. **Understand the Business and Needs:** review current business processes, prioritize needs/functionality, create Request for Information (RFI)
3. **Create Initial Short List of Potential Solutions & Vendors**
4. **In-depth Review of Short Listed Vendors:** have vendors conduct realistic product demonstrations, references from current users.
5. **Create and Distribute Final Request for Proposal (RFP)**
6. **Make the Decision:** negotiate contract, price, and service level agreements (SLAs) and establish an implementation plan.

While cost is one of the primary factors in decision-making, there are many other criteria that need to be evaluated on top of cost. They are:

- **Functionality** – does the system features fit the firm's processes and needs?
- **Ease of Use** – how fast is the initial learning curve and on-going use?
- **Performance** – what are the processing speeds?
- **Scalability** – how well can the system expand and grow with the firm?
- **Interoperability** – how well does the system integrate with other systems?
- **Extendibility** – how easily can the system be extended or customized?
- **Stability** – how reliable is the system in terms of bugs and up-time?

- **Security** – how well does the system restrict access, control confidential data, and prevent cyber hacking?
- **Support** – how is the quality of the vendor in terms of implementation, support, training, thought leadership etc.?
- **Vendor Viability** – how is the vendor's financial strength and willingness to supply updates and enhancements? Will they be here in 3 years?

Because there are a variety of criteria that firms will be evaluating vendors on, a scorecard is a popular way to capture financial and non-financial attributes. The criteria can be scored as rank, ratings, and grades. Scorecards tend to be very detailed and can even be broken down by specific features. The selection can be made between vendors or between alternative hosting platforms.

Total Cost of Ownership

Software License: Direct cost of the software system itself – assuming ownership.
Maintenance: Ongoing annual costs to guarantee upgrades and bug fixes.
Platform/Hardware: Cost of needed hardware to run the new software.
Training: Cost of training initial and ongoing personnel
Implementation: Cost of getting the system to go live. These vary widely between systems and firms.
Customization: Cost of modifying the system itself to fit the firm's processes. Nothing in SCM is used straight out of the box (vanilla).
System Integration Cost of interfacing this system with other modules and modifying existing systems to fit.

	# Modules Converted	# Locations Converted	Comments
Direct or Big Bang	All	All	• Switch from the old to new system occurs on one day • Pain of switch concentrated for entire firm • Fastest implementation time, but highest risk • Post-implementation productivity drop • High potential for system wide failures due to insufficient testing/training
Parallel	All/Some	All/Some	• Old and new systems kept on for testing period • Lowest risk of failure, but highest cost and longest implementation time • Employees do double entry work
Pilot	All	One	• Full implementation of all modules at one location • Identify bugs or issues that are corrected prior to larger rollout • Contains any potential failure from infecting all locations • Tests individual modules and integration simultaneously
Phased or Rolling	One	All	• Implementation of one module at a time across the network • Longer implementation duration than direct, but with lower risk • Users have more time & learn as they go - no dip in performance after • Learn and fix as you go – better process for later implementations • Loss of managerial focus over time and a continuous state of change • Potential for missing data during transitional implementation period • Might require temporary bridges from old to new systems during transition

FIGURE 5.15: Software Implementation Options

5.10.5 Implementation

While selecting a vendor can be difficult and time consuming, the actual process of implementation can take an even more significant amount of time and consume a lot of resources. There are a few different approaches to implementation. They include Direct (or Big Bang), Parallel, Pilot, and Phased (or Rolling). Each of these has its own positives and negatives, but the approach must suit the needs of the business.

There are a few **best practices** to keep in mind when going about implementation. They include:

1. Secure senior executive commitment: ability to gather and use resources, empower team.
2. Form interdisciplinary team(s)
3. Create a clear and specific scope document
4. Build extensive testing into the project plan (you can't test too much)
5. Include extensive user training into the project plan

Learning Objectives

- Recognize selecting a software vendor is an intertwined decision between architecture and source.
- Understand tradeoffs between On-Premise and Cloud based systems.
- Know the differences between In-House, Best of Breed, ERP Extensions, and Outsourced forms of software systems.
- Review the selection process, recognizing there are multiple attributes, and the total cost of ownership is complex.
- Understand the challenge of implementation and the various approaches to implementing systems within a firm.
- Review best practices of implementation.

Part VI

SCx - Formulae

6

SCx Formulae

CONTENTS

6.1 SC0x Formulae

Description	Formula	Lesson Topic
Linear Function	y=ax+b	Page 7, section 1.2.2
Profit Equation	Profit = (Revenue - Cost) * Volume - Fixed Cost	Module 1, Unit 2
Quadratic Function	$y = ax^2 + bx + c$	Page 8, section 1.2.2
Quadratic Formula	$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Page 9, section 1.2.2
Power Function	$y = f(x) = ax^b$	Page 9, section 1.2.2
Exponential Function	$y = ab^x$	Page 10, section1.2.2
Euler's Number	$e = limn \to \infty \left(1 + \frac{1}{n}\right)^n = 2.71828$	Page 10, section 1.2.2
Logarithmic Relationships	$y = b^x \leftrightarrow log_b(y) = x$ $a = e^x \leftrightarrow ln(a) = x$	Page 10, Section 1.2.2
Properties of Logarithms	$log(xy) = log(x) + log(y)$, $log\left(\frac{x}{y}\right) = log(x) - log(y)$, $log(x^a) = a \cdot log(x)$	Module 1, Unit 2
Multivariate Functions	$y = f(x_1, x_2, x_3, \ldots, x_n)$	Page 10, Section 1.2.2

Description	Formula	Lesson Topic
Total Logistics Cost Equation	$cost = cD + \frac{AD}{Q}$, D = annual demand (items), c = cost per item (\$/item), A = cost per order (\$/order), Q = order size (items/order)	Module 1, Unit 2
Mean	$E[X] = \bar{x} = \mu = \sum_{i=1}^{n} p_i x_i$	Page 12, section 1.4.2
Population Variance	$Var[X] = \sigma^2 = \sum_{i=1}^{n} p_i (x_i - \bar{x})^2$ $= \sum_{i=1}^{n} p_i (x - \mu)^2$	Page 13, section 1.4.2
Standard Deviation	$\sigma = \sqrt{Var[X]} = \sqrt{\sigma^2}$	Page 13, section 1.4.2
Coefficient of Variation	$\frac{\sigma}{\mu}$	Page 13, section 1.4.2
Four Laws of probability	• $0 \leq P(A) \leq 1$ • P(A or B) = P(A$\cup B) = P(A) + P(B)$, where A and B are mutually exclusive events • P(A$\vert B) = P(A\ and\ B)/P(B) = P(A \cap B)/P(B)$ • P(A$\vert B) = P(A)$ and $P(A\ and\ B) = P(A \cap B)$ $= P(A\vert B)/P(B) = P(A) \times P(B)$, where A and B are independent events	Page 12, section 1.4.2

Description	Formula	Lesson Topic
Discrete Uniform Distribution	• PMF : if a≤ $x \leq\ b, P\left[X = x\right] = f\left(x\vert a, b\right) = 1/n$ Otherwise P$[X = x] = f\left(x\vert a, b\right) = 0$ • $Metrics$ Mean = (a + b) / 2 Median = (a + b) / 2 Mode N/A Variance = ((b-a+1)2 − 1)/12 • $Where$: a = Minimum b = Maximum n = # of values = b − a + 1	Page 13, section 1.4.3
Poisson Distribution	• PMF : if x=0,1,2,..,P$[X = x] = f\left(x\vert\lambda\right) = e^{-\ \lambda}\lambda^{x}/x!$ Otherwise P$[X = x] = f\left(x\vert a, b\right) = 0$ • CDF : P$[X \leq x] = \sum_{k=0}^{x}$ e$^{-\lambda}\ \ \lambda^{k}\ /\ k!$ • $Metrics$ $\lambda = mean = variance$	Page 14, section 1.4.3

Description	Formula	Lesson Topic
Continuous Uniform Distribution	• PDF : if a≤ $t \leq\ b, f\left(t\right|\ a, b) = 1/\left(a - b\right)$ Otherwise f(t| $a, b) = 0$ • CDF : if t ¡ a, F(t — a,b)=0) if a≤ $t \leq\ b, F\left(t\right|\ a, b) = \left(t - a\right)/\left(b - a\right)$ if t >b,F(t| $a, b) = 1$ • $Metrics$ Mean = (a + b) / 2 Median = (a + b) / 2 Mode = any value in range [a,b] Variance = (b - a)²/12	Page 15, section 1.4.4
Sample Variance	$s^2_{sample} = \frac{\sum_{i=1}^{n}(x_i - \underline{x})^2}{n-1}$	Page 13, section 1.4.2
Triangle Distribution	pdf = f(x) = $\begin{cases} 0\ \ if\ \ x < a \\ \dfrac{2(x-a)}{(b-a)(c-a)}\ \ if\ \ a \leq x \leq c \\ \dfrac{2(b-x)}{(b-a)(b-c)}\ \ if\ \ c \leq x \leq b \\ 0\ \ if\ \ x > b \end{cases}$ cdf = f(x) = $\begin{cases} 0\ \ if\ \ x \leq a \\ \dfrac{(x-a)^2}{(b-a)(c-a)}\ \ if\ \ a < x \leq c \\ 1 - \dfrac{(b-x)^2}{(b-a)(b-c)}\ \ if\ \ c \leq x \leq b \\ 1\ \ if\ \ b \leq x \end{cases}$	Page 17, section 1.4.4
Discrete Probability Distribution	$\mu = E\left(X\right) = \sum_{i=1}^{n} p_i x_i$ $\sigma^2 = \sum_{i=1}^{n} p_i \left(x_i - \mu\right)^2$	Page 18, section 1.4.4

Description	Formula	Lesson Topic
Continuous Probability Distribution	$\mu = \int_a^b t \cdot\ f(t)\, dt$ $\sigma^2 = \int_a^b (t-\mu)^2 \cdot\ f(t)\, dt$	Page 18, section 1.4.4
Confidence Interval	$[\bar{x} - \ c *\ s/\sqrt{n}\ , \bar{x}\ +\ c *\ s/\sqrt{n}\ \ ]$	Page 22, section 1.5.2
Prediction Interval	$[\bar{x} - \ c *\ s, \bar{x}\ +\ c *\ s]$	Page 22, section 1.5.2
Interval	$L = c * s/\sqrt{n}$	Page 21, section 1.5.2
Chi-Square Test	$\chi^2 = \sum_{i=1}^{c} \frac{(Observed_i - Expected_i)^2}{Expected_i}$ P- value = 1- CHISQ.DIST(test statistic,n-1,1)	Page 24, section 1.5.3
Covariance & Correlation	$Cov(X,Y) = \frac{\sum_{i=1}^{n} (x_i \ \ -\mu_x\ \ )*(y_i \ \ -\mu_y\ \ )}{}$ CORR(X,Y) $= \frac{Cov(x,y)}{\sigma_x * \sigma_y}$	Page 25, section 1.5.4
Linear Functions of Random Variables	$Y = a * X + b$ $E[Y] = \mu_y = a * \mu_x + b$ $VAR[Y] = \sigma^2{}_y = a^2 \sigma_x^2$ $\sigma_Y\ = \|a\| \sigma_X$	Page 25, section 1.5.4
Sum of Two Random Variables	$Z = a * X + b * Y$ $E[Z] = a * E[X] + b * E[Y] = a * \mu_x + b * \mu_y$ $VAR[Z] =$ $= x^2 * VAR[X] + b^2 * VAR[Y] + 2 * a * bCOV[X,Y]$ $= a^2\sigma_x^2 + b^2\sigma_y^2 + 2 * a * b * COV[X,Y]$ $= a^2\sigma_x^2 + b^2\sigma_y^2 + 2 * a * b * \sigma_x * \sigma_y CORR[X,Y]$ $If X and Y are independent, then$ $VAR[Z] = x^2 * VAR[X] + b^2 * VAR[Y] = a^2\sigma_x^2 + b^2\sigma_y^2$	Page 25, section 1.5.4
Linear regression model	y$_i = \beta_0 + \beta_1 x_i$ Y$_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ for i=1,2,.....n	Page 26, section 1.6.2

Description	Formula	Lesson Topic
Residuals	$= b_0 + b_1x_i$ for i=1,2,.....n $e_i = y_i - \hat{y_i} = y_i - b_0 - b_1x_i$ for i=1,2,.....n	Page 27, section 1.6.2
Ordinary Least Squares (OLS) regression	$\sum_{i=1}^{n}(e_i^2) = \sum_{i=1}^{n}(y_i - \hat{y_i})^2$ $= \sum_{i=1}^{n}(y_i - b_0 - b_1x_i)^2 = \bar{y} - b_1\bar{x}$ $b_1 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}(x_i-\bar{x})^2}$	Page 27, section 1.6.2
Multiple variables	$Y_i = \beta_0 + \beta_0x_{1i} + ... + \beta_kx_{ki} + \varepsilon_i$ for i=1,2,....n $E(Y\|x_1, x_2, ..., x_k) = \beta_0 + \beta_0x_1 + ... + \beta_kx_k$ $StdDev(Y\|x_1, x_2, ..., x_k) = \sigma$ $\sum_{i=1}^{n}(e_i^2) = \sum_{i=1}^{n}(y_i - \hat{y_i})^2$ $= \sum_{i=1}^{n}(y_i - b_0 - b_1x_{1i} - .... - b_kx_{ki})^2$	Page 27, section 1.6.2
Overall fit , Total Sum of Squares(TSS)	$TSS = \Sigma_{\mathrm{i=1}}^{\mathrm{n}}(y_i - \bar{y})^2$	Page 27, section 1.6.2
Error or Residual Sum of Squares	$ei = yi - \hat{y}$ $\mathrm{RSS} = \sum e_i^2 = \sum(y_i - \hat{y_i})^2$	Page 27, section 1.6.2
Coefficient of Determination or Goodness of Fit (R^2)	$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\Sigma\ (yi-\hat{y}i)^2}{\Sigma\ (yi-\bar{y}i)^2}$	Page 28, section 1.6.2
Adjusted R2 corrects	$adj\ R^2 = 1 - \frac{RSS}{TSS}(\frac{n-1}{n-k-n})$	Page 28, section 1.6.2
Standard error (sbi)	$b_1 \pm t_{\alpha/2}s_{b_1}$ $v = n - 2$ $s_e = \sqrt{\frac{\sum(y_i-\hat{y_i})^2}{N-2}}$ $s_{b_1} = \frac{s_\varepsilon}{\sqrt{(n-1)s_x^2}}$	Page 28, section 1.6.2
t- statistic	$t = \frac{b_1 -\ \beta_1}{s_{b1}}$	Page 28, section 1.6.2
First derivative	(delta) = rate of change y' = f' (x) = dy/dx	Page 30, section 1.7.2
Product rule	$y = f(x) = a \rightarrow y' = f'(x) = 0$	Page 30, section 1.7.2
Power rule	$y = f(x) = ax^n \rightarrow y' = f'(x) = anx^{n-1}$	Page 30, section 1.7.2

Description	Formula	Lesson Topic
First order conditions	First Order (necessary)condition - the slope must be 0 i.e. $f'(x*) = 0$	Page 31, section 1.7.2
Second Order Conditions	Second order (sufficiency) condition - determines where extreme point is min or max by taking the second derivative, f"(x). – If f"(x) >0 extreme point is a local min – If f"(x) <0 extreme point is a local max – If f"(x) = 0 it is inconclusive • Special cases – If f(x) is convex – >global min – If f(x) is concave – >global max	Page 31, Section 1.7.2
Single distribution center	$d_{TOUR} \approx 2d_{LineHaul} + d_{Local}$ $d_{LineHaul} =$ Distance from origin to center of gravity (centroid) of delivery district $d_{Local} =$ Local delivery between customers in one district	Page 49 , section 1.9.10
Estimating point to point distances	$EuclideanSpace:$ $d_{A-B} = \sqrt{[(x_A - x_B)^2 + (y_A - y_B)^2]}$ $Grid: d_{A-B} = \|x_A - x_B\| + \|y_A - y_B\|$	Page 50 , section 1.9.10
Random (real) networks	$D_{A-B} = k_{CF} d_{A-B}$	Page 50 , section 1.9.10
Great circle (for location within the same hemisphere)	$d_{A-B} = 3959(arccos[sin[LAT_A]sin[LAT_B]$ $+cos[LAT_A]cos[LAT_B]cos[LONG_A - LONG_B]])$	Page 50 , section 1.9.10
One to many system	$d_{TSP} \approx k_{TSP}\sqrt{nA} = k_{TSP}\sqrt{n(\frac{n}{\delta})} = k_{TSP}\frac{n}{\sqrt{\delta}}$ $d_{stop} \approx \frac{k_{TSP}\sqrt{nA}}{n} = k_{TSP}\sqrt{\frac{A}{n}}$	Page 51 , section 1.9.11

Description	Formula	Lesson Topic
One to many system	Euclidean Networks $-k_{TSP} = 0.57$ to 0.99 depending on clustering and size of N $(MAPE \approx 4\%, MPE \approx -1\%)$ $-k_{TSP} = 0.765$ commonly used and is a good approximation! Grid Networks $k_{TSP} = 0.97$ to 1.15 depending on clustering and partitioning of district	Page 51 , section 1.9.11
Estimating Vehicle Tour Distances	Total distance travelled on all tours $d_{TOUR} = 2d_{LineHaul} + \frac{ck_{TSP}}{\sqrt{\delta}}$ $d_{AllTours} = ld_{TOUR} = 2ld_{LineHaul} + \frac{nk_{TSP}}{\sqrt{\delta}}$	Page 52 , section 1.9.12
Minimize number of tours by maximising vehicle capacity	$l = [\frac{D}{Q_{MAX}}] + d_{AllTours}$ $= 2[\frac{D}{Q_{MAX}}] + d_{LineHaul} + \frac{nkTSP}{\sqrt{\delta}}$	Page 52 , section 1.9.12
Transport Cost	$c_s[n + \frac{D}{Q_{MAX}} + \frac{1}{2}] +$ $c_d(2[\frac{D}{Q_{MAX}} + \frac{1}{2}]d_{LineHaul} + \frac{nkTSP}{\sqrt{\delta}}) + c_{vs}D$	Page 52 , section 1.9.12
Generating random variables	$UniformDistribution$ $\sim U(a,b) = a + (b–a) * RAND()$ $NormalDistribution$ $\sim N(\mu, \sigma) = NORM.INV(RAND(), \mu, \sigma)$	Page 54 , section 1.10.2

6.2 SC1x Formulae

Description	Formula	Lesson Topic
Power Law	$y = ax^k$	Page 61, section 2.1.2
Revenue	Average sales * Unit sales price	Page 61, section 2.1.2
Profit	Average sales * Margin	Page 61, section 2.1.2
Margin	Unit sales price – Unit cost	Page 61, section 2.1.2
Probabilities under the Normal Distribution N(μ, σ) ($\mu = mean$ & $\sigma = standard\ deviation$)	The probability that a random variable is less than or equal to x under the Normal Distribution (μ, σ) : Excel 2007 and above : NORM.DIST(x, $\mu, \sigma, true$) Excel 2003 : use the above function without the period The value of x where the probability that a random variable is less than or equal to it is the specified probability: Excel 2007 and above = NORM.INV(probability, μ, σ) Excel 2003 : use the above function without the period	Page 61/62, section 2.1.2
K-value (for Unit Normal Distribution $\sim N(0,1)$)	k=(x - μ)/σ	Page 62, section 2.1.2
Probabilities under the Unit Normal Distribution $\sim N(0,1)$	The probability that a random variable is less than k units above (or below) mean: NORM.S.DIST(k,1) The value corresponding to the given probability: NORM.S.INV(probability)	Page 62,section 2.1.2
Probabilities under Poisson distribution $\sim Poisson(\lambda)$	**POISSON(**x_0, λ**, false)** $\Longrightarrow P(\mathrm{x} = \mathrm{x}_0)$ **POISSON(**x_0, λ**, true)** $\Longrightarrow P(\mathrm{x} \leq x_0)$	Page 63, section 2.1.2
Uniform distribution U (a,b)	standard deviation is = $(b-a)/\sqrt{12}$.	Page 63, section 2.1.2
Demand Forecasting		

Description	Formula	Lesson Topic
Key concepts NOTATION	A_t: Actual value for observation t F_t: Forecasted value for observation t e_t: Error for observation t, $e_t = A_t - F_t$ n: number of observations μ: mean σ: standard deviation CV: Coefficient of Variation – a measure of volatility – CV=$\frac{\sigma}{\mu}$	Page 65, section 2.2.2
Mean Deviation	Mean Deviation: MD= $\frac{\sum_{t=1}^{n} e_t}{n}$	Page 65, Section 2.2.2
Mean Absolute Deviation	Mean Absolute Deviation: MAD= $\frac{\sum_{t=1}^{n} \lvert e_t \rvert}{n}$	Page 65, Section 2.2.2
Mean Squared Error	MSE=$\frac{\sum_{t=1}^{n} {e_t}^2}{n}$	Page 65, Section 2.2.2
Root Mean Squared Error	RMSE=$\sqrt{\frac{\sum_{t=1}^{n} {e_t}^2}{n}}$	Page 65, Section 2.2.2
Mean Percent Error	MPE=$\frac{\sum_{t=1}^{n} \frac{e_t}{A_t}}{n}$	Page 66, Section 2.2.2
Mean Absolute Percent Error	MAPE=$\frac{\sum_{t=1}^{n} \frac{\lvert e_t \rvert}{A_t}}{n}$	Page 66, Section 2.2.2
Statistical Aggregation	Statistical Aggregation:$\sigma_{agg}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + ... + \sigma_n^2$ $\sigma_{agg} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + ... + \sigma_n^2}$ $\mu_{agg} = \mu_1 + \mu_2 + \mu_3 + ... + \mu_n$	Page 66, Section 2.2.2
Statistical Aggregation of n Distributions of Equal Mean and Variance	$\sigma_{agg} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + ... + \sigma_n^2} = \sigma_{ind}\sqrt{n}$ $\mu_{agg} = \mu_1 + \mu_2 + \mu_3 + ... + \mu_n = n\mu_{ind}$ $CV_{agg} = \frac{\sigma\sqrt{n}}{\mu n} = \frac{\sigma}{\mu\sqrt{n}} = \frac{CV_{ind}}{\sqrt{n}}$	Page 66, Section 2.2.2

Description	Formula	Lesson Topic
Time Series Analysis NOTATION	x_t: Actual demand in period t $\hat{x}_{t,t+1}$: Forecast for time t+1 made during time t a: Level component b: Linear trend component Ft: Season index appropriate for period t e_t: Error for observation t, $e_t = A_t - F_t$ t: Time period (0, 1, 2,... n) Level Model: $x_t = a + e_t$ Trend Model: $x_t = a + bt + e_t$ Mix Level-Seasonality Model: $x_t = aF_t + e_t$ Mix Level-Trend-Seasonality Model:$x_t = (a + bt)F_t + e_t$	Page 68, Section 2.2.3
Cumulative Model (Stationary Demand only)	$\hat{x}_{t,t+1} = \frac{\sum_i^t x_i}{t}$	Page 68, Section 2.2.3
Naive Model (Stationary Demand only)	$\hat{x}_{t,t+1} = x_t$	Page 68, Section 2.2.3
M-Period Moving Average Forecast Model (Stationary Demand only)	$\hat{x}_{t,t+1} = \frac{\sum_{i=t+1-M}^t x_i}{M}$	Page 68, Section 2.2.3
Exponential Smoothing NOTATION	x_t : Actual demand in period t $x_{t,t+1}$: Forecast for time t+1 made during time t α :Exponential smoothing factor forlevel $(0 \leq \alpha \leq 1)$ β : Exponentialsmoothing factor for trend $(0 \leq \beta \leq 1)$ ϕ :Exponential smoothing factor for dampening $(0 \leq \phi \leq 1)$ ω : Mean Square Error trending factor $(0.01 \leq \omega \leq 0.1)$	Page 68 and 69, Section 2.2.4
Simple Exponential Smoothing Model (Level Only)	$\hat{x}_{t,t+1} = \alpha x_t + (1-\alpha)\hat{x}_{t-1,t}$	Page 69, Section 2.2.4
Damped Trend Model with Level and Trend	$\hat{x}_{t,t+\tau} = \hat{a}_t + \sum_i^\tau \varphi^i \hat{b}_t$ $\hat{a}_t = \alpha x_t + (1-\alpha)(\hat{a}_{t-1} + \varphi \hat{b}_{t-1})$ $\hat{b}_t = \beta(\hat{a}_t - \hat{a}_{t-1}) + (1-\beta)\varphi \hat{b}_{t-1}$	Page 69, Section 2.2.4

Description	Formula	Lesson Topic
Exponential Smoothing for Level & Trend	$\hat{x}_{t,t+\tau} = \hat{\alpha}_t + \tau\hat{b}_t$ $\hat{a}_t = \alpha x_t + (1-\alpha)(\hat{a}_{t-1} + \hat{b}_{t-1})$ $\hat{b}_t = \beta(\hat{a}_t - \hat{a}_{t-1}) + (1-\beta)\hat{b}_{t-1}$	Page 69, Section 2.2.4
Mean Square Error Estimate	$MSE_t = \omega(x_t - \hat{x}_{t-1,t})^2 + (1-\omega)MSE_{t-1}$	Page 69, Section 2.2.4
Exponential Smoothing with Holt-Winter NOTATION	x_t : Actual demand in period t $x_{t,t+1}$: Forecast for time t+1 made during time t α : Exponential smoothing factor $(0 \leq \alpha \leq 1)$ β : Exponential smoothing trend factor $(0 \leq \beta \leq 1)$ γ : Seasonality smoothing factor $(0 \leq \gamma \leq 1)$ F_t : Multiplicative seasonal index appropriate for period t P : Number of time periods within the seasonality (note: $\sum_{i=1}^{P} \hat{F}_i = P$)	Page 70, Section 2.2.5
Holt-Winter Exponential Smoothing Model (Level, Trend, and Seasonality)	$\hat{x}_{t,t+\tau} = (\hat{a}_t + \tau\hat{b}_t)\hat{F}_{t+\tau-P}$ $\hat{a}_t = \alpha(\frac{x_t}{\hat{F}_{t-P}}) + (1-\alpha)(\hat{a}_{t-1} + \hat{b}_{t-1})$ $\hat{b}_t = \beta(\hat{a}_t - \hat{a}_{t-1}) + (1-\beta)\hat{b}_{t-1}$ $\hat{F}_t = \gamma(\frac{x_t}{\hat{a}_t}) + (1-\gamma)\hat{F}_{t-P}$	Page 70, Section 2.2.5
Double Exponential Smoothing (Seasonality and Level)	$\hat{x}_{t,t+\tau} = \hat{a}_t\hat{F}_{t+\tau-P}$ $\hat{a}_t = \alpha(\frac{x_t}{\hat{F}_{t-P}}) + (1-\alpha)(\hat{a}_{t-1})$ $\hat{F}_t = \gamma(\frac{x_t}{\hat{a}_t}) + (1-\gamma)\hat{F}_{t-P}$	Page 71, Section 2.2.5
Normalizing Seasonality Indices	$\hat{F}_t^{NEW} = \hat{F}_t^{OLD}\left(\frac{P}{\sum_{i=t+1-P}^{t} \hat{F}_i^{OLD}}\right)$	Page 71, Section 2.2.5

Description	Formula	Lesson Topic
Intermittent or Sparse Demand–Croston's Method NOTATION	x_t : Demand in period t y_t : 1 if transaction occurs in period t, =0 otherwise z_t : Size (magnitude) of transaction in time t n_t : Number of periods since last transaction α : Smoothing parameter for magnitude β : Smoothing parameter for transaction frequency	Page 73, Section 2.2.6

Intermittent or
Sparse Demand – Croston's
Method
FORMULA
	$\text{Prob}(\text{y}_t = 1) = \frac{1}{n} \quad and \quad Prob(y_t = 0) = 1 - \frac{1}{n}$
	Updating Procedure for $\hat{z}_t$ and $\hat{n}_t$:
	If $x_t = 0$ (no transaction occurs), then
	$\hat{z}_t = \hat{z}_{t-1} \quad and \quad \hat{n}_t = \hat{n}_{t-1}$
	If $x_t > 0$ (transaction occurs), then
	$\hat{z}_t = \alpha x_t + (1-\alpha)\hat{z}_{t-1}$
	$\hat{n}_t = \beta n_t + (1-\beta)\hat{n}_{t-1}$
	Updating Procedure for n_t:
	$n_t = 1$ if $x_{t-1} > 0$
	$n_{t-1} + 1$ if $xt - 1 = 0$
	Forecast:
	$\hat{x}_{t,t+1} = \frac{\hat{z}_t}{\hat{n}_t}$
		Page 73,
		Section 2.2.6

EOQ Formulas		
Total cost	TC= Purchase (Unit Value) Cost + Order (Set Up) Cost + Holding (Carrying) Cost + Shortage (stock-out) Cost c : Purchase cost (\$/unit) c_t : Ordering Costs (\$/order) h : Holding rate – usually expressed as a percentage (\$/\$ value/time) c_e : Excess holding Costs (\$/unit-time); also equal to ch c_s : Shortage costs (\$/unit) TRC : Total Relevant Costs – the sum of the relevant cost components TC : Total Costs – the sum of all four cost elements	Page 76,77 section 2.3.2

Description	Formula	Lesson Topic
EOQ Notations	c : Purchase cost (\$/unit) c_t : Ordering Costs (\$/order) c_e : Excess holding Costs (\$/unit/time); equal to ch c_s : Shortage costs (\$/unit) D : Demand (units/time) DA : Actual Demand (units/time) DF : Forecasted Demand (units/time) h : Carrying or holding cost (\$/inventory \$/time) Q : Replenishment Order Quantity (units/order) Q^* : Optimal Order Quantity under EOQ (units/order) Q^*_A : Optimal Order Quantity with Actual Demand (units/order) Q^*_F : Optimal Order Quantity with Forecasted Demand (units/order) T : Order Cycle Time (time/order) T^* : Optimal Time between Replenishments (time/order) N : Orders per Time or 1/T (order/time) TRC(Q) : Total Relevant Cost (\$/time) TC(Q) : Total Cost (\$/time)	Page 77, section 2.3.3
Total Costs(Generic)	$TC(Q) = cD + c_t\frac{D}{Q} + c_e\frac{Q}{2} + c_sE[UnitsShort]$	Page 78, section 2.3.3
TRC (EOQ Model)	$TRC(Q) = c_t\frac{D}{Q} + c_e\frac{Q}{2}$	Page 78, section 2.3.3
Optimal Order Quantity (Q) (EOQ Model)	$Q^* = \sqrt{\frac{2c_tD}{c_e}}$	Page 78, section 2.3.3
Optimal Time between Replenishments (EOQ Model)	$T^* = Q^*/D$ $T^* = \sqrt{\frac{2c_t}{Dc_e}}$	Page 78, section 2.3.3
Number of replenishments per year (EOQ Model)	$N^* = 1/T^* = D/Q^*$	Page 78, section 2.3.3
Optimal Total Costs (EOQ Model)	$TC(Q^*) = cD + \sqrt{2c_tc_eD}$	Page 78, section 2.3.3

Description	Formula	Lesson Topic
Optimal Total Relevant Costs (EOQ Model)	$TRC(Q^*) = \sqrt{2c_tc_eD}$	Page 79, section 2.3.3
EOQ Sensitivity with Respect to Order Quantity	$\frac{TRC(Q)}{TRC(Q^*)} = \left(\frac{1}{2}\right)\left(\frac{Q^*}{Q} + \frac{Q}{Q^*}\right)$	Page 79, section 2.3.3
EOQ Sensitivity with Respect to Demand	$\frac{TRC(Q_F^*)}{TRC(Q_A^*)} = \left(\frac{1}{2}\right)\left(\sqrt{\frac{D_A}{D_F}} + \sqrt{\frac{D_F}{D_A}}\right)$	Page 79, section 2.3.3
EOQ Sensitivity with Respect to Time Interval between Orders	$\frac{TRC(T)}{TRC(T^*)} = \left(\frac{1}{2}\right)\left(\frac{T}{T^*} + \frac{T^*}{T}\right)$	Page 79, section 2.3.3
EOQ Extensions Non-Instantaneous Lead Time (Lead time greater than 0)	Inventory Policy: Order Q^* units when IP=D*L Order Q^* units every T^* time periods	Page 80, section 2.3.4
Inventory Position	Inventory Position (IP) = Inventory on Hand (IOH) + Inventory on Order (IOO) – Back Orders (BO) – Committed Orders (CO) (Inventory on Order is also known as Pipeline Inventory (PI))	Page 80, section 2.3.4
Average Pipeline Inventory	$API = DL$	Page 81, section 2.3.4
Total Cost including Pipeline Inventory (non-zero lead time and include the pipeline inventory)	TC(Q)=cD+$c_t\left(\frac{D}{Q}\right) + c_e\left(\frac{Q}{2} + DL\right) + c_sE[UnitsShort]$ $Q^* = \sqrt{\frac{2c_tD}{c_e}}$	Page 81, section 2.3.4
All Units Discounts	c=c_0 $\quad for \quad 0 \le Q \le Q_1 \quad and \quad c = c_1 \quad for \quad Q_1 \le Q$ TRC= $Dc_0 + c_t\left(\frac{D}{Q}\right) + c_0\left(\frac{hQ}{2}\right) \quad for \quad 0 \le Q \le Q_1$ TRC=$Dc_1 + c_t\left(\frac{D}{Q}\right) + c_1\left(\frac{hQ}{2}\right) \quad for \quad Q_1 \le Q$	Page 81, section 2.3.4

Description	Formula	Lesson Topic
Incremental Discounts	$F_0 = 0; F_i = F_{i-1} + (c_{i-1} - c_i)Q_i$ $Q^* = \sqrt{\frac{2D(c_t+F_i)}{hc_i}}$ $c_i^e = c_i + \frac{F_i}{Q_i^*}$	Page 82, section 2.3.4
One Time Discount	TC=(CycleTime)(TC$^* + PurchaseCost$) $= \left(\frac{Q_g}{D}\right)\sqrt{2c_t hcD} + \left(\frac{Q_g}{D}\right)cD$ Savings=TC-TC$_{SP}$ Savings= $\left(\left(\frac{Q_g}{D}\right)\sqrt{2c_t hcD} + \left(\frac{Q_g}{D}\right)cD\right)$ - $\left(c_g Q_g + hc_g \left(\frac{Q_g}{2}\right)\left(\frac{Q_g}{D}\right) + c_t\right)$ $Q_g^* = \frac{Q^*ch + D(c-c_g)}{hc_g}$	Page 82, section 2.3.4
Economic Production Quantity	TRC[Q] $= \frac{c_t D}{Q} + \frac{Q\left(1-\frac{D}{P}\right)hc}{2}$ EPQ $= \sqrt{\frac{2c_t D}{hc(1-\frac{D}{P})}} = \frac{EOQ}{\sqrt{\left(1-\frac{D}{P}\right)}}$	Page 82, section 2.3.4
EOQ with Planned Backorders (penalty of cs)	TRC(Q,b) $= c_t\left(\frac{D}{Q}\right) + c_e\left(\frac{(Q-b)^2}{2Q}\right) + c_s\left(\frac{b^2}{2Q}\right)$ $Q_{PBO}^* = \sqrt{\frac{2c_t D}{c_e}}\sqrt{\frac{c_s+c_e}{c_s}} = Q^*\sqrt{\frac{(c_s+c_e)}{c_s}} = Q^*\sqrt{\frac{1}{CR}}$ $b^* = \frac{c_e Q_{PBO}^*}{(c_s+c_e)} = \left(1 - \frac{c_s}{(c_s+c_e)}\right)Q_{PBO}^*$ $T_{PBO}^* = \frac{Q^*PBO}{D}$	Page 83, section 2.3.4
Single Period (Newsvendor) Model		
Probability of NOT stocking out	$$P[x \leq Q] = \frac{c_s}{(c_e + c_s)}$$	Page 86, Section 2.3.6
Critical Ratio in simple case	c_s = p – c c_e = c $CR = \frac{c_s}{c_s+c_e} = \frac{(p-c)}{(p-c+c)} = \frac{p-c}{p}$	Page 87, Section 2.3.6
Critical Ratio with salvage value(g) and shortage penalty(B)	c_s = p – c + B c_e = c – g $CR = \frac{c_s}{c_s+c_e} = \frac{(p-c+B)}{(p-c+B+c-g)} = \frac{p-c+B}{p+B-g}$	Page 87, Section 2.3.6
Multi Period Inventory Models		

Description	Formula	Lesson Topic
Safety Stock	$SS = c_e k \sigma_{DL}$	Page 90, section 2.3.8
Optimal Base Stock	$S^* : S^* = \mu_{DL} + k_{LOS}\sigma_{DL}$	Page 91, section 2.3.8
Level of Service (LOS)	$LOS = P[\mu_{DL} \leq S^*] = CR = \frac{c_s}{(c_s+c_e)}$	Page 91, section 2.3.8
Reorder Point	$s = \mu_{DL} + k\sigma_{DL}$	Page 91, section 2.3.8
Level of Service Metrics Cycle Service Level (CSL)	CSL=1-P[Stockout]=1-P[X >s]=P[X $\leq s$] s = NORM.INV (CSL, Mean, StandardDeviation) and k=NORM.S.INV(1-P[X >s]) and k=NORM.S.INV(CSL).	Page 91, section 2.3.8
Level of Service Metrics Cost Per Stock out Event (CSOE) or B1 Cost	If $\frac{B_1 D}{c_e \sigma_{DL} Q \sqrt{2\pi}} > 1$, then we should set: k= $\sqrt{2ln\frac{B_1 D}{c_e \sigma_{DL} Q \sqrt{2\pi}}}$ If $\frac{B_1 D}{c_e \sigma_{DL} Q \sqrt{2\pi}} < 1$ we should set k as low as management allows.	Page 91, section 2.3.8
Item Fill Rate (IFR)	$IFR = 1 - \frac{E[US]}{Q} = 1 - \frac{\sigma_{DL} G[k]}{Q}$ $G(k) = \frac{Q}{\sigma_{DL}}(1 - IFR)$ G(k) is the Unit Normal Loss Function, which can be calculated in Spreadsheets as New Excel Version: $G(k) = NORM.S.DIST(k,0) - k * (1 - NORM.S.DIST(k,1))$ Old Excel Version: $G(k) = NORMDIST(k,0,1,0) - k * (1 - NORMSDIST(k))$	Page 92, section 2.3.8
Cost per Item Short (CIS)	If $\frac{Qc_e}{Dc_s} \leq 1$, then P[StockOut]=P[x $\geq k$] $= \frac{Qc_e}{Dc_s}$ Otherwise, we should set k as low as management allows. In a spreadsheet, this becomes k=NORM.S.INV(1 - $\frac{Qc_e}{Dc_s}$) or (k = NORMSINV(1 - $\frac{Qc_e}{Dc_s}$) for older versions)	Page 92, section 2.3.8

Description	Formula	Lesson Topic
Converting long to short	$E[D_S] = E[D_L]/n$ $VAR[D_S] = VAR[D_L]/n$ $\sigma_s = \sigma_L/\sqrt{n}$	Page 93, section 2.3.8
Converting from short to long:	$E[D_L] = nE[D_S]$ $VAR[D_L] = nVAR[D_S]$ $\sigma_L = \sqrt{n}\sigma_s$	Page 93, section 2.3.8
Inventory special cases		
Grouping Like Items—Break Points	Pick a base time period, w_0, (typically a week) Create a set of candidate ordering periods (w_1, w_2, etc.) Find $D_i c_i$ values where TRC(w_j)=TRC(w_{j+1}) Group SKUs that fall in common value ($D_i c_i$) buckets	Page 97, Section 2.3.9
Managing Multiple Locations:Power of Two Policy	$T^* = \frac{Q^*}{D} = \sqrt{\frac{\frac{2c_t D}{c_e}}{D}} = \sqrt{\frac{2c_t}{Dc_e}}$ In a spreadsheet this is: $T_{practical}$ = 2ˆ(ROUNDUP(LN($T_{optimal}$ / SQRT(2)) /LN(2)))	Page 99, Section 2.3.9
Exchange Curves: Cycle Stock	$TACS = \sum_{i=1}^{n} \frac{QiCi}{2} = \sqrt{\frac{c_t}{h}} \frac{1}{\sqrt{2}} \sum_{i=1}^{n} \sqrt{D_i C_i}$ $N = \sum_{i=1}^{n} \frac{D_i}{Q_i} = \sqrt{\frac{h}{c_t}} \frac{1}{\sqrt{2}} \sum_{i=1}^{n} \sqrt{D_i C_i}$	Page 100 Section 2.3.9
Exchange Curves: Safety Stock	$TSS = \sum_{i=1}^{n} k_i \sigma_{DLi} c_i$ $TVIS = \sum_{i=1}^{n} (\frac{D_i}{Q_i} c_i \sigma_{DLi} G(k_i))$	Page 100, section 2.3.9

Description	Formula	Lesson Topic
Independent and pooled inventories	Independent & Cycle Stock: $q_i^* = \sqrt{\frac{2c_t d_i}{c_e}} = \sqrt{\frac{2c_t D}{c_e n}}$ $\overline{IOH} = \sum_{i=1}^{n} \frac{q_i^*}{2} = \sqrt{n}(\frac{Q^*}{2})$ Independent & Safety Stock: $\overline{SS}_{independent} = k\sigma_{d_i} = k\sigma_D\sqrt{n}$ Pooled & Cycle Stock: $Q^* = \sqrt{\frac{2c_t D}{c_e}}$ $\overline{IOH} = \frac{Q^*}{2}$ Pooled & Safety Stock: $\overline{SS}_{pooled} = k\sigma_D$	Page 101, Section 2.3.10
Fast Moving A Items	$TRC = c_t(\frac{D}{Q}) + c_e(\frac{Q}{2} + k\sigma_{DL}) + B_1(\frac{D}{Q})P[x > k]$ $Q^* = EOQ\sqrt{(1 + \frac{B_1 P[x>k]}{c_t}}$ $k^* = \sqrt{2ln\frac{DB_1}{\sqrt{2\pi}Qc_e\sigma_{DL}}}$	Page 104, Section 2.3.10
Slow Moving A Items	$p[x_0] = Prob[x = x_0] = \frac{e^{-\lambda}\lambda^{x_0}}{x_0!}$ for x_0 $F[x_0] = Prob[x \leq x_0] = \sum_{x=0}^{x_0} \frac{e^{-\lambda}\lambda^x}{x!}$ for x_0 For a discrete function, the loss function $L[X_i]$ can be calculated as follows (Cachon & Terwiesch) $L[X_i] = L[X_{i-1}] - (X_i - X_{i-1})(1 - F[X_{i-1}])$	Page 104, Section 2.3.10
Transportation		
Mean Demand over Lead Time	$\mu_{DL} = \mu_L\mu_D$	Page 109, section 2.4.4
Standard Deviation of Demand over Lead Time (where lead time is variable)	$\sigma_{DL} = \sqrt{\mu_L\sigma_D^2 + (\mu_D)^2\sigma_L^2}$	Page 109, section 2.4.4

Description	Formula	Lesson Topic
Total Cost Per Shipment	$(c_{LF} + c_{DF} + c_{UF}) + (c_{LVu} + c_{DVu} + c_{UVu})(Q) + c_{DVd}(distance)$ $Where:$ $Loading: c_{LF} + c_{LVu}(Q)$ $Unloading: c_{UF} + c_{UVu}(Q)$ $Direct\ Linehaul: c_{DF} + c_{DVd}(distance) + c_{DVu}(Q)$	Page 111, section 2.4.7
Total Cost Per Year (of Shipments)	$(c_{LVu} + c_{DVu} + c_{UVu})(Q)(S/Q) + [(c_{LF} + c_{DF} + c_{UF}) + c_{DVd}(distance)][S/Q] + (h)(Q/2)$	Page 111, section 2.4.7
Euclidean Distance (If Euclidean Space)	$dist((x,y),(a,b)) = \sqrt{(x-a)^2 + (y-b)^2}$	Page 111, section 2.4.7
Manhattan distance: (If Grid)	$dist((x,y),(a,b)) = \|x-a\| + \|y-b\|$	Page 111, section 2.4.7
Distance (If Random Network)	$D_{A-B} = k_{cf} \times d_{A-B}$, where k_{cf} is the circuity factor or k factor. For short distances, d_{A-B} can be considered to be the Euclidean distance. The k factor ranges from 1 to 2, or rarely a higher number, depending on the density or sparsity of the network.	Page 111, section 2.4.7

6.3 SC2x Formulae

Description	Formula	Lesson Topic
Total Cost Equation	$TC = cD + c_t\frac{D}{Q} + c_e(\frac{Q}{2} + k\sigma_{DL} + DL) + c_sP[StockOutType]$	Page 122, section 3.1.3
The transportation problem	$\text{Min } z = \sum_i \sum_j c_{ij}x_{ij}$ $\sum_j x_{ij} \leq S_i \quad \forall i \in S$ $\sum_i x_{ij} \geq D_j \quad \forall j \in D$ $x_{ij} \geq 0 \quad \forall ij$	Page 125, section 3.2.3
The transshipment problem	$\text{Min } z = \sum_i \sum_j c_{ij}x_{ij}$ $\sum_j x_{ij} \leq S_i \quad \forall i \in S$ $\sum_i x_{ij} \geq D_j \quad \forall j \in D$ $\sum_i x_{ij} - \sum_i x_{ji} = 0 \quad \forall j \notin D, \notin S$ $x_{ij} \geq 0 \quad \forall ij$	Page 125, section 3.2.3
Center of gravity	$x = \sum_{k \in K} w_k x_k$ $y = \sum_{k \in K} w_k y_k$	Page 126, section 3.2.4
Weber method	$\text{Min } z = \sum_{k \in K} w_k d_k(x, y)$ $= \sum_{k \in K} w_k \sqrt{(x - x_k)^2 + (y - y_k)^2}$	Page 127, section 3.2.4
Network facility location problem	$\text{Min } z = \sum_i \sum_j c_{ij}x_{ij} + \sum_i f_i Y_i$	Page 127, section 3.2.4.1
Simulate a uniform distribution with mean of X, plus or minus Y%	$X * (1 + (RAND() - 0.50) * (2 * Y))$	Page 131, section 3.2.6
Simulate a Normal Distribution with a mean of X and a standard deviation of Y	$NORM.INV(RAND(), (X, Y))$	Page 131, section 3.2.6
Multi-commodity flow model	$\text{Min } z = \sum_i \sum_j \sum_k c_{ijk}x_{ijk}$	Page 131, section 3.2.6

Description	Formula	Lesson Topic
Multiple time period models	$\sum_i \sum_j \sum_t c_{ijt}x_{ijt} + \sum_j \sum_t hI_{jt}$	Page 132, section 3.2.6
Master Production Schedule - Level production strategy	$t = P_t = \frac{\sum F_t - I_0}{n}$	Page 135, section 3.3.3
Master Production Schedule - Chase demand strategy	$t = P_t = F_t$	Page 136, section 3.3.3
Fixed Order Quantity (FOQ) - Fixed Planning Horizon (FPH) Problem	$Q^* = EOQ$ if $F_t > IOH$	Page 137, section 3.3.3
Periodic Order Quantity (POQ) - Fixed Planning Horizon (FPH) Problem	Order sum of forecasts every ~T* periods T is stable, Q varies	Page 139, section 3.3.3
Optimal Time between Replenishments	$T^* = \sqrt{\frac{2c_t}{Dc_e}}$	Page 139, section 3.3.3
Fixed Planning Horizon (FPH) Problem - Optimal Method (MILP)	Min $z = \sum_i c_{setup}Z_t + \sum_t hI_t$ s.t. $I_0 = 0$ $Q_t - D_t + I_{t-1} - I_t = 0 \quad \forall t \in T$ $MZ_t - Q_t \geq 0 \quad \forall t \in T$ $Q_t \leq CAP_t \quad \forall t \in T$ $I_t, Q_t \geq 0 \quad \forall t \in T$ $Z_t = \{0,1\} \quad \forall t \in T$	Page 140, section 3.3.3
Aggregate Planning Model	Min $z = \sum_t (c_w W_t + c_H H_t + c_F F_t + c_O O_t + c_I I_t + c_B B_t$ $+ c_M P_t + c_C C_t)$ s.t. $P_t - (\frac{HW_t + O_t}{L}) \leq 0 \quad \forall t \in T$ $W_t - W_{t-1} - H_t + F_t = 0 \quad \forall t \in T$ $I_t - I_{t-1} - P_t - C_t + D_t + B_{t-1} - B_t = 0 \quad \forall t \in T$ $O_t - MW_t \leq 0 \quad \forall t \in T$ $W_t, \quad H_t, \quad F_t, \quad O_t, \quad I_t, \quad B_t, \quad P_t, \quad C_t \geq 0 \quad \forall t$	Page 143, section 3.4.3

Description	Formula	Lesson Topic
Demand elasticity	$e = \frac{percent\ change\ in\ demand}{percent\ change\ in\ price} = \frac{\frac{D_{new}-D_{old}}{D_{old}}}{\frac{P_{new}-P_{old}}{P_{old}}}$ $= (\frac{P_{old}}{D_{old}})(\frac{D_{new}-D_{old}}{P_{new}-P_{old}})$	Page 144, section 3.4.3
Working capital	Working capital = current assets - current liabilities	Page 163, section 3.6.4
Current ratio	Current ratio = current assets / current liabilities	Page 164, section 3.6.4
Acid test	Acid test = (cast + accounts receivables + short term investments) / current liabilities	Page 164, section 3.6.4
Working capital turnover	Working capital turnover = sales / working capital	Page 164, section 3.6.4
Days of Inventory Outstanding (DIO)	DIO = (average inventory / cost of sales) x 365	Page 164, section 3.6.4
Days of Sales Outstanding (DSO)	DSO = (average accounts receivable / total sales) x 365	Page 164, section 3.6.4
Days of Payables Outstanding (DPO)	DPO = (average accounts payable / cost of sales) x 365	Page 164, section 3.6.4
Cash-to-Cash Cycle (CCC)	CCC = DIO + DSO - DPO	Page 164, section 3.6.4
Net Operating Profit After Taxes (NOPAT)	$NOPAT_t$ = (1- tax rate) * $EBIT_t$	Page 166, section 3.6.5
Free Cash Flows (FCF)	$FCF_t = NOPAT_t + DA_t - CapEx_t - \Delta NetWorkingCapital$ $FCF_t =$(1- tax rate) * $EBIT_t + DA_t - CapEx_t$ $-\Delta NetWorkingCapital$	Page 166, section 3.6.5
Return on Assets (ROA)	ROA = net income / assets	Page 167, section 3.6.6
Present Value (PV)	$PV = \frac{FV}{(1+r)^n}$	Page 168, section 3.6.6
Net Present Value (NPV)	$NPV = c_0 + \frac{c1}{(1+r)^1} + \frac{c2}{(1+r)^2} + ... + \frac{c_T}{(1+r)^T}$	Page 168, section 3.6.6
Internal Rate of Return (IRR)	$c_0 + \frac{c1}{(1+IRR)^1} + \frac{c2}{(1+IRR)^2} + ... + \frac{c_T}{(1+IRR)^T} = 0$	Page 168, section 3.6.6
Annuity (PV)	$PV_{annuity} = \frac{C}{r} - \frac{C}{r}\frac{1}{(1+r)^T}$	Page 169, section 3.6.5
Return on Equity (ROE)	ROE = Net Income/Equity	Page 175, section 3.6.8

Description	Formula	Lesson Topic
Return on Assets	ROA = Net Income / Total Assets	Page 176, section 3.6.8
Sales growth	Sales growth = ($Sales_t$ / $Sales_{t-1}$) - 1	Page 176, section 3.6.8
Compound Annual Growth Rate (CAGR)	CAGR = $(Sales_t/Sales_{t-n})^{1/n}$ - 1	Page 176, section 3.6.8
Gross margin	Gross margin = Gross Income / Sales	Page 176, section 3.6.8
Operating margin	Operating margin = Operating Income / Sales	Page 176, section 3.6.8
Net margin	Net margin = Net Income / Sales	Page 176, section 3.6.8
Asset turnover	Asset turnover = Sales / Total Assets	Page 176, section 3.6.8
Inventory turnover based on COGS	Inventory turnover = COGS / Average Inventory	Page 176, section 3.6.8
Acc. receivable turnover	Accounts receivable turnover = Credit Sales / Average Accounts Receivable	Page 176, section 3.6.8
Acc. payable turnover	Account payable turnover = COGS / Average Accounts Payable	Page 176, section 3.6.8
DuPont Analysis	R = T × P	Page 176, section 3.6.8
Gross Margin Return on Investment(GMROI)	GMROI = Gross Margin × Inventory Turnover	Page 177, section 3.6.8
Inventory turnover based on Sales	Inventory turnover = sales / inventory	Page 177, section 3.6.8
Return on Invested Capital (ROIC)	ROIC = NOPAT / Invested Capital	Page 177, section 3.6.8
Invested Capital (IC)	IC = Interest – Bearing Debt + Equity	Page 177, section 3.6.8
Return on Net Assets (RONA)	RONA = NOPAT/ Net Assets	Page 177, section 3.6.8
Net Assets (NA)	Net assets = Fixed Assets + Non-cash Working Capital	Page 177, section 3.6.8
Net Assets (NA)	Net assets = Total Assets – Current Liabilities – Cash	Page 177, section 3.6.8

Description	Formula	Lesson Topic
Utilization metric	Utilization = Actual Input/Normal Input	Page 180, section 3.7.4
Productivity metric	Productivity = Actual Output/Actual Input	Page 180, section 3.7.4
Effectiveness metric	Effectiveness = Actual Output/Normal Output	Page 180, section 3.7.4